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MEDIUM EFFECTS AND QUANTUM CONDENSATES IN LOW-DENSITY NUCLEAR MATTER*

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The equation of state of nuclear matter at finite temperature and density with various proton fractions is considered at subsaturation densities and finite temperatures. The formation of few-body correlations, in particular bound clusters is taken into account considering free nucleons, as well as clusters, like quasiparticles. Medium modification of the clusters is described by self-energy and Pauli blocking effects. Benchmarks such as the nuclear statistical equilibrium, virial expansion and the relativistic mean field approximation are considered. An interesting effect is the formation of a two-nucleon or four-particle quantum condensate, showing the crossover from Cooper pairing to Bose–Einstein condensation. The resulting thermodynamic properties are of interest for heavy-ion collisions and astrophysical applications. Quantum condensates and the Mott effect are also of relevance for the structure of finite nuclei, especially dilute excited states like the Hoyle state of ^{12}C.

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1. Introduction

The equation of state (EoS) in the subsaturation region, the composition and possible occurrence of phase transitions in nuclear matter are widely discussed topics not only in nuclear theory, but are also of great interest in astrophysics and cosmology, see [1]. Experiments on heavy ion collisions, performed over the last decades, gave new insight into the behavior of nuclear systems in a broad range of densities and temperatures, see [2]. The observed cluster abundances, their spectral distribution and correlations in momentum space can deliver information about the state of dense, highly excited matter.

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Within a quantum statistical approach to the thermodynamic properties of nuclear matter, the main quantity to be evaluated is the single-nucleon self-energy. Different approximations are obtained by partial summations within a diagram representation. The formation of bound states is taken into account considering ladder approximations [3], leading in the low-density limit to the solution of the Schrödinger equation. The effects of the medium can be included in a self-consistent way within the cluster-mean field approximation [4], where the influence of the correlated medium on the single particle states as well as on the clusters is considered in first order with respect to the interaction. As a point of significance, besides the single-particle self-energy shifts of the constituents, the bound state energies are also modified by the Pauli blocking due to the correlated medium. Due to the in-medium quasiparticle shift, bound states will merge with the continuum of scattering states at increasing density and are dissolved. Within a generalized Beth–Uhlenbeck approach, the relation to the nuclear statistical equilibrium (NSE), to the extended virial expansions [5] and to the Brueckner Hartree–Fock and relativistic mean-field theory (RMF) can be given. An extended discussion of the two-particle problem is found in Refs. [6, 7].

The EoS can be applied to different situations. In astrophysics, the relativistic EoS of nuclear matter for supernova explosions was investigated recently [8]. If nuclei are considered as inhomogeneous nuclear matter, within a local density approximation the EoS can serve for comparison to estimate the role of correlations. In nuclear reactions a nonequilibrium theory is needed, but within a simple approach such as the freeze out concept or the coalescence model the results from the EoS may be used to describe heavy ion reactions. The EoS including the contribution of light clusters has been evaluated recently [9], and the inclusion of heavier clusters is under consideration [10]. As example, the symmetry energy at subsaturation densities is sensitive to the formation of clusters [2].

The inclusion of light cluster formation describing nuclear matter has some yet unsolved implications to be discussed here. The account of correlations means, besides the contribution of bound states that can be treated in quasiparticle approximation, also the contribution of scattering states. The EoS contains contributions from the scattering phase shifts as seen, for example, from the Beth–Uhlenbeck formula [5, 6]. In contrast, cluster yields from HIC do not contain the scattering contributions that quickly decay during the nonequilibrium expansion process. Only the bound states remain because ternary particles are needed for energy and momentum conservation.

There is not unique definition of a bound state and composition in a dense system. From the spectral function in the corresponding A-particle channel, a peak structure can be used to define the quasiparticles, but due to the interactions in dense systems, sharp peaks in the spectral function become broadened and can become resonances in the continuum. We introduce the spectral function in the following section. The quasiparticle approach is introduced to reproduce significant contributions of the clusters to the total nucleon density. Further items to be discussed are the occurrence of quantum condensates, phase instabilities, the role of higher clusters, and the treatment of alpha matter at low temperatures and densities.

2. Quantum statistical approach to the equation of state

Using the finite-temperature Green function formalism, a non-relativistic quantum statistical approach can be given to describe the equation of state of nuclear matter including the formation of bound states [3, 6].

The single-nucleon spectral function $S_1(1,\omega)$ is related to the self-energy $\Sigma(1,z)$ according to

$$S_1(1,\omega) = \frac{2\mathrm{Im}\,\Sigma(1,\omega-i0)}{[\omega - E(1) - \mathrm{Re}\,\Sigma(1,\omega)]^2 + [\mathrm{Im}\,\Sigma(1,\omega-i0)]^2}\,, \tag{1}$$

where $E(1) = p_1^2/(2m_1)$ is the kinetic energy of the free nucleon. The solution of the relation

$$E_1^{\mathrm{qu}}(1) = E(1) + \mathrm{Re}\,\Sigma\left[1, E_1^{\mathrm{qu}}(1)\right] \tag{2}$$

defines the single-nucleon quasiparticle energies $E_1^{\mathrm{qu}}(1) = E(1) + \Delta E^{\mathrm{SE}}(1)$. In mean-field approximations for the self-energy, the EoS for an ideal Fermi gas of quasiparticles results. Expressions for the single-nucleon quasiparticle energy $E_\tau^{\mathrm{qu}}(p)$ can be given by the Skyrme parametrization or by more sophisticated approaches such as relativistic mean-field approaches [11,12] and relativistic Dirac–Brueckner Hartree–Fock calculations. The fit to properties of nuclei implies the appropriate description of nuclear matter near saturation density.

As shown in Refs. [3,6], the bound state contributions are obtained from the poles of Im $\Sigma(1,z)$ which cannot be neglected in expanding the spectral function with respect to Im $\Sigma(1,z)$. A cluster decomposition of the self-energy has been proposed, see Ref. [4]. The self-energy is expressed in terms of the A-particle Green functions which contain the A-particle wave function $\psi_{A\nu P}(1\ldots A)$ and the corresponding eigenvalues $E_{A,\nu}^{\mathrm{qu}}(P)$ from solving the in-medium Schrödinger equation, see below. P denotes the center of mass momentum of the A-nucleon system, the internal quantum number ν refers to bound states as well as to scattering states.

Considering only the bound-state contributions, we obtain the result

$$n_p^{\mathrm{tot}}(T,\mu_p,\mu_n) \;=\; \frac{1}{\Omega}\sum_{A,\nu,P} Z f_{A,Z}\left[E_{A,\nu}^{\mathrm{qu}}(P;T,\mu_p,\mu_n)\right]\,,$$

$$n_n^{\rm tot}(T,\mu_p,\mu_n) = \frac{1}{\Omega}\sum_{A,\nu,P}(A-Z)f_{A,Z}\left[E^{\rm qu}_{A,\nu}(P;T,\mu_p,\mu_n)\right],$$
$$f_{A,Z}(\omega) = \left(\exp\left\{\beta\left[\omega - Z\mu_p-(A-Z)\mu_n\right]\right\} - (-1)^A\right)^{-1} \quad (3)$$

for the EoS describing a mixture of components (cluster quasiparticles) obeying Fermi or Bose statistics, $n(T,\mu_p,\mu_n)=n_n^{\rm tot}(T,\mu_p,\mu_n)+n_p^{\rm tot}(T,\mu_p,\mu_n)$ is the total baryon density. To derive the extended Beth–Uhlenbeck formula, see Ref. [3], we restrict the summation to $A\le 2$, but extend the summation over the internal quantum numbers ν, not only to excited bound states, but also the scattering states. Note that at low temperatures Bose–Einstein condensation may occur.

The NSE is obtained in the low-density limit if the in-medium energies $E^{\rm qu}_{A,\nu}(P;T,\mu_p,\mu_n)$ can be replaced by the binding energies of the isolated nuclei. Recent progress of the description of clusters in low density nuclear matter [8, 12, 13] enables us to evaluate the properties of deuterons, tritons, helions and helium nuclei in a non-relativistic microscopic approach, taking the influence of the medium into account.

For nuclei imbedded in nuclear matter, an effective wave equation can be derived, using a quantum statistical approach [3, 13]. The A-particle wave function $\psi_{A\nu P}(1\dots A)$ and the corresponding eigenvalues $E^{\rm qu}_{A,\nu}(P)$ follow from solving the in-medium Schrödinger equation

$$\left[E^{\rm qu}(1)+\ldots+E^{\rm qu}(A)-E^{\rm qu}_{A,\nu}(P)\right]\psi_{A\nu P}(1\dots A)$$
$$+\sum_{1'\dots A'}\sum_{i<j}\left[1-\tilde f(i)-\tilde f(j)\right]V(ij,i'j')\prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu P}(1'\dots A')=0\,. \quad (4)$$

This equation contains the effects of the medium in the single-nucleon quasiparticle shifts as well as in the Pauli blocking terms. The A-particle wave function and energy depend on the total momentum P relative to the medium.

The effective Fermi distribution function $\tilde f(1)=(\exp\{\beta[E^{\rm qu}(1)-\tilde\mu_1]\}+1)^{-1}$ contains the non-relativistic effective chemical potential $\tilde\mu_1$. It is determined by the normalization condition that the total proton or neutron density is reproduced in quasiparticle approximation, $n_\tau^{\rm tot}=\Omega^{-1}\sum_1\tilde f(1)\delta_{\tau_1,\tau}$ for the particles inside the volume Ω. It describes the occupation of the phase space neglecting any correlations in the medium. This means that the nucleons bound in clusters, *e.g.* deuterons, will also occupy phase space. The total amount of occupied phase space is correctly given, but the form factor will deviate from the Fermi distribution as given by the bound state wave function. A more rigorous treatment can be given within the cluster mean-field approximation [4, 13].

The solution of the in-medium Schrödinger equation (4) can be obtained in the low-density region by perturbation theory. At higher densities, a variational approach can be used. In particular, the quasiparticle energy of the A-nucleon cluster with Z protons in the ground state is

$$E^{\mathrm{qu}}_{A,\nu}(P) \;=\; E^{\mathrm{qu}}_{A,Z}(P) = E^{(0)}_{A,Z} + \frac{P^2}{2Am} + \Delta E^{\mathrm{SE}}_{A,Z}(P) + \Delta E^{\mathrm{Pauli}}_{A,Z}(P) + \Delta E^{\mathrm{Coul}}_{A,Z}(P) + \dots \qquad (5)$$

with the cluster binding energy in the vacuum, and the kinetic term, the self-energy shift $\Delta E^{\mathrm{SE}}_{A,Z}(P)$, the Pauli shift $\Delta E^{\mathrm{Pauli}}_{A,Z}(P)$ and the Coulomb shift.

The self-energy contribution to the quasiparticle shift is determined by the contribution of the single-nucleon shift

$$\Delta E^{\mathrm{SE}}_{A,Z}(0) = (A-Z)\Delta E^{\mathrm{SE}}_{n}(0) + Z\Delta E^{\mathrm{SE}}_{p}(0) + \Delta E^{\mathrm{SE,eff.mass}}_{A,Z}\,. \qquad (6)$$

The contribution to the self-energy shift due to the change of the effective nucleon mass can be calculated from perturbation theory using the unperturbed wave function of the clusters [8].

The most important effect in the calculation of the abundances of light elements comes from the Pauli blocking terms in Eq. (4) in connection with the interaction potential. This contribution is restricted only to the bound states so that it may lead to the dissolution of the nuclei if the density of nuclear matter increases. The corresponding shift

$$\Delta E^{\mathrm{Pauli}}_{A,Z}(P) \approx \Delta E^{\mathrm{Pauli}}_{A,Z}(0)\,\exp\left(-\frac{P^2}{g_{A,Z}}\right) \qquad (7)$$

can be evaluated in perturbation theory provided the interaction potential and the ground state wave function are known. A variational approach gives the dispersion $g_i(T,n,Y_p)$ [9,12].

The shift of the binding energy of light clusters at zero total momentum has been calculated recently [13]. The light clusters deuteron ($d = {}^2$H), triton ($t = {}^3$H), helion ($h = {}^3$He) and the α particle (^{4}He) have been considered. The interaction potential and the nucleonic wave function of the few-nucleon system have been fitted to the binding energies and the rms radii of the corresponding nuclei. With the neutron number $N_i = A_i - Z_i$, it can be written as

$$\Delta E^{\mathrm{Pauli}}_{A_i,Z_i}(0;n_p,n_n,T) = -\frac{2}{A_i}\left[Z_i n_p + N_i n_n\right]\delta E^{\mathrm{Pauli}}_{i}(T,n)\,, \qquad (8)$$

where the temperature dependence and higher density corrections are contained in the functions $\delta E^{\mathrm{Pauli}}_{i}(T,n)$ given in Refs. [9,12].

3. Contribution of the continuum correlations

Now, the nucleon number densities (3) can be evaluated as in the non-interacting case, with the only difference that the number densities of the particles are calculated with the quasiparticle energies. In the light cluster-quasiparticle approximation, the total densities of neutrons $n_n^{\rm tot} = n_n + \sum_{i=d,t,h,\alpha} N_i n_i$ and of protons $n_p^{\rm tot} = n_p + \sum_{i=d,t,h,\alpha} Z_i n_i$ contain the densities of the free neutrons and protons n_n and n_p, respectively, and the contributions due to the correlations in the corresponding few-nucleon channels. The summation over the internal quantum number ν, Eq. (3), covers the bound state part (ground state and possibly excited states) as well as the contribution of the continuum (for example resonances). Thus, the contribution n_d of the deuteron channel is given by the Beth–Uhlenbeck formula that contains besides the bound state contribution also the contribution of the scattering states [3, 5, 6].

We suppose that the densities $n_i^{\rm bound}$ give the cluster yields within a freeze-out approach to heavy ion collisions. In expanding nuclear matter, continuum correlations such as n–n or p–p will decay. The formation or decay of a bound state, *e.g.* in the deuteron channel, demands a further nucleon as spectator to obey energy and momentum conservation. At decreasing density, such processes become suppressed. The continuum correlations can be included into the free nucleon part n_n, n_p of $n_n^{\rm tot,\, bound}$, $n_p^{\rm tot,\, bound}$. A more systematic approach to heavy ion collisions can be given by transport codes based on coupled kinetic equations for the different constituents of nuclear matter (n, p, d, t, h, α in the case considered here).

For the evaluation of the equation of state, the account of scattering states needs further consideration. Investigations on the two-particle level have been performed and extensively discussed [3, 5, 6].

The contribution of scattering states $n_i^{\rm scatt}$ is necessary to obtain the second virial coefficient according to the Beth–Uhlenbeck equation, see Refs. [5, 6]. This leads also to corrections in comparison with the NSE that accounts only for the bound state contributions, neglecting all effects of scattering states. These corrections become important at increasing temperatures for weakly bound clusters. Thus, the corrections which lead to the correct second virial coefficient are of importance for the deuteron system, when the temperature is comparable or large compared with the binding energy per nucleon.

To take the contributions of these continuum correlations into account, we propose the following approximation

$$n_i^{\rm scatt} = \frac{1}{\Omega} \sum_P^{\rm bound} b_i \frac{E_i^{\rm qu}(0)}{E_i^{(0)}}\, e^{-P^2/(2A_i m T)} e^{\left[(A_i - Z_i)\left(\mu_n - \Delta E_n^{\rm SE}(0)\right) + Z_i\left(\mu_p - \Delta E_p^{\rm SE}(0)\right)\right]/T}, \quad (9)$$

where $b_i(T)$ is the second virial coefficient after subtraction of $e^{-E_i^{(0)}/T} - 1$. We have used the values for the deuteron as given in Ref. [5]. Of course, the inclusion of the scattering states can be improved, *e.g.* by comparing with the Beth–Uhlenbeck formula. To approach the low-density limit of the EoS correctly, one has to reproduce the virial coefficients of the cluster-virial expansion.

4. α matter and quartetting

In the low-temperature region, below the Mott density the α particles yield the dominating contribution to the composition of symmetric matter, if we restrict us to clusters with $A \leq 4$. Below densities of the order 10^{-4} fm^{-3}, nuclear matter can be considered as ideal mixture of free nucleons and clusters. The interaction between the constituents can be neglected. The mass action law gives increasing yield of α particles at decreasing temperature for fixed density, but decreasing bound state concentration for decreasing density at fixed temperature (entropy dissociation). The ideal mixture of free nucleons and clusters has to be considered as ideal quantum gas where the components have a Fermi or Bose distribution function in momentum space, and Bose–Einstein condensation can occur at low temperatures.

The low-density limit at fixed temperature is the ideal classical nucleon gas. Corrections are given by the virial expansion. In particular, within a cluster-virial expansion the empirical scattering phase shifts can be used to evaluate density corrections for the ideal mixture of the different components. Thus, the virial coefficient for α–α interaction is obtained from the corresponding scattering phase shifts [5]. Alternatively, one can use the phase shifts to introduce an effective interaction. The corresponding equation of state was reconsidered recently [14], also taking into account the formation of a quantum condensate. However, such effective interactions become questionable if the density is increasing so that the wave functions of the clusters overlap. Then, Pauli blocking leads to the dissolution of the clusters.

In our approach, these medium effects are included. Results for the internal energy per nucleon are shown in Fig. 1. At very low densities, the internal energy pro nucleon takes the value $U = \frac{3}{2}T$ for the classical ideal gas of free nucleons. As soon as cluster are formed, what happens for low temperatures already at very low densities, the binding energy of the nucleons in clusters determines the internal energy. In the case of α particles this contribution to the internal energy amounts -7 MeV. It determines the internal energy at zero temperature in the low-density limit.

With increasing density, the Pauli blocking leads to a reduction of the binding energy of the α particles and its dissolution. This is the reason for the increase of internal energy until the Mott density (≈ 0.006 fm^{-3}) is reached. Above the Mott density, after the bound states are dissolved, the free nucleon RMF approach gives the behavior of the internal energy.

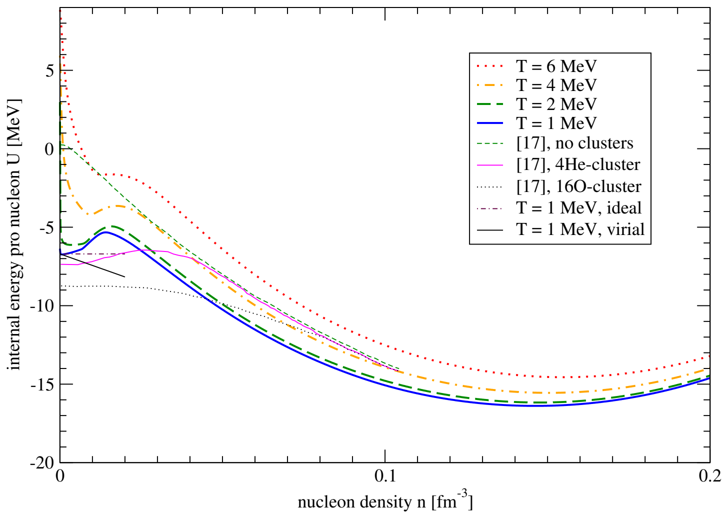

Fig. 1. Internal energy of symmetric matter as function of density for different temperatures. Thin lines: Results of a fully antisymmetrized calculation at $T = 0$ MeV, considering different clusters [17]. For comparison, values for the ideal gas of α particles as well as the virial expansion [5] are also shown.

The formation of quantum condensates will give further contributions to the EoS. However, in the region considered here the formation of quantum condensates does not appear. On the other hand, quartetting is expected to occur at low temperatures [15]. With increasing density, due to correlations the condensate is decreasing [14, 16]. First steps to describe the quartetting at zero temperature have been given in [17]. The quantum condensate of α particles was discussed recently for finite nuclei [18].

5. Conclusions and outlook

We presented a quantum statistical approach to evaluate the EoS of warm nuclear matter at subsaturation densities. Investigating the self-energy within a cluster decomposition, a quasiparticle approach is given for the free nucleons as well as for the nuclei. The account of medium effects allows to obtain a general approach that combines different well-known cases in the low-density region and near saturation density. The role of continuum correlations is discussed, that contribute to the cluster yields in heavy ion collisions and to the EoS in different ways.

The approximation of the uncorrelated medium can be improved considering the cluster mean-field approximation [3,4,13]. This would also improve the correct inclusion of α matter as discussed here. The formation of quantum condensates (quartetting) and its disappearance with increasing density demands further investigations.

We do not consider the formation of heavy clusters here. This limits the parameter range $n_n^{\rm tot}$, $n_p^{\rm tot}$, T in the phase diagram to that area where the abundances of heavier clusters are small. For a more general approach to the EoS, which takes also the contribution of heavier cluster into account, see Refs. [10, 19]. Future work will include the contribution of the heavier clusters.

Phase separation occurs when thermodynamic stability is violated. If phase instability occurs, droplet formation has to be considered. Due to the Coulomb interaction, a neutralizing charged background has to be taken into account to avoid divergencies in the EoS. Demanding local neutrality, the Wigner–Seitz approximation can be used for the Coulomb energy. The determination of the optimal droplet can be performed in the Thomas–Fermi approximation or the local density approach, but will not detailed here. It is expected that the formation of droplets corresponds to the account of higher clusters. The critical point for the phase transition from the nuclear quantum liquid to nuclear gas is influenced by the formation of clusters [9]. Due to the Coulomb interaction and local neutrality, the phase transition becomes smooth in the thermodynamic limit.

Phase transitions are of high importance for the evolution the early universe [20, 21], because they can produce inhomogeneities. To calculate the primordial distribution of elements, medium effects have to be included if densities near the saturation density occur. This allows for the primordial production of heavy elements.

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REFERENCES

[1] T. Klähn *et al.*, *Phys. Rev.* **C74**, 035802 (2006).

[2] S. Kowalski *et al.*, *Phys. Rev.* **C75**, 014601 (2007); J. Natowitz *et al.*, `arXiv:nucl-th/1001.1102` to be published in *Phys. Rev. Lett.*

[3] G. Röpke, L. Münchow, H. Schulz, *Nucl. Phys.* **A379**, 536 (1982); G. Röpke, M. Schmidt, L. Münchow, H. Schulz, *Phys. Lett.* **B110**, 21 (1982).

[4] G. Röpke, T. Seifert, H. Stolz, R. Zimmermann, *Phys. Status Solidi* **B100**, 215 (1980); G. Röpke, M. Schmidt, L. Münchow, H. Schulz, *Nucl. Phys.* **A399**, 587 (1983).

[5] C.J. Horowitz, A. Schwenk, *Nucl. Phys.* **A776**, 55 (2006).

[6] M. Schmidt, G. Röpke, H. Schulz, *Ann. Phys. (NY)* **202**, 57 (1990).

[7] H. Stein, A. Schnell, T. Alm, G. Röpke, *Z. Phys.* **A351**, 295 (1995).

[8] K. Sumiyoshi, G. Röpke, *Phys. Rev.* **C77**, 055804 (2008); *Phys. Rev.* **C77**, 055804 (2008) [`arXiv:0801.0110[astro-ph]`].

[9] S. Typel, G. Röpke, T. Klähn, D. Blaschke, H.H. Wolter, *Phys. Rev.* **C81**, 015803 (2010) [`arXiv:0908.2344[nucl-th]`].

[10] M. Hempel, J. Schaffner-Bielich, `arXiv:0911.4073[nucl-th]`.

[11] S. Typel, *Phys. Rev.* **C71**, 064301 (2005) [`arXiv:nucl-th/0501056`].

[12] G. Röpke, *Int. J. Mod. Phys.*, to be published.

[13] G. Röpke, *Phys. Rev.* **C79**, 014002 (2009) [`arXiv:0810.4645[nucl-th]`].

[14] S. Misicu, F. Carstoiu, *Phys. Lett.* **B682**, 33 (2009) [`arXiv:0909.1719[nucl-th]`].

[15] G. Röpke, A. Schnell, P. Schuck, P. Nozières, *Phys. Rev. Lett.* **80**, 3177 (1998).

[16] Y. Funaki, H. Horiuchi, G. Röpke, P. Schuck, A. Tohsaki, T. Yamada, *Phys. Rev.* **C77**, 064312 (2008).

[17] H. Takemoto, M. Fukushima, S. Chiba, H. Horiuchi, Y. Akaishi, A. Tohsaki, *Phys. Rev.* **C69**, 035802 (2004).

[18] A. Tohsaki, H. Horiuchi, P. Schuck, G. Röpke, *Phys. Rev. Lett.* **87**, 192501 (2001); Y. Funaki *et al.*, *Phys Rev.* **C80**, 064326 (2009).

[19] G. Röpke, M. Schmidt, H. Schulz, *Nucl. Phys.* **A424**, 594 (1984).

[20] G. Röpke, *Clusters in Nuclear Matter*, in: *Condensed Matter Theories*, vol. 16, Ed. S. Hernandez, J.W. Clark, Nova Sciences Publ., New York 2001.

[21] G. Röpke, *Phys. Lett.* **B185**, 281 (1987).