

THE PHASE STRUCTURE OF DENSE QCD FROM CHIRAL MODELS*

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(Received January 27, 2010)

A new phase of dense QCD proposed in the limit of large number of colors, Quarkyonic Phase, is discussed in chiral approaches. The interplay between chiral symmetry breaking and confinement together with the N_c dependence of the phase diagram are dealt with in the PNJL model. We also discuss a possible phase at finite density where chiral symmetry is spontaneously broken while its center remains unbroken. The quark number susceptibility exhibits a strong enhancement at the restoration point of the center symmetry rather than that of the chiral symmetry. This is reminiscent of the quarkyonic transition.

PACS numbers: 11.30.Rd, 11.30.Ly, 25.75.Nq, 21.65.Qr

1. Introduction

Model studies of dense baryonic and quark matter have suggested a rich phase structure of QCD at temperatures and quark chemical potentials being of order Λ_{QCD} . Our knowledge on the phase structure is however still limited and the description of the matter around the phase transitions does not reach a consensus because of the non-perturbative nature of QCD [1].

Possible phases and spectra of excitations are guided by symmetries and their breaking pattern in a medium. Dynamical chiral symmetry breaking and confinement are characterized by strict order parameters associated with global symmetries of the QCD Lagrangian in two limiting situations: the quark bilinear $\langle \bar{q}q \rangle$ in the limit of massless quarks, and the Polyakov loop $\langle \Phi \rangle$ in the limit of infinitely heavy quarks. The system at finite density could also allow other symmetries, which are not manifest in the QCD Lagrangian but might emerge in a dense medium. In this contribution we discuss the phases in dense QCD from chiral models.

* Talk presented at the EMMI Workshop and XXVI Max Born Symposium “Three Days of Strong Interactions”, Wrocław, Poland, July 9–11, 2009.

2. From $N_c = \infty$ to $N_c = 3$

A novel phase of dense quarks, Quarkyonic Phase, was recently proposed based on the argument using large N_c counting where N_c denotes number of colors [2–4]: in the large N_c limit there are three phases which are rigorously distinguished using $\langle\Phi\rangle$ and the baryon number density $\langle N_B\rangle$. The quarkyonic phase is characterized by $\langle\Phi\rangle = 0$ indicating the system confined and non-vanishing $\langle N_B\rangle$ above $\mu_B = M_B$ with a baryon mass M_B . The phase structure in large N_c is shown in Fig. 1.

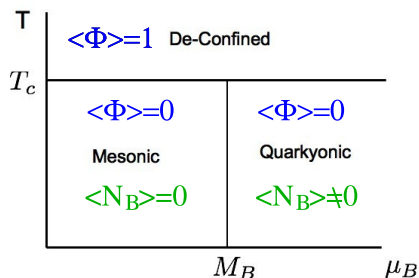


Fig. 1. The phase diagram in large N_c proposed in [2].

A possible deformation of the phase boundaries in Fig. 1 together with the chiral phase transition can be described using a chiral model coupled to the Polyakov loop [5]. The Nambu–Jona-Lasinio model with Polyakov loops (PNJL model) has been developed to deal with chiral dynamics and “confinement” simultaneously [6]. The model describes that only three-quark states are thermally relevant below the chiral critical temperature, which is reminiscent of confinement. Fig. 2 shows the two transition lines for $N_c = \infty$ and for $N_c = 3$ in the two-flavored PNJL model. In the large N_c limit assuming that the system is confined, the gap equations for the order parameters $\langle\bar{q}q\rangle$ and $\langle\Phi\rangle$ become two uncorrelated equations. Consequently, the quark dynamics carries only a μ dependence and the Polyakov loop sector does only a T dependence. Finite N_c corrections make the transition lines bending down. The crossover for deconfinement shows a weak dependence on μ which is a remnant of the phase structure in large N_c . One finds that for $N_c = 3$ deconfinement and chiral crossover lines are on top of each other in a wide range of μ . A critical point associated with chiral symmetry appears around the junction of those crossovers.

The clear separation of the quarkyonic from hadronic phase is lost in a system with finite N_c . Nevertheless, an abrupt change in the baryon number density would be interpreted as the quarkyonic transition which separates meson dominant from baryon dominant regions. In fact, a steep increase in the baryon number density and the corresponding maximum in

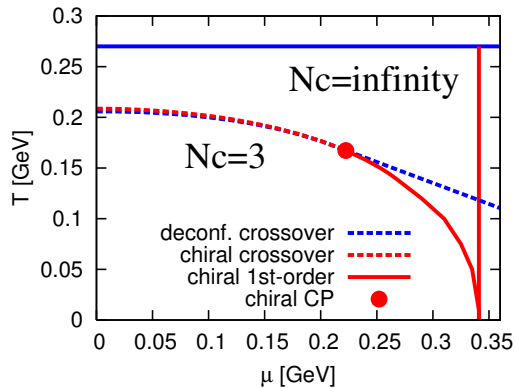


Fig. 2. The phase diagram of a PNJL model for different N_c [5]. Two straight lines indicate the deconfinement and chiral phase transitions for $N_c = \infty$ and the lower curves for $N_c = 3$.

its susceptibility χ_B are driven by a phase transition from chirally broken to restored phase in most model-approaches using constituent quarks. One might then consider the chirally symmetric confined phase as the quarkyonic phase.

The constituent quarks are however unphysical in confined phase. It is not obvious to have a realistic description of hadrons from chiral quarks. In particular, chiral symmetry restoration for baryons must be worked out. Two alternatives for chirality assignment are known [7] and it remains an open question which scenario is preferred by nature [8]: (i) in the naive assignment, dynamical chiral symmetry breaking generates a baryon mass which thus vanishes at the restoration. (ii) in the mirror assignment, dynamical chiral symmetry breaking generates a mass difference between parity partners and the chiral symmetry restoration does not necessarily dictate the chiral partners being massless. If the chiral invariant mass is not very small, the baryon number density is supposed to be insensitive to the quarkyonic transition.

Besides, it seems unlikely that the chirally-restored confined phase is realized in QCD on the basis of the anomaly matching: external gauge fields, *e.g.* photons, interacting with quarks lead to anomalies in the axial current. Since there are no Nambu–Goldstone bosons in chiral restored phase, the anomalous contribution must be generated from the triangle diagram in which the baryons are circulating. In three flavors, however, the baryons forming an octet do not contribute to the pole in the axial current because of the cancellations [9]. The mirror scenario has nothing to do with this problem because the sign of the axial couplings to the positive and negative parity states are relatively opposite. It is indispensable to any rigorous argu-

ment for this taking account of the physics around the Fermi surface, which could lead to a possibility of the chirally restored phase with confinement. The anomaly matching conditions at finite temperature and density are in fact altered, see *e.g.* [10].

3. Role of the tetra-quark at finite density

There is a possibility of two different phases with broken chiral symmetry distinguished by the baryon number density. An alternative pattern of spontaneous chiral symmetry breaking was suggested in the context of QCD at zero temperature and density [11–13]. This pattern keeps the center of chiral group unbroken, *i.e.*

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \rightarrow \mathrm{SU}(N_f)_V \times (Z_{N_f})_A, \quad (1)$$

where a discrete symmetry $(Z_{N_f})_A$ is the maximal axial subgroup of $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$. The center Z_{N_f} symmetry protects a theory from condensate of quark bilinears $\langle \bar{q}q \rangle$. Spontaneous symmetry breaking is driven by quartic condensates which are invariant under both $\mathrm{SU}(N_f)_V$ and Z_{N_f} transformation. Although meson phenomenology with this breaking pattern seems to explain the reality reasonably [11], this possibility is strictly ruled out in QCD both at zero and finite temperatures but at zero density since a different way of coupling of Nambu–Goldstone bosons to pseudo-scalar density violates QCD inequalities for density–density correlators [14]. However, this does not exclude the unorthodox pattern in the presence of dense matter. In a system with the breaking pattern (1) the quartic condensate is the strict order parameter which separates different chirally-broken phases.

It has been shown that the phase, where the symmetry is spontaneously broken due to the higher-dimensional operator, is realized as a meta-stable state in an $O(2)$ scalar model [15]. Another interesting observation came out from the Skyrme model on crystal: a new intermediate phase where a skyrmion turns into two half skyrmions was numerically found [16]. This phase is characterized by a vanishing quark condensate $\langle \bar{q}q \rangle$ and a non-vanishing pion decay constant. Although the above non-standard pattern of symmetry breaking (1) was not imposed in the Skyrme Lagrangian, the result could suggest a dynamical emergence of new symmetries in dense environment.

4. The phase diagram and observables

Assuming the symmetry breaking pattern (1) at finite density, it has been shown that an intermediate phase between chiral symmetry broken and its restored phases can be realized using a general Ginzburg–Landau

free energy [17]. A 2-quark state M in the fundamental and a 4-quark state Σ in the adjoint representation are introduced as¹

$$M_{ij} = \frac{1}{\sqrt{2}} (\sigma \delta_{ij} + i \phi^a \tau_{ij}^a), \quad \Sigma_{ab} = \frac{1}{\sqrt{3}} \chi \delta_{ab} + \frac{1}{\sqrt{2}} \varepsilon_{abc} \psi_c, \quad (2)$$

where the flavor indices run $(i, j) = 1, 2$ and $(a, b, c) = 1, 2, 3$ and Pauli matrices $\tau^a = 2T^a$ with $\text{Tr}[T^a T^b] = \delta^{ab}/2$. σ and χ represent scalar fields and ϕ and ψ pseudoscalar fields, and ε_{ijk} is the total anti-symmetric tensor with $\varepsilon_{123} = 1$. The pion decay constant is read from the Noether current as

$$F_\pi = \sqrt{\sigma_0^2 + \frac{8}{3} \chi_0^2}, \quad (3)$$

with χ_0 and σ_0 being the expectation values of χ and σ , determined from the gap equations. One can deduce a potential in the mean field approximation, which with an explicit breaking term is obtained as²

$$V(\sigma, \chi) = A\sigma^2 + B\chi^2 + \sigma^4 + \chi^4 - h\sigma + C\sigma^2\chi + D\chi^3 + F\sigma^2\chi^2. \quad (4)$$

A phase diagram of this model is shown in Fig. 3. There are three distinct phases characterized by two order parameters: Phase I represents the system where both chiral symmetry and its center are spontaneously broken due to non-vanishing expectation values χ_0 and σ_0 . The center symmetry

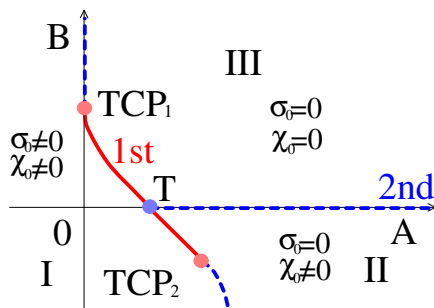


Fig. 3. Phase diagram with $C = -1$, $D = F = 0$ and $h = 0$ [17]. The solid and dashed lines indicate first and second order phase boundaries, respectively. One tricritical point, TCP_1 , is located at $(A, B) = (0, 1/4)$ and another, TCP_2 , at $(A, B) = (1/4, -1/8)$. The triple point represented by T is at $(A, B) = (1/8, 0)$.

¹ We restrict ourselves to a two-flavor case.

² A similar potential was considered for a system with 2- and 4-quark states under the symmetry breaking pattern without unbroken center symmetry in [18] where their 4-quark states are chiral singlet and the potential does not include quartic terms in fields.

is restored when σ_0 becomes zero. However, chiral symmetry remains broken as long as χ_0 is non-vanishing, indicated by phase II where the pure 4-quark state is the massless Nambu–Goldstone boson. The chiral symmetry restoration takes place under $\chi_0 \rightarrow 0$ which corresponds to phase III. The phases II and III are separated by a second-order line, while the broken phase I from II or from III is by both first- and second-order lines. Accordingly, there exist two tricritical points (TCPs) and one triple point. One of these TCP, TCP₂ in Fig. 3, is associated with the center Z_2 symmetry restoration rather than the chiral transition. The other coefficients D and F change the topology of the phase diagram and a TCP₁ turns to be a critical point depending on its sign even for $h = 0$.

With an explicit breaking of chiral symmetry one would draw a phase diagram mapped onto (T, μ) plane as in Fig. 4. The intermediate phase remains characterized by a small condensation $|\sigma_0| \ll |\chi_0|$. One would expect a new critical point associated with the restoration of the center symmetry, CP₂, rather than that of the chiral symmetry if dynamics prefers a negative coefficient of the cubic term in χ . Multiple critical points in principle can be observed as singularities of the quark number susceptibility.

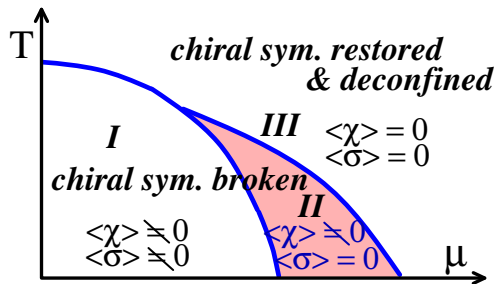


Fig. 4. Schematic phase diagram mapped onto (T, μ) plane. The lines do not distinguish the order of phase transitions. The critical point can appear on the boundary that separates the phase I from III at low μ and/or the phase I from II at intermediate μ [17].

It has been suggested that a similar critical point in lower temperature could appear in the QCD phase diagram based on the two-flavored Nambu–Jona-Lasinio model with vector interaction [19] and a Ginzburg–Landau potential with the effect of axial anomaly in three flavors [20]. There the interplay between the chiral (2-quark) condensate and BCS pairings plays an important role. In our framework without diquarks, the critical point is driven by the interplay between the 2-quark and 4-quark condensates, where anomalies have nothing to do with its appearance. Besides, the universality

class which the critical point in our model belongs to is expected to be different from the anomaly-induced one since spontaneous breaking of $U(1)_B$ is not imposed in (1).

Appearance of the above intermediate phase seems to have a similarity to the notion of Quarkyonic Phase. The transition from hadronic to quarkyonic world can be characterized by a rapid change in the net baryon number density. In our model this feature is driven by the restoration of center symmetry and is due to the fact that the Yukawa coupling of χ to baryons is not allowed by the Z_2 invariance. Fig. 5 shows an expected behavior of the quark (baryon) number susceptibility which exhibits a maximum when across the Z_2 cross over. This can be interpreted as the realization of the quarkyonic transition in $N_c = 3$ world. How far μ_{z2} from μ_{chiral} depends crucially on its dynamical-model description. Thus, the present analysis does not exclude the possibility that both transitions take place simultaneously and in such case enhancement of χ_B is driven by chiral phase transition. The phase with $\chi_0 \neq 0$ and $\sigma_0 = 0$ does not seem to appear in the large N_c limit [13–15]. It would be expected that including $1/N_c$ corrections induce a phase with unbroken center symmetry.

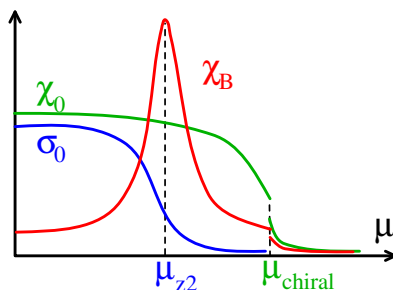


Fig. 5. The behavior of the baryon number susceptibility as a function of chemical potential.

5. Conclusions

We have discussed the phases in dense QCD from chiral approaches along with the anomaly matching which is a field-theoretical requirement. A possibility of a non-standard breaking pattern leads to a new phase where chiral symmetry is spontaneously broken while its center symmetry is restored. This might appear as an intermediate phase between chirally broken and restored phases in (T, μ) plane. The appearance of this phase also suggests a new critical point in low temperatures. A tendency of the center symmetry restoration is carried by the net baryon number density which shows a rapid increase indicating baryons more activated, and this is reminiscent of the

quarkyonic transition. Dynamical breaking of chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ down to $SU(N_f)_V \times (Z_{N_f})_A$ should be addressed in microscopic calculations using the Swinger–Dyson equations or Nambu–Jona-Lasinio type models with careful treatment of the quartic operators. The properties of baryons near the chiral phase transition are also an issue to be clarified. Depending on the chirality assignment to baryons, equations of state may be altered. In this respect, it attracts an interest that a top–down holographic QCD model predicts the same sign of the axial couplings to the parity partners and $g_A^{(+)} / g_A^{(-)} \sim 2$ [21].

I am grateful for fruitful collaboration with M. Harada, L. McLerran, K. Redlich and S. Takemoto. The work has been supported in part by the DFG cluster of excellence “Origin and Structure of the Universe”.

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