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# EOS AND BULK VISCOSITY OF COLD QUARK MATTER IN A RUNNING COUPLING NJL MODEL*

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We analyze the thermodynamical properties of color superconducting quark matter in the isotropic color spin locking (iso-CSL) phase at zero temperature and finite chemical potential. We perform calculations in the NJL model with quark–quark and quark–antiquark coupling for two parametrizations of the coupling strength: (a) constant value and (b) logarithmic dependence on the chemical potential. The bulk viscosity of the iso-CSL phase is calculated for both parametrizations for electrically neutral two-flavor matter in $\beta$-equilibrium. We discuss an extension of this model to the three-flavor case where we find that behaviour of the strange quark mass is qualitatively different for the cases (a) and (b). In this context we examine the influence of the value of QCD momentum scale $\Lambda_{\rm QCD}$ and investigate the stability of three-flavor quark matter in the iso-CSL phase under compact star (CS) constraints.

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## 1. Quark matter in the iso-CSL phase with running coupling

A sensible diagnostic tool for the state of matter in the interiors of compact stars are the thermal and transport properties which determine, *e.g.*, the cooling and spin evolution of CS. Based on a Nambu–Jona-Lasinio (NJL) type model, a consistent determination of the density and temperature dependence of quark masses, pairing gaps and chemical potentials under neutron star constraints has been performed first in [1, 2]. The resulting phase diagram suggests that for moderate diquark coupling strength the three-flavor phases of the CFL-type occur only at rather high densities and render

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hybrid star configurations gravitationally unstable [3, 4]. Moreover, due to large pairing gaps in CFL quark matter, the r-mode instabilities cannot be damped [5] and cooling is inhibited [6]. The two-flavor quark matter at moderate densities might be in the normal state since the formation of a spin-0 pairing gap (2SC phase) is inhibited by a large flavor asymmetry. Therefore, we will focus here on the discussion of single-flavor spin-one pairing [7–9] in quark matter under compact star constraints.

We consider here the iso-CSL phase [10, 11] for which in the pairing gap matrix $\hat{\Delta} = \Delta(\gamma_3\lambda_2 + \gamma_2\lambda_5 + \gamma_1\lambda_7)$ the three antisymmetric color matrices $(\lambda_2, \lambda_5, \lambda_7)$ are locked to the three spin matrices $(\gamma_3, \gamma_2, \gamma_1)$. It has been demonstrated [10] that an extension of the NJL model by a running coupling ansatz leads to a density dependence of the pairing gaps which mimicks that of the so-called 2SC+X phase [12], for which a detailed investigation of the cooling phenomenology for hybrid stars has been worked out [13, 14], see also [15] for a recent discussion. While neutrino emissivity and bulk viscosity of the iso-CSL phase have been investigated in the constant coupling NJL model before [16], we discuss here for the first time the behaviour of the bulk viscosity for the running coupling model.

We base our discussion on the thermodynamical potential for which (without KMT interaction, see [17]) the flavor channels decouple in the iso-CSL phase

$$\Omega_q(T,\mu) = \sum_{f=u,d,s} \Omega(T,\mu_f)\,, \tag{1}$$

and the contribution of a single flavor in mean field approximation is

$$\begin{aligned}\Omega(T,\mu_f) \;=\; & \frac{(M_f-m_f)^2}{8G_S(T,\mu_f)} + \frac{\Delta_f^2}{8G_D(T,\mu_f)} \\ & - \sum_{r=1}^{6} \int \frac{d^3\,p}{(2\pi)^3}\left[E_{f,r}(p) + 2T\ln\left(1+e^{-E_{f,r}(p)/T}\right)\right] - \Omega(0,0)\,, \end{aligned} \tag{2}$$

where $M_f$ and $m_f$ are the dynamical and current quark masses, respectively, and the dispersion relations of all modes $E_{f,r}(p)$ have nonvanishing gaps [10] with the lowest excitation energy being of the order of 1 MeV, as required from cooling phenomenology. Here, the vacuum contribution $\Omega_0$ is subtracted in order to guarantee vanishing pressure and energy density of the vacuum. For the running coupling strengths $G_S(T,\mu_f)$ in the scalar meson channel and $G_D(T,\mu_f) = 3G_S(T,\mu_f)/8$ in the spin-one diquark channel we use two parameterizations: (a) constant coupling $G_S(T,\mu_f) = G_S = \text{const.}$ and (b) chemical potential dependent coupling [10]

$$
\begin{aligned}
G_S(\mu_f) &= G_S\, R_{\Lambda_{\rm QCD}}(\mu_f)\,, \\
R_{\Lambda_{\rm QCD}}(\mu_f) &= \ln\left(\frac{\mu_{\rm c}}{\Lambda_{\rm QCD}}\right) \Big/ \ln\left(\frac{\mu_f}{\Lambda_{\rm QCD}}\right) .
\end{aligned} \tag{3}
$$

The function $R(\mu_f)$ is motivated by the logarithmic running of the one-loop beta function of QCD, where $\Lambda_{\rm QCD}$ is the QCD momentum scale and $\mu_{\rm c}$ is the critical chemical potential for chiral phase transition of up and down quarks. For the calculations of the masses and iso-CSL pairing gaps we employ the parameters of the NJL model for $M_{u,d}(p=0) = 380\,{\rm MeV}$ from Ref. [18]. Results for $\Lambda_{\rm QCD} = 300\,{\rm MeV}$ are shown in Fig. 1. The difference in the behavior of the chiral condensate for the two parametrizations is not altered qualitatively. One can notice that the dynamical mass decreases more rapidly for the case of chemical potential dependent coupling. The behavior of the pairing gap is qualitatively different in the two cases. When the coupling is kept constant the gap is an increasing function of chemical potential while for the running coupling ansatz (3) the gap is decreasing with $\mu_f$. We can also notice that in the second case the gap is by an order of magnitude smaller than in constant coupling case. It has been argued in [10, 15] that such a behavior of pairing gap is required to explain the observed cooling curves for neutron stars.

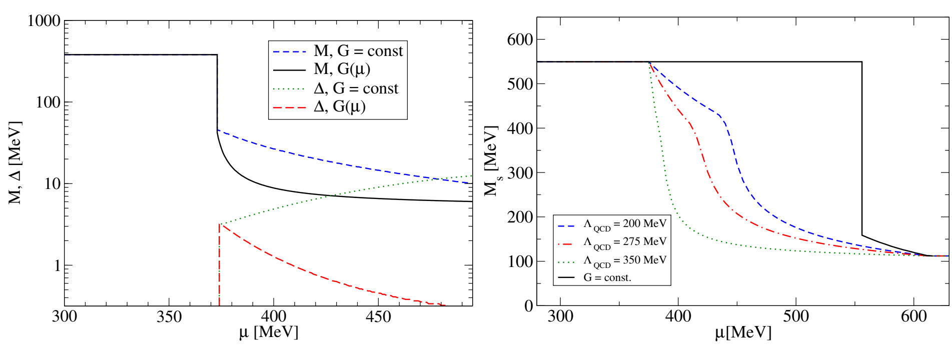


Fig. 1. Left panel: Comparison of dynamical mass functions ($M = M_u,\ M_d$) and iso-CSL diquark gaps ($\Delta = \Delta_u, \Delta_d$) for constant and chemical potential dependent coupling as a function of the chemical potential $\mu = \mu_u, \mu_d$. Right panel: Strange quark mass $M_{\rm s}$ as a function of the strange chemical potential $\mu = \mu_{\rm s}$ in the constant coupling model (solid line) and in the running coupling model for three values of $\Lambda_{\rm QCD}$.

## 2. Bulk viscosity in the two-flavor iso-CSL phase

For investigations of the stability of fastly rotating millisecond pulsars against r-modes [19, 20] one can derive constraints [5] which are based on the bulk and shear viscosities as key quantities. We discuss here the bulk viscosity for the iso-CSL phase applying an approach given in Ref. [21]

$$\zeta = \frac{\lambda C^2}{\omega^2 + (\lambda B)^2}\,, \tag{4}$$

with the coefficients functions

$$\begin{aligned} C &= \frac{M_u^2}{3\mu_u} - \frac{M_d^2}{3\mu_d} + \frac{4\alpha_\mathrm{s}}{3\pi}\left[\frac{M_d^2}{\mu_d}\left(\ln\frac{2\mu_d}{M_d} - \frac{2}{3}\right) - \frac{M_u^2}{\mu_u}\left(\ln\frac{2\mu_u}{M_u} - \frac{2}{3}\right)\right]\,, \\ B &\simeq \frac{\pi^2}{3}\left(\frac{1}{\mu_u^2} + \frac{1}{\mu_d^2} + \frac{1}{\mu_e^2}\right)\,, \\ \lambda &= \frac{4}{\pi^6}\alpha_\mathrm{s} G_F^2\ \mu_e\mu_u\mu_d\ T^4 \int_0^\infty dz\ z\left[\mathcal{F}_1(z) + \mathcal{F}_3(z) + \mathcal{F}_5(z)\right]\,, \end{aligned} \tag{5}$$

where the functions

$$\begin{aligned} \mathcal{F}_r(z) &= \sum_{e_1,e_2=\pm}\int_0^\infty\int_0^\infty dxdy\left(e^{-e_1\sqrt{y^2+a_{u,r}\Delta_u^2}} + 1\right)^{-1}\left(e^{e_2\sqrt{x^2+a_{d,r}\Delta_d^2}} + 1\right)^{-1} \\ &\quad\times\left(e^{z+e_1\sqrt{y^2+a_{u,r}\Delta_u^2}-e_2\sqrt{x^2+a_{d,r}\Delta_d^2}} + 1\right)^{-1}\,, \end{aligned} \tag{6}$$

characterize the influence of the superconducting gaps $\Delta_f$ on the bulk viscosity. The coefficients $a_{f,r}$ for $r = 1, 3, 5$ stem from the corresponding dispersion relations $E_{f,r}(p)$ in Eq. (2) and are defined in Ref. [10]. The gaps, obtained from the minimization of (1) fulfill in general $\Delta_u \neq \Delta_d$. In Fig. 2 we compare bulk viscosities for our two parameterizations. Firstly, we observe that for the running coupling model the location of the maximum is slightly shifted towards lower temperatures. Secondly, we notice that the lower pairing gap for the running coupling model results in a lower temperature ($\sim 2.5$ MeV) for the transition to the "normal" behavior of the bulk viscosity when compared with the constant coupling case ($\sim 6.5$ MeV). Nevertheless, the influence of the small pairing gaps on the bulk viscosity is minor when compared to that of the mass function which is responsible for the fact that the bulk viscosity in the iso-CSL phase is sufficiently large to circumvent r-mode instability.

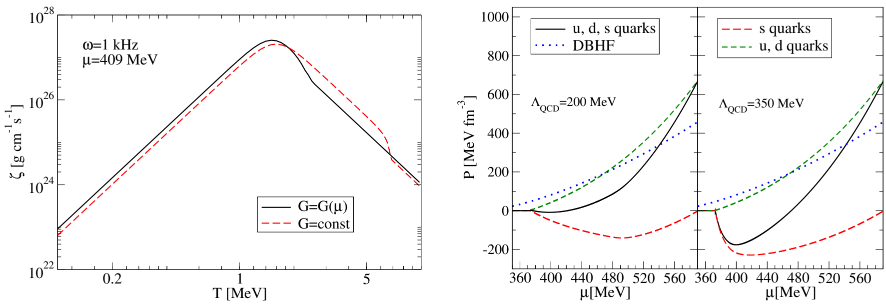


Fig. 2. Left panel: Bulk viscosity in 2 flavor CSL phase for constant and running coupling cases at the chemical potential $\mu = 409$ MeV for a typical millisecond pulsar frequency $\omega = 1$ kHz. Right panel: Pressure of quark matter for two values of $\Lambda_{\rm QCD}$ in comparison to a modern EoS for nuclear matter (DBHF). The crossing point of a quark matter and the DBHF curves indicates a deconfinement phase transition.

## 3. Strange quarks and the EoS

In order to answer the question whether the phases discusssed above could play a role for the interiors of compact stars one has to evaluate the EoS and solve the Tolman–Oppenheimer–Volkoff equations for hydrodynamic stability. From the meanfield thermodynamical potential (1) we obtain the equation of state in the quark sector of QCD. The present running coupling model leads to a behavior of the strange mass shown in the right panel of Fig. 1. It results in a lowering of the onset of strange quark degrees of freedom above the critical chemical potential $\mu_{\rm c} = 375$ MeV for the chiral transition in the light quark sector. The discussion of the thermodynamics we restrict to the case $\Lambda_{\rm QCD} = 200$ MeV and $\Lambda_{\rm QCD} = 350$ MeV shown in the right panel of Fig. 2. We can observe that the quark pressure exhibits an instability associated with the running of the coupling. Thermodynamic instabilities of such kind have been observed before in confining quark models. They can be circumvented by a phase transition construction to the hadronic phase. In Fig. 2 we provide as an example the Dirac–Brueckner–Hartree-Fock (DBHF) EoS under neutron star constraints, taken from Ref. [4]. The critical baryochemical potential where the corresponding deconfinement phase transition should occur is too large to be relevant for compact star applications.

## 4. Summary

We can conclude that the iso-CSL phase with the suggested running coupling generalization of the NJL model yields pairing gaps consistent with the cooling phenomenology of NS. It also leads to marginal modifications of the bulk viscosity which remains large enough to prevent r-mode instabilities of millisecond pulsars.

The question whether strange quarks appear in NS interiors remains open. The present running coupling model leads to a thermodynamic instability in the quark sector caused by a negative partial pressure for strange quarks. Circumventing this problem by a phase transition construction to hadronic matter leads to rather high values of the critical baryochemical potential for the deconfinement transition which would exclude quark matter in compact star interiors. Nevertheless, the chiral phase transition might very well occur at moderate densities so that a chirally symmetric hadronic phase ("quarkyonic matter" [22, 23]) could be expected.

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