DIQUARK BOSE–EINSTEIN CONDENSATION AT STRONG COUPLING*

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We investigate the phase structure of the $\mathrm{SU}_f(2) \otimes \mathrm{SU}_c(3)$ Nambu–Jona-Lasinio model as a function of the scalar diquark coupling strength. Above a critical coupling, the binding energy is sufficiently large to overcompensate the quark masses and a massless scalar diquark bound state emerges which leads to Bose condensation already in the vacuum.

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1. Introduction

Recent laboratory experiments with ultracold gases of fermionic atoms allow to investigate dense Fermi systems with their coupling strength tunable via Feshbach resonances by applying external magnetic fields. After the preparation of fermionic dimers in 2003, now also their Bose–Einstein condensation (BEC) [1, 2] and superfluidity of these dimers has been observed [3, 4]. Besides this strong coupling regime, for weak attractive interactions at low enough temperatures the condensation of bosonic correlations (Cooper pairs) in the continuum of unbound states occurs according to the Bardeen–Cooper–Schrieffer (BCS) theory.

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The BEC–BCS crossover is physically related [5] to the bound state dissociation or Mott–Anderson delocalization transition [6] where the modification of the effective coupling strength is caused by electronic screening and/or Pauli blocking effects. The Mott transition is a very general effect expected to occur in a wide variety of dense Fermi systems with bound states such as deuterons in nuclear matter [8–10] or diquarks in quark matter [11–14]. Below a critical temperature, bosonic correlations form a condensate and this transition appears as BEC-BCS crossover, which in quark matter is of particular theoretical interest due to the ultrarelativistic regime for massless (Goldstone) bosons [15–17].

A systematic treatment of these effects is possible within the path integral formulation for finite-temperature quantum field theories. This approach is rather general as it is relativistic and is especially suited to take into account the effects of spontaneous symmetry breaking. In this contribution we sketch the basics of this approach on the example of a model field theory of the Nambu–Jona-Lasinio type for a relativistic strongly interacting Fermi system, see [18] for a recent review. These investigations are also motivated by the analogies of the strongly coupled quark–gluon plasma (sQGP) at Relativistic Heavy Ion Collider (RHIC) in Brookhaven [19] with the experiments on BEC of atoms in traps. The further development of the approach may provide qualiative insights into the phases of QCD at high densities like the recently suggested quarkyonic phase [20–22]. Possible evidence for a triple point related to this new phase comes from hadron production in heavy-ion collision experiments [23] to be further investigated at upcoming dedicated facilities, *e.g.*, CBM at FAIR Darmstadt, NICA at JINR Dubna.

2. Formalism

2.1. Model Lagrangian and mean field approximation

Our starting point is a NJL-type Lagrangian for three colors and two flavors, motivated from Fierz transformed one-gluon exchange

$$\begin{aligned}\mathcal{L} \;=\; & \bar{q}(\mathrm{i}\partial_\mu\gamma^\mu - m_0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma_5\boldsymbol{\tau}q)^2\right] \\ & + G_D \sum_{A=2,5,7}\left(\bar{q}\mathrm{i}\gamma_5 C\tau_2\lambda_A\bar{q}^T\right)\left(q^T\mathrm{i}C\gamma_5\tau_2\lambda_A q\right), \qquad (1)\end{aligned}$$

with $C = \mathrm{i}\gamma_2\gamma_0$ being the charge conjugation matrix, $\tau = (\tau_1, \tau_2, \tau_3)$ and τ_i the Pauli matrices in flavor space and λ_A the anti-symmetric Gell-Mann matrices in color space. The parameter choice $m_0 = 5$ MeV, $G_S\Lambda^2 = 2.1$ and $\Lambda = 653\,\mathrm{MeV}$ reproduces the vacuum pion mass and decay constant.

For this model Lagrangian we can give the partition function in its bosonized form

$$\mathcal{Z} = \int \mathcal{D}\Delta^* \mathcal{D}\Delta \mathcal{D}\sigma \mathcal{D}\pi \exp\left\{ -\int^{\beta} d^4x \frac{\sigma^2+\pi^2}{4G_S} + \frac{|\Delta|^2}{4G_D} \right\} \det S^{-1}\,, \tag{2}$$

where diquark (Δ^*, Δ) and meson (σ, π) degrees of freedom appear as collective fields instead of the quark ones which have been integrated out leading to the determinant of the quark propagator S in Nambu–Gorkov representation. For details of the further calculation we refer to [24]. By minimizing the thermodynamical potential, we obtain gap equations which need to be solved self-consistently. The corresponding order parameters then characterize the phase structure of the model.

2.2. Gaussian fluctuations and polarization matrix

We expand around the mean field Nambu–Gorkov quark propagator $S_{\rm MF} \equiv \begin{pmatrix} G^+ & F^- \\ F^+ & G^- \end{pmatrix}$, up to 2nd order in the matrix $\Sigma \equiv \begin{pmatrix} -(\sigma+\pi^+) & \delta^- \\ \delta^+ & -(\sigma+\pi^-) \end{pmatrix}$, and obtain

$$\ln\det S^{-1} = {\rm Tr}\ln S^{-1}_{\rm MF} + {\rm Tr}\left(S_{\rm MF}\Sigma - \tfrac{1}{2}\, S_{\rm MF}\Sigma S_{\rm MF}\Sigma\right) + \mathcal{O}\left(\Sigma^3\right)\,. \tag{3}$$

The fluctuations of the collective fields can be decomposed according to: $\pi^{+/-} \equiv {\rm i}\,\gamma_5 \tau^{/t}\cdot\pi$, $\delta^{+/-} \equiv {\rm i}\,\gamma_5\tau_2\lambda_2\delta^{*/}$, and their amplitudes can be arranged in the vector $\vec{\phi} \equiv \{\pi,\sigma,\delta,\delta^*\}$. Performing the trace operations over Nambu–Gorkov, flavor, color, Dirac and momentum space we introduce the elements of the polarization matrix $\Pi(k_0,\boldsymbol{k})$ (for details see [25, 26])

$$\tfrac{1}{2}\ {\rm Tr}\left(S_{\rm MF}\Sigma S_{\rm MF}\Sigma\right) = \phi_i \Pi_{ij}\phi_j\ , \tag{4}$$

with $i,j = \{\pi,\sigma,\delta,\delta^*\}$ denoting the channels. Some matrix elements are pairwise equal, *e.g.*, $\Pi_{\sigma\delta} = \Pi_{\delta^*\sigma}, \Pi_{\delta\sigma} = \Pi_{\sigma\delta^*}$ and for real Δ even $\Pi_{\delta\delta} = \Pi_{\delta^*\delta^*}$. Thus, the pions are degenerate, as expected for isospin symmetric matter. We explicitly include the mixing terms between the σ and the diquarks in our investigation, which has been omitted in the literature so far [12, 14, 27]. Performing the Gaussian path integral over the fluctuation fields results in an expression for the thermodynamical potential

$$\Omega(T,\mu) = -T\ln Z = \ln\det S_{\rm MF} + \ln\det\left[\delta_{ij}/(2G_i) - \Pi_{ij}(\omega,\boldsymbol{k})\right]\,, \tag{5}$$

where $G_i = \{G_S, G_S, G_D, G_D\}$. The mass spectrum of quasiparticle modes can be found from the condition of the vanishing determinant in Eq. (5) at $\boldsymbol{k}=0$ for $\omega = \omega_i(\boldsymbol{k}=0) = \{m_\pi, m_\sigma, m_\delta - \mu, m_{\delta^*}+\mu\}$.

3. Results and discussion

We want to discuss first the vacuum case $\mu = T = 0$, where diquark and anti-diquark are degenerate, the general discussion will be given in [26]. In the normal phase, for the dimensionless diquark coupling strength $\eta_D = G_D/G_S$ in the range [17]

$$\frac{\pi^2}{4G_S\left(\Lambda\sqrt{\Lambda^2+m^2}+m^2\ln\frac{\Lambda+\sqrt{\Lambda^2+m^2}}{m}\right)} \leq \eta_D \leq \frac{3}{2}\,\frac{m}{m-m_0} = \eta_D^*\,, \tag{6}$$

the polarization matrix is diagonal and thus (5) for the diquarks gives

$$1 = 8G_D \int \frac{d^3p}{(2\pi)^3}\left(\frac{1}{E_{\boldsymbol{p}}+m_D/2}+\frac{1}{E_{\boldsymbol{p}}-m_D/2}\right). \tag{7}$$

If $\eta_D > \eta_D^*$ the matrix is not diagonal anymore. Neglecting the mixing terms, we get two solutions for (5), namely

$$1 \;=\; 8G_D \int \frac{d^3p}{(2\pi)^3}\left(\frac{1}{E_{\boldsymbol{p}}^{\Delta}+m_D/2}+\frac{1}{E_{\boldsymbol{p}}^{\Delta}-m_D/2}\right) \tag{8}$$

$$1 \;=\; 4G_D \int \frac{d^3p}{(2\pi)^3}\left(1+\frac{E_{\boldsymbol{p}}^2-\Delta^2}{E_{\boldsymbol{p}}^2+\Delta^2}\right)\left(\frac{1}{E_{\boldsymbol{p}}^{\Delta}+m_D/2}+\frac{1}{E_{\boldsymbol{p}}^{\Delta}-m_D/2}\right), \tag{9}$$

where $E_{\boldsymbol{p}}^{\Delta} = \sqrt{E_{\boldsymbol{p}}^2+\Delta^2}$, $E_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2+m^2}$. A Goldstone mode $m_D = 0$ solves the first equation, which in this case coincides with the gap equation for the pairing gap. The solution of the second equation gives a massive mode. The results are shown in the left panel of Fig. 1 as a function of η_D. In the right panel of Fig. 1 the phase structure of the model is shown for four cases of coupling strengths: $\eta_D = 0.9, 1.2, 1.5, 1.8$. While for $\eta_D < 0.9$ there is no coexistence of chiral symmetry breaking and diquark condensation, in the range $0.9 < \eta_D < \eta_D^*$ one obtains Bose condensation of bound diquarks in such regions of coexistence. At $\eta_D > \eta_D^*$ a still more spectacular effect occurs: the vacuum state itself is a Bose condensate of diquarks! While this model description of a relativistic Fermi system at arbitrary coupling is surely of methodological interest in the context of experiments with Bose condensates of atoms in traps, its relevance for the discussion of the phase structure of QCD requires a careful analysis of the corresponding hadron spectrum.

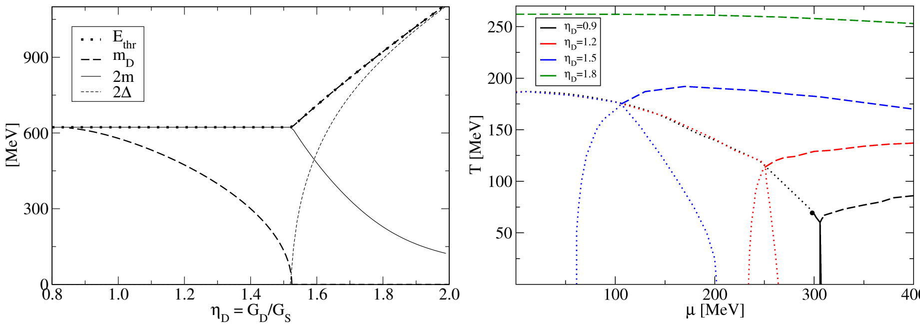

Fig. 1. Left panel: Vacuum diquark masses m_D and threshold energies $E_{\rm thr} = 2\sqrt{m^2+\Delta^2}$ as function of the coupling strength η_D. Right panel: Phase diagram for different values of the coupling strength $\eta_D = 0.9, 1.2, 1.5, 1.8$. First and second order phase transitions are indicated by solid and dashed lines, respectively. Dotted lines denote crossover transitions. The black dot for $\eta_D = 0.9$ indicates the critical endpoint for first order phase transitions. For the discussion see text.

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