# THE QCD PHASE DIAGRAM: LARGE $N_c$, QUARKYONIC MATTER AND THE TRIPLE POINT*

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I discuss the phase diagram of QCD in the large $N_c$ limit. Quarkyonic Matter is described. The properties of QCD matter as measured in the abundance of produced particles are shown to be consistent with this phase diagram. A possible triple point of Hadronic Matter, Deconfined Matter and Quarkyonic Matter is shown to explain various behaviors of ratios of particle abundances seen in CERN fixed target experiments.

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## 1. The phase diagram in the large $N_c$ limit

In the large $N_c$ limit of QCD, the nucleon mass is of the order of $N_c$ [1–3]. This means that in the confined phase of hadronic matter, for baryon chemical potential $\mu_B \leq M_N$, the baryon number density is essentially zero

$$\langle N_B \rangle \sim e^{(\mu_B - M_N)/T} \sim e^{-N_c}\,. \tag{1}$$

For temperatures above the de-confinement phase transition the baryon number is non-zero since there the baryon number density is controlled by $e^{-M_q/T} \sim 1$, and quark masses are independent of $N_c$. For sufficiently large chemical potential the baryon number density can be also nonzero. The Hadronic Matter phase of QCD is characterized in large $N_c$ by zero baryon number density, but at higher density there is a new phase.

In the large $N_c$ limit, fermion loops are suppressed by a factor of $1/N_c$. Therefore, the contribution to Debye screening from quarks cannot affect the quark potential until

$$M^2_{\rm Debye} \sim \alpha_{\rm t'Hooft}\ \mu^2_{\rm quark}/N_c \sim \Lambda^2_{\rm QCD}\,. \tag{2}$$

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Here the quark chemical potential is $\mu_B = N_c\mu_{\text{quark}}$. The relationship involving the Debye mass means there is a region parametrically large in chemical potential $M_N \leq \mu_B \leq \sqrt{N_c}M_N$ where matter is confined, and has finite baryon number. This matter is different than either the Hadronic Matter or the De-Confined Phases. It is called Quarkyonic because it exists at densities parametrically large compared to the QCD scale, where quark degrees of freedom are important, but it is also confined so the degrees of freedom may be thought of also as those of confined baryons [4–5].

The width of the transition region between the Hadronic phase and the Quarkyonic phase is estmated by requiring that the baryon number density becomes of the order of $N_B/V \sim k_{\text{Fermi}}^3 \sim \Lambda_{\text{QCD}}^3$. Recall that the baryon chemical potential is $\mu_B \sim M_N + k_f^2/2M_N$ for small $k_f$, so that the width of the transition in $\mu_B$ is very narrow, of the order of $1/N_c$. This is $\delta\mu_{\text{quark}} \sim 1/N_c^2$ when expressed in terms of $\mu_{\text{quark}}$ which is the finite variable in the large $N_c$ limit.

The transition from Hadronic Matter to that of the Quark Gluon Plasma may be thought of as a change in the number of degrees of freedom of matter. Hadronic Matter at low temperatures has 3 pion degrees of freedom. The quark gluon plasma has of the order of $2(N_c^2-1)$ degrees of freedom corresponding to gluons and $4N_c$ degrees of freedom for each light mass quark. The change in degrees of freedom is of the order of $N_c^2$ in the large $N_c$ limit. At very high baryon number densities, the quarks in the Fermi sea interact at short distances, and although strictly speaking are confined, behave like free quarks. The number of degrees of freedom is therefore of the order of $N_c$. Each phase has different numbers of degrees of freedom, and is presumably separated from the other by a rapid crossover.

Quarkyonic Matter is confined and therefore thermal excitations such as mesons, glueballs, and Fermi surface excitations must be thought of as confined. The quarks in the Fermi sea are effectively weakly interacting since their interactions take place at short distances. So in some sense, the matter is "de-confined" quarks in the Fermi sea with confined glueball, mesons and Fermi surface excitations [6].

In Hadronic Matter, chiral symmetry is broken and in Deconfined Matter it is broken. In Quarkyonic Matter chiral symmetry is broken by the formation of charge density waves from binding of quark and quark hole excitations near the Fermi surface [7]. In order that the quark hole have small relative momentum to the quark, the quark hole must have momentum opposite to that of the quark. This means the quark–quark hole excitation has total net momentum, and therefore the finite wavelength of the corresponding bound state leads to a breaking of translational invariance. The chiral condensate turns out to be a chiral spiral where the chiral condensate rotates between different Goldstone boson as one moves through the condensate [8]. Such condensation may lead to novel crystalline structures [9].

A figure of the hypothetical phase diagram of QCD is shown in Fig. 1 for $N_c = 3$. Also shown is the weak liquid-gas phase transition, and the phase associated with color superconductivity. Although the color superconducting phase cannot coexist with quarkyonic matter in infinite $N_c$, for finite $N_c$ there is such possibility. The lines on this phase diagram might correspond to true phase transitions or rapid cross overs. The confinement–deconfinement transition is known to be a cross over. In the FPP-NJL model [10–12], the Hadronic–Quarkyonic transition is first order [13], but nothing is known from lattice computations.

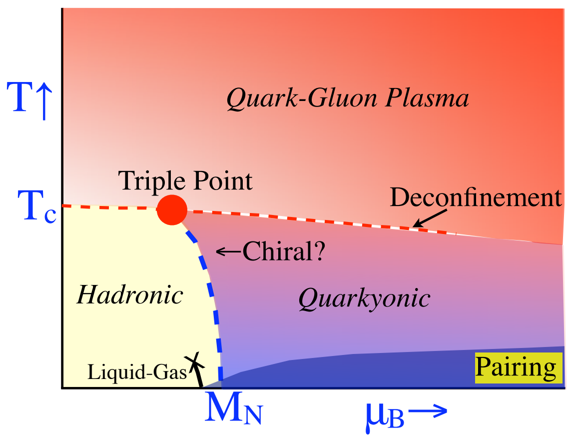


Fig. 1. The revised phase diagram of QCD.

A remarkable feature of this plot is the triple point where the Hadronic Matter, Deconfined Matter and Quarkyonic Matter all meet [14]. This triple point is reminiscent of the triple point for the liquid, gas and vapor phases of water.

Since we expect a rapid change in the number of degrees of freedom across the transitions between these forms of matter, an expanding system crossing such a transition would undergo much dilution at a fixed value of temperature or baryon chemical potential [16–18]. One might expect in heavy ions to see decoupling of particle number changing processes at this transition, and the abundances of produced particles will be characteristic of the transition.

In Fig. 2, the expectations for the confinement–deconfinement transition are shown with the dotted (red) line. It is roughly constant with the baryon chemical potential, and the constant value of temperature is taken from lattice estimates. The dark dashed curve represents $\mu_B - T = \text{const.} \times M_N$, corresponding to a simple model for the Quarkyonic transition. Such a very simple description does remarkably well.

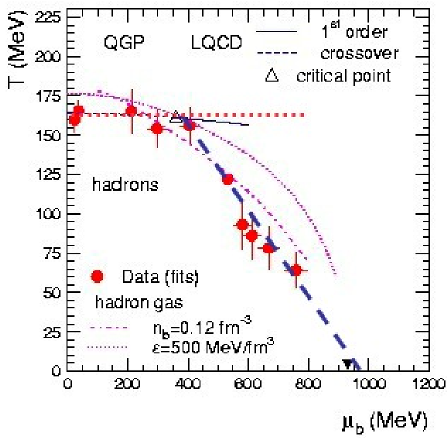


Fig. 2. Chemical potentials and temperatures at decoupling.

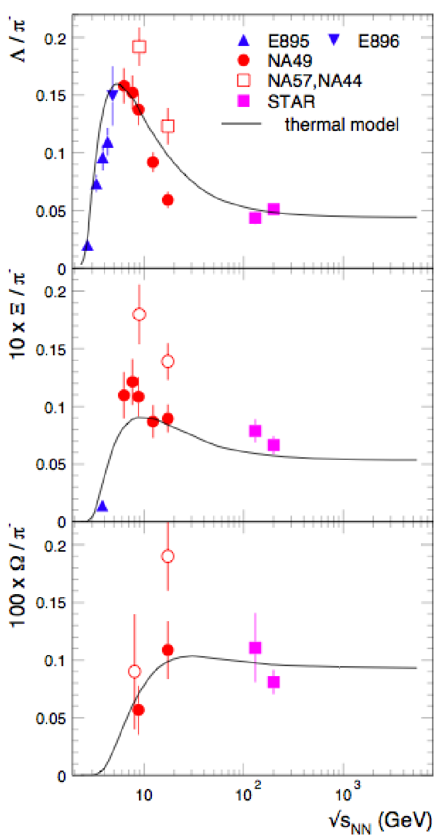


Fig. 3. Ratios of abundances of various particles.

A triple point is suggested at a baryon chemical potential near 400 MeV, and temperature near 160 MeV. This corresponds to a center of mass energy for Pb–Pb collisions of 9–10 GeV. This is near the point where there are anomalies in the abundances of rations of particles [15], as shown in Fig. 3. Shown are fits using statistical models of abundances of particles using chemical potentials and temperature extracted from experimental data. The sharp peak reflects the change in behavior as one proceeds along the dashed line of Fig. 2 corresponding to the Quarkyonic transition and joins to the dotted red line of the deconfinement transition.

It is remarkable that the value of beam energy, where this occurs, corresponds to the hypothetical triple point of Fig. 2, and that this is the density where the energy density stored in baryons becomes equal to that stored in mesons, Fig. 4.

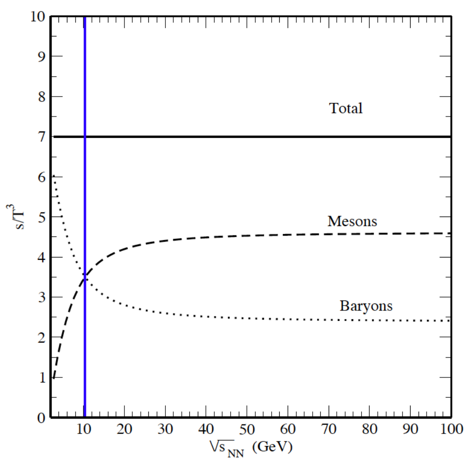


Fig. 4. Energy density stored in baryons compared to that stored in mesons.

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