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CONFINEMENT MODELS AT FINITE TEMPERATURE AND DENSITY*

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In-medium chiral symmetry breaking in confining potential models of QCD is examined. Ring diagrams are proposed as a resolution to the infrared divergence problem in the gap equations. We present the first determination of the temperature-density phase diagram for two model systems. We find that observables and the phase structure of the confinement models depend strongly on whether vacuum polarisation is accounted for. Finally, it appears that standard confinement models cannot adequately describe both hadron phenomenology and in-medium properties of QCD.

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1. Introduction

The properties of QCD at finite temperature and density find applications in topics as diverse as the nature of proto-neutron stars, early universe cosmology, and experiment at RHIC and the LHC. Unfortunately, many of the properties of interest are nonperturbative, and, with the exception of lattice techniques, tools for dealing with nonperturbative field theory remain rudimentary. Here we apply Schwinger–Dyson techniques to simple confinement models in an attempt to determine their applicability to in-medium QCD and to shed light on a longstanding issue concerning IR divergences.

2. Confinement models and the gap equation

We examine the properties of two simple models of confinement that are motivated by QCD in Coulomb gauge. Upon neglecting transverse gluons, the QCD Hamiltonian takes the form [1, 2]

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$$H = \int \bar{\psi}(-i\vec{\gamma}\cdot\nabla + m)\psi + \tfrac{1}{2}\int \rho^a(x)V(x-y)\rho^a(y)\,, \tag{1}$$

where $\rho^a = \psi^\dagger T^a \psi$ is the colour quark current and T^a is a generator of SU(N). We consider the linear confinement potential

$$V(\vec{r}) = -\frac{3}{4}br\,, \qquad V(\vec{q}) = \frac{6\pi b}{q^4}\,, \tag{2}$$

and the Richardson potential

$$V(\vec{q}) = \frac{3}{4}\frac{4\pi}{q^2\beta_0 \log(1+q^2/\Lambda^2)} \tag{3}$$

with $\beta_0 = 11 - \frac{2}{3}n_f$, $\Lambda^2 = 2b\beta_0$, and n_f is the number of quark flavours. The string tension is denoted b and its phenomenological value is approximately 0.2 GeV^2.

The Schwinger–Dyson equation for the full fermion propagator is represented in Fig. 1.

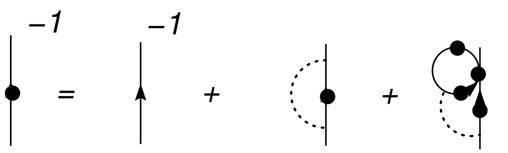

Fig. 1. Schwinger–Dyson equation for the full fermion propagator in potential models. Minus signs are not made explicit.

A simple substitution of the tree order approximation to the fermion four-point vertex yields correction terms due to dressed vacuum polarisation and vertex correction. If one neglects vertex correction diagrams, it is possible to sum all dressed vacuum polarisation insertions by rewriting the equation These coupled equations, called the gap equations, form the starting point for our investigation of dynamical mass generation (see Fig. 2). Note that the vacuum polarisation fermion loop of the second equation utilises dressed fermion propagators.

Fig. 2. Gap equations: summing polarisation insertions in the truncated Schwinger–Dyson equations.

We shall employ the imaginary time formalism to perform the finite temperature and density calculation of the gap equations. In order to solve the coupled integral equations consistently, we make the following ansatz for the in-medium inverse fermion propagator

$$S^{-1}(k) = i(\omega_n - i\tilde{\mu})\gamma_0 - \vec{\gamma}\cdot\vec{k}A - B\,. \tag{4}$$

The scalars $\tilde{\mu}$, A, and B are functions of k_0 and $|\vec{k}|$. The gap equations then read

$$\begin{aligned}
A(\vec{p}) &= 1 + \frac{C_{\rm F}}{2}\int \frac{d^3q}{(2\pi)^3} V_{\rm ring}(\vec{p}-\vec{q})\frac{A_q}{E_q}\frac{\vec{p}\cdot\vec{q}}{p^2}[1-n(q)-\bar{n}(q)]\,,\\
B(\vec{p}) &= m + \frac{C_{\rm F}}{2}\int \frac{d^3q}{(2\pi)^3} V_{\rm ring}(\vec{p}-\vec{q})\,\frac{B_q}{E_q}\,[1-n(q)-\bar{n}(q)]\,,\\
\tilde{\mu}(\vec{p}) &= \mu + \frac{C_{\rm F}}{2}\int \frac{d^3q}{(2\pi)^3} V_{\rm ring}(\vec{p}-\vec{q})[n(q)-\bar{n}(q)]\,,\\
E_p^2 &= A_p^2 p^2 + B_p^2\,,
\end{aligned} \tag{5}$$

where we have introduced the colour factor $C_{\rm F} = (N^2-1)/(2N)$, and

$$n(p) = \frac{1}{\exp(\beta(E_p - \tilde{\mu})) + 1}\,, \tag{6}$$

$$\bar{n}(p) = \frac{1}{\exp(\beta(E_p + \tilde{\mu})) + 1}\,, \tag{7}$$

with

$$V_{\rm ring}(q_0,\vec{q}) = \frac{V(\vec{q})}{1 - \Pi(q_0,\vec{q})V(\vec{q})}\,, \tag{8}$$

and

$$\Pi(k_0,k) = \frac{1}{2\beta} n_f \sum_n \int \frac{d^3p}{(2\pi)^3}\,{\rm Tr}[\gamma_0 S(k)\gamma_0 S(p+k)]\,. \tag{9}$$

We remark that the gap equation takes on this form only in the case where the frequency dependence of the ring potential is neglected.

3. Numerical results

Our results will be presented as plots of the dynamical mass at zero momentum as a function of chemical potential and temperature: $M[k \to 0, T, \mu]$. Thus we determine the phase structure of the contact and confinement models. As far as we know these are the first computations of these phase diagrams, even in the case of the bare potential. The computation for the ring potentials are also new.

As mentioned above, the gap equations contain an IR divergence in the scalars A and B. The ratio is IR finite, however, the individual scalar functions appear in the expressions for n and $\bar{n}$, and hence the IR divergence cannot be avoided at finite temperature. With the prescription suggested by Alkofer *et al.* [4] (which amounts to replacing A in n and $\bar{n}$ by 1) we obtain the results of Fig. 3.

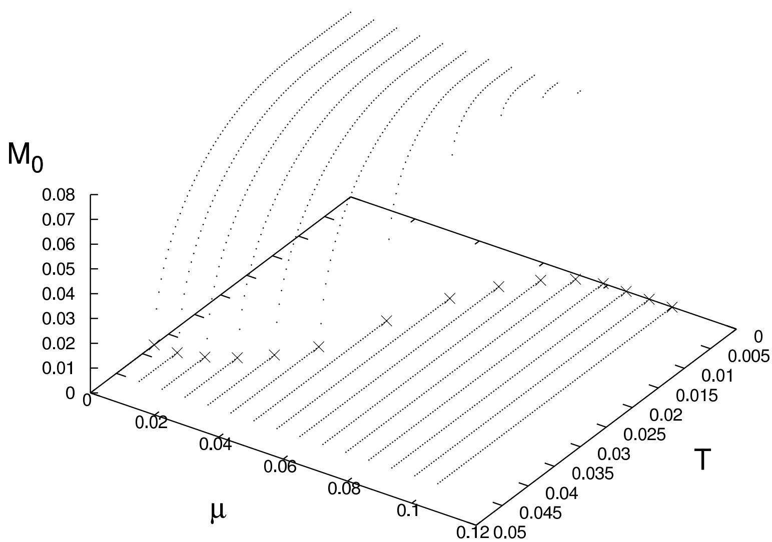

Fig. 3. Dynamical mass *versus* temperature and density for the bare linear AAL confinement model. Quantities in GeV.

A more physical way to deal with the infrared divergence problem is to include the ring contribution in our gap equations [5]. As demonstrated in Fig. 4, including the ring diagrams causes the dynamical mass, critical temperature, and critical chemical potential to drop to even more unrealistic values. More details and further studies are contained in Ref. [3].

We remark that the vacuum polarisation function introduces explicit temperature and density dependence to the quark interaction, which raises the possibility of explicit quark deconfinement in the model. It is possible that this dependence causes the potential to deconfine at a critical temperature. However, it is more likely that the potential deconfines for all nonzero temperature. Indeed, in perturbation theory one can approximate the ring potential as

$$V_{\rm ring}(q,T,\mu=0) \approx \frac{6\pi b}{q^4+\pi b T^2}\,. \tag{10}$$

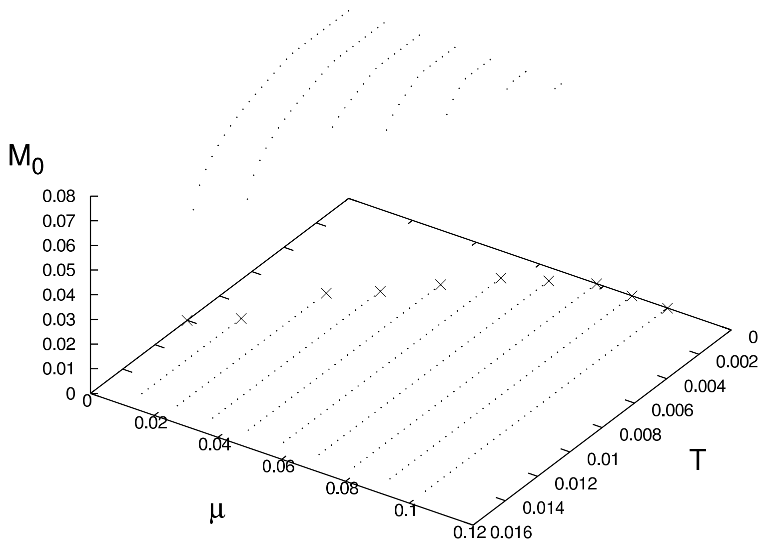

Fig. 4. Dynamical mass *versus* temperature and density for the linear static long wavelength ring approximation. Quantities in GeV.

The Fourier transform of this potential is linear when $T = 0$. When $T > 0$ the potential is linear at small distances, has a transition region at $r \sim (\pi b T^2)^{-1/4}$, and approaches zero at large distances, so that deconfinement is natural, although not sudden. A careful analysis of deconfinement awaits a study of QCD.

4. Discussion and conclusion

For the bare linear model with the AAL infrared prescription we confirm the existence of a second order phase transition at small chemical potential. For chemical potential larger than $\mu_\star \approx 43$ MeV the phase transition becomes first order. The appearance of *any* phase transition is somewhat surprising, since it is in conflict with the reasonable expectations of Davis and Matheson [6]. The numerical values for the dynamical mass, chiral restoration temperature and density, and chiral condensate are all in agreement with QCD expectations if the string tension is increased to a value of 1.8 GeV^2. Unfortunately, this is in severe conflict with well-established quark model phenomenology and lattice gauge results that require a string tension of approximately 0.2 GeV^2. It is thus apparent that the simplest confinement models cannot both reproduce thermodynamic and spectroscopic quantities with any reliability. Of course, this conclusion depends on the approximations we have made. However, the large discrepancy seems difficult to overcome and we expect that simple confinement models are incapable of describing in-medium properties of QCD.

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