THERMODYNAMICS OF DENSE MATTER IN CHIRAL APPROACHES*

Chihiro Sasaki

Frankfurt Institute for Advanced Studies
60438 Frankfurt am Main, Germany

(Received September 23, 2010)

We discuss phases in dense hadronic and quark matter from chiral model approaches. Within PNJL models the phase diagram for various number of colors N_c is studied. How phases are constrained in quantum field theories is also discussed along with the anomaly matching. An exotic phase with unbroken center symmetry of chiral group has a characteristic feature in the thermodynamics, which can be interpreted as one realization of the quarkyonic phase in QCD for $N_c = 3$.

PACS numbers: 11.30.Rd, 11.30.Ly, 25.75.Nq, 21.65.Qr

1. Phase diagram: from $N_c = \infty$ to $N_c = 3$

Model studies of dense baryonic and quark matter have suggested a rich phase structure of QCD at temperatures and quark chemical potentials being of the order of Λ_{QCD}. Our knowledge on the phase structure is however still limited and the description of the matter around the phase transitions does not reach a consensus because of the non-perturbative nature of QCD [1]. Possible phases and spectra of excitations are guided by symmetries and their breaking pattern in a medium. Dynamical chiral symmetry breaking and confinement are characterized by strict order parameters associated with global symmetries of the QCD Lagrangian in two limiting situations: the quark bilinear $\langle \bar{q}q \rangle$ in the limit of massless quarks, and the Polyakov loop $\langle \Phi \rangle$ in the limit of infinitely heavy quarks.

A novel phase of dense quarks, Quarkyonic Phase, was recently proposed based on the argument using large N_c counting, where N_c denotes number of colors [2]: in the large N_c limit there are three phases which are rigorously

---

* Presented at the Workshop "Excited QCD 2010", Tatranská Lomnica/Stará Lesná, Tatra National Park, Slovakia, January 31–February 6, 2010.

distinguished using $\langle \Phi \rangle$ and the baryon number density $\langle N_B \rangle$. The quarkyonic phase is characterized by $\langle \Phi \rangle = 0$ indicating the system confined and non-vanishing $\langle N_B \rangle$ above $\mu_B = M_B$ with a baryon mass M_B.

A possible deformation of the phase boundaries in large N_c together with the chiral phase transition can be described using a chiral model coupled to the Polyakov loop [3]. The Nambu–Jona-Lasinio model with Polyakov loops (PNJL model) has been developed to deal with chiral dynamics and "confinement" simultaneously [4]. The model describes that only three-quark states are thermally relevant below the chiral critical temperature, which is reminiscent of confinement. Figure 1 shows the two transition lines for $N_c = \infty$ and for $N_c = 3$ in the two-flavored PNJL model. In the large N_c limit assuming that the system is confined, the gap equations for the order parameters $\langle \bar{q}q \rangle$ and $\langle \Phi \rangle$ become two uncorrelated equations. Consequently, the quark dynamics carries only a μ dependence and the Polyakov loop sector does only a T dependence. Finite N_c corrections make the transition lines bending down. The crossover for deconfinement shows a weak dependence on μ which is a remnant of the phase structure in large N_c. One finds that for $N_c = 3$ deconfinement and chiral crossover lines are on top of each other in a wide range of μ. A critical point associated with chiral symmetry appears around the junction of those crossovers.

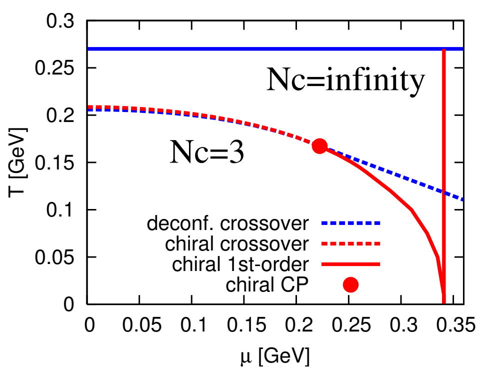

Fig. 1. The phase diagram of a PNJL model for different N_c [3].

The clear separation of the quarkyonic from hadronic phase is lost in a system with finite N_c. Nevertheless, an abrupt change in the baryon number density would be interpreted as the quarkyonic transition which separates meson dominant from baryon dominant regions [5]. In fact, a steep increase in the baryon number density and the corresponding maximum in its susceptibility χ_B are driven by a phase transition from chirally broken to restored phase in most model-approaches using constituent quarks. One might then consider the chirally symmetric confined phase as the quarkyonic phase.

The constituent quarks are, however, unphysical in confined phase. It is not obvious to have a realistic description of hadrons from chiral quarks. In particular, chiral symmetry restoration for baryons must be worked out. Two alternatives for chirality assignment are known [6] and it remains an open question which scenario is preferred by nature: *(i)* in the naive assignment, dynamical chiral symmetry breaking generates a baryon mass which thus vanishes at the restoration; *(ii)* in the mirror assignment, dynamical chiral symmetry breaking generates a mass difference between parity partners and the chiral symmetry restoration does not necessarily dictate the chiral partners being massless. If the chiral invariant mass is not very small, the baryon number density is supposed to be insensitive to the quarkyonic transition.

Besides, it is unsettled that the chirally-restored confined phase is realized in QCD on the basis of the anomaly matching: external gauge fields, *e.g.* photons, interacting with quarks lead to anomalies in the axial current. Since there are no Nambu–Goldstone bosons in chiral restored phase, the anomalous contribution must be generated from the triangle diagram in which the baryons are circulating. In three flavors, however, the baryons forming an octet do not contribute to the pole in the axial current because of the cancellations [7]. It is indispensable to any rigorous argument for this taking account of the physics around the Fermi surface, which could lead to a possibility of the chirally restored phase with confinement. The anomaly matching conditions at finite temperature and density are in fact altered, see *e.g.* [8].

2. Role of the tetra-quark at finite density

There is a possibility of two different phases with broken chiral symmetry distinguished by the baryon number density. An alternative pattern of spontaneous chiral symmetry breaking was suggested in the context of QCD at zero temperature and density [9–11]. This pattern keeps the center of chiral group unbroken, *i.e.*

$$\mathrm{SU}(N_f)_\mathrm{L} \times \mathrm{SU}(N_f)_\mathrm{R} \to \mathrm{SU}(N_f)_\mathrm{V} \times (Z_{N_f})_\mathrm{A}\,, \tag{1}$$

where a discrete symmetry $(Z_{N_f})_\mathrm{A}$ is the maximal axial subgroup of $\mathrm{SU}(N_f)_\mathrm{L} \times \mathrm{SU}(N_f)_\mathrm{R}$. The center Z_{N_f} symmetry protects a theory from condensate of quark bilinears $\langle\bar{q}q\rangle$. Spontaneous symmetry breaking is driven by quartic condensates which are invariant under both $\mathrm{SU}(N_f)_\mathrm{V}$ and Z_{N_f} transformation. Although meson phenomenology with this breaking pattern seems to explain the reality reasonably [9], this possibility is strictly ruled out in QCD both at zero and finite temperatures but at zero density since a different way of coupling of Nambu–Goldstone bosons to pseudo-scalar density violates QCD inequalities for density-density correlators [12]. However, this does not exclude the unorthodox pattern in the presence of dense matter. In a system with the breaking pattern (1) the quartic condensate is the strict order parameter which separates different chirally-broken phases[1].

Assuming (1) at finite density, it has been shown that an intermediate phase between chiral symmetry broken and its restored phases can be realized using a general Ginzburg–Landau free energy [15]. The pion decay constant is read from the Noether current as

$$F_\pi = \sqrt{\sigma_0^2 + \tfrac{8}{3}\chi_0^2}\,, \tag{2}$$

with χ_0 and σ_0 being the expectation values of 4-quark and 2-quark scalar fields, determined from the gap equations.

The effective potential deduced in the mean field approximation describes three distinct phases characterized by the two order parameters: Phase I represents the system where both chiral symmetry and its center are spontaneously broken due to non-vanishing expectation values χ_0 and σ_0. The center symmetry is restored when σ_0 becomes zero. However, chiral symmetry remains broken as long as χ_0 is non-vanishing, where the pure 4-quark state is the massless Nambu–Goldstone boson (phase II). The chiral symmetry restoration takes place under $\chi_0 \to 0$ which corresponds to phase III. The phases II and III are separated by a second-order line, while the broken phase I from II or from III is by both first- and second-order lines. Accordingly, there exist two tricritical points (TCPs) and one triple point. One of these TCP is associated with the center Z_2 symmetry restoration rather than the chiral transition. With an explicit breaking of chiral symmetry one would draw a phase diagram mapped onto (T, μ) plane as in Fig. 2.

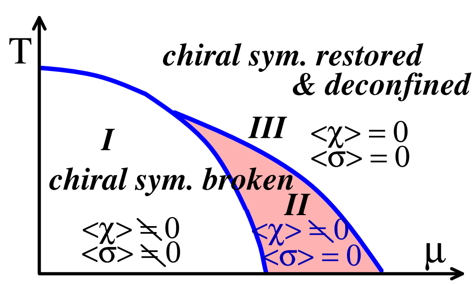

Fig. 2. Schematic phase diagram mapped onto (T, μ) plane.

Appearance of the above intermediate phase seems to have a similarity to the notion of Quarkyonic Phase. The transition from hadronic to quarkyonic world can be characterized by a rapid change in the net baryon number

[1] A similar phase structure was discussed in [13, 14].

density. In our model this feature is driven by the restoration of center symmetry and is due to the fact that the Yukawa coupling of χ to baryons is not allowed by the Z_2 invariance. Consequently, the baryon number susceptibility exhibits a maximum when across the Z_2 cross over. This can be interpreted as the realization of the quarkyonic transition in $N_c = 3$ world. The phase with $\chi_0 \neq 0$ and $\sigma_0 = 0$ does not seem to appear in the large N_c limit [11–13]. It would be expected that including $1/N_c$ corrections induce a phase with unbroken center symmetry.

3. Conclusions

We have discussed the phases in dense QCD from chiral approaches along with the anomaly matching which is a field-theoretical requirement. Although the chiral restored phase below "deconfinement" seems to be a common feature in PNJL models, this might be an artifact of this toy model in which the temporal gluon field is treated as a constant background and thus confinement dynamics is lost. A possibility of a non-standard breaking pattern leads to a new phase where chiral symmetry is spontaneously broken while its center symmetry is restored. This might appear as an intermediate phase between chirally broken and restored phases in (T, μ) plane. The appearance of this phase also suggests a new critical point in low temperatures. A tendency of the center symmetry restoration is carried by the net baryon number density which shows a rapid increase indicating baryons more activated, and this is reminiscent of the quarkyonic transition.

The properties of baryons near the chiral phase transition are also an issue to be clarified. Depending on the chirality assignment to baryons, equations of state may be altered. The chirality assignment becomes more involved when one introduces axial-vector mesons [16]. In this case the axial-vector meson gives a non-trivial contribution to the axial couplings and eventually it is not clear that the sign of the axial coupling to the negative parity state does distinguish two scenarios. In this respect, it attracts an interest that the same sign of the axial couplings to the parity partners is predicted in a top-down holographic QCD model [17] and the lattice QCD with dynamical quarks [18].

I am grateful for fruitful collaboration with M. Harada, L. McLerran, K. Redlich and S. Takemoto. Parital support by the Hessian LOEWE initiative through the Helmholtz International Center for FAIR (HIC for FAIR) is acknowledged.

REFERENCES

[1] C. Sasaki, *Nucl. Phys.* **A830**, 649C (2009).

[2] L. McLerran, R.D. Pisarski, *Nucl. Phys.* **A796**, 83 (2007); Y. Hidaka, L.D. McLerran, R.D. Pisarski, *Nucl. Phys.* **A808**, 117 (2008).

[3] L. McLerran, K. Redlich, C. Sasaki, *Nucl. Phys.* **A824**, 86 (2009).

[4] K. Fukushima, *Phys. Lett.* **B591**, 277 (2004); C. Ratti, M.A. Thaler, W. Weise, *Phys. Rev.* **D73**, 014019 (2006).

[5] A. Andronic *et al., Nucl. Phys.* **A837**, 65 (2010).

[6] C.E. Detar, T. Kunihiro, *Phys. Rev.* **D39**, 2805 (1989); D. Jido, M. Oka, A. Hosaka, *Prog. Theor. Phys.* **106**, 873 (2001) and references therein.

[7] M.A. Shifman, *Phys. Rep.* **209**, 341 (1991) [*Sov. Phys. Usp.* **32**, 289 (1989 UFNAA, 157, 561-598.1989)].

[8] H. Itoyama, A.H. Mueller, *Nucl. Phys.* **B218**, 349 (1983); R.D. Pisarski, *Phys. Rev. Lett.* **76**, 3084 (1996); R.D. Pisarski, T.L. Trueman, M.H.G. Tytgat, *Phys. Rev.* **D56**, 7077 (1997); S.D.H. Hsu, F. Sannino, M. Schwetz, *Mod. Phys. Lett.* **A16**, 1871 (2001).

[9] M. Knecht, J. Stern, `arXiv:hep-ph/9411253`; J. Stern, `arXiv:hep-ph/9712438`; `arXiv:hep-ph/9801282`.

[10] B. Holdom, G. Triantaphyllou, *Phys. Rev.* **D51**, 7124 (1995); *Phys. Rev.* **D53**, 967 (1996); B. Holdom, *Phys. Rev.* **D54**, 1068 (1996).

[11] P. Maris, Q. Wang, *Phys. Rev.* **D53**, 4650 (1996); F.S. Roux, T. Torma, B. Holdom, *Phys. Rev.* **D61**, 056009 (2000).

[12] I.I. Kogan, A. Kovner, M.A. Shifman, *Phys. Rev.* **D59**, 016001 (1999).

[13] Y. Watanabe, K. Fukushima, T. Hatsuda, *Prog. Theor. Phys.* **111**, 967 (2004).

[14] M. Rho, `arXiv:0711.3895[nucl-th]`; B.Y. Park, J.I. Kim, M. Rho, *Phys. Rev.* **C81**, 035203 (2010).

[15] M. Harada, C. Sasaki, S. Takemoto, *Phys. Rev.* **D81**, 016009 (2010); C. Sasaki, `arXiv:0910.4375[hep-ph]`.

[16] S. Gallas, F. Giacosa, D.H. Rischke, `arXiv:0907.5084[hep-ph]`.

[17] K. Hashimoto, T. Sakai, S. Sugimoto, *Prog. Theor. Phys.* **120**, 1093 (2008).

[18] T.T. Takahashi, T. Kunihiro, *Phys. Rev.* **D78**, 011503 (2008).