# GLUEBALLS, GLUON CONDENSATE, AND PURE GLUE QCD BELOW $T_{\rm c}$*

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A quasiparticle description of pure glue QCD below $T_{\rm c}$ is presented. It is shown that the strong decrease of the gluon condensate combined with the increasing thermal width of the lightest glueballs at $T \lesssim T_{\rm c}$ might be the trigger of the phase transition. The proposed model compares very well with recent lattice data.

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## 1. Introduction

A key observable of finite temperature QCD is its equation of state (EoS), that can be computed by resorting to either effective models or lattice QCD calculations. In particular, the EoS of pure glue QCD has been computed on the lattice using gauge groups ranging from SU(3) to SU(8) [1, 2]. Many other calculations of the QCD EoS have also been performed at nonzero quarks flavors and chemical potential, see *e.g.* the review [3]. Currently, lattice data have a particular status since they are often used to fit other model's parameters when experimental data are lacking.

Among the existing phenomenological approaches, quasiparticle models rely on the assumption that the quark gluon plasma (QGP) can be seen as a gas containing the relevant hadronic degrees of freedom: deconfined quarks and gluons above the critical temperature, $T_{\rm c}$, but rather a hadron gas before the phase transition. The present work aims at describing the pure glue EoS below $T_{\rm c}$, which has not been as intensively studied as above $T_{\rm c}$.

Below $T_{\rm c}$, the pure glue hadronic matter might be similar to a noninteracting glueball gas — notice that the scattering amplitudes between glueballs scales in $1/N_c^2$ instead of $1/N_c$ for mesons. Following Bose–Einstein statistics, the only relevant contribution to the EoS is thus the one of the

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lowest-lying glueballs because of the statistical suppression in $e^{-m_g/T}$, $m_g$ being the glueball mass. However, by using typical values for the low-lying glueball masses in a glueball gas model, one fails to reproduce the strong increase of the EoS near the phase transition [4,5]. The only proposal leading to a model in agreement with the lattice EoS is so far the one of [5], where a high-lying glueball spectrum of Hagedorn-type is assumed. But, the Hagedorn spectrum relies on a string-theoretical picture of hadrons, *e.g.* closed strings for glueballs. In view of the many models reproducing the lattice spectrum at zero temperature by assuming totally different frameworks, it might be of interest to see whether an alternative way of understanding the early stages of the pure glue phase transition can be found or not. As it will be shown, giving a summary of [6], two ingredients are needed. First, a significant reduction of the glueball masses near $T_{\rm c}$. Second, the nontrivial contribution of the gluon condensate to the trace anomaly, especially its brutal reduction near the phase transition.

## 2. Glueball masses and the pressure

Glueball masses at finite temperature have already been computed on the lattice, see in particular [7]. The basic conclusion of this work is twofold: On one hand, if the temporal glueball correlator is fitted assuming glueball states with zero width, then the $0^{++}$ and $2^{++}$ pole masses are found to significantly decrease approaching $T_{\rm c}$. On the other hand, when a Breit–Wigner fit is chosen, the glueball masses are found to be constant from $T = 0$ to $T_{\rm c}$ with an increasing thermal width near $T_{\rm c}$. Such a behavior might be a general feature of hadrons: Their progressive "dissolution" in the medium near the deconfinement temperature should enhance their width. The pole mass, $m_g(T)$, and the Breit–Wigner mass, $\bar{m}_g(T)$, and thermal width, $\Gamma_g(T)$, are actually linked as follows [7]

$$m_g(T) \approx \bar{m}_g(T) - 2T + \sqrt{4T^2 - \Gamma_g(T)^2}\,. \tag{1}$$

As shown in the left panel of Fig. 1, the pole masses computed in [7] are well described by the form (1) with $\bar{m}_g(T) = m_g^0$ and $\Gamma_g(T) = 0$ for $T$ lower that some temperature $T_g$ but equal to $b_g\,(T - T_g)$ for $T_g < T < T_{\rm c}$. So the thermal broadening of the glueballs generates a pole-mass reduction.

From large $N_c$ arguments, the interactions between glueballs can be neglected in a first approximation. The pressure of a noninteracting gas of glueballs with mass $m_g$ and spin $J_g$ corresponds to that of an ideal Bose–Einstein gas

$$p_g = -\frac{(2J_g+1)T}{2\pi^2}\int_0^\infty dk\, k^2 \ln\left(1 - e^{-\sqrt{k^2+m_g^2}/T}\right)\,. \tag{2}$$

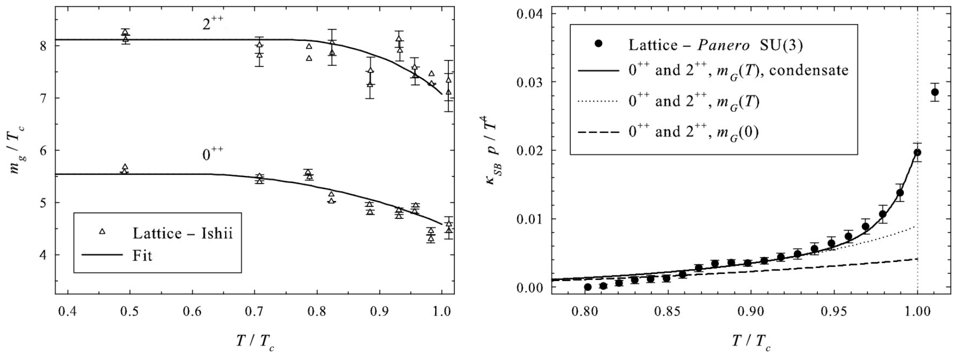


Fig. 1. Left: Scalar and tensor glueball masses computed through a pole-mass fit in [7] (symbols), compared to a fit of the form (1). See [6] for the values of the fit parameters. Right: Pressure computed in pure glue SU(3) lattice QCD and normalized to $\kappa_{\rm SB} = 45/8\pi^2$, taken from [2] (full circles). The lattice data are compared to Eq. (3) using $m_g = m_g^0$ (dashed line), $m_g = m_g(T)$ given by Eq. (1) (dotted line), and the full model in which the term (6) is added (solid line).

In such a framework, only the glueball mass appears in the computation of the EoS, but the effects of the increasing thermal width should be taken into account. We choose here to adopt a procedure that is widespread in quasiparticle models: We consider that all physical effects can be absorbed in a redefinition of the quasiparticle masses. Since a glueball with a constant mass and an increasing thermal decay width can be described effectively as a glueball with a zero width and a decreasing pole mass, the pole mass $m_g(T)$ will be used hereafter.

Starting from Eq. (2), the pressure of the QCD matter below $T_c$ is then given by $p = \sum_g p_g$, the sum running over all the glueball states. A Hagedorn spectrum is not assumed, thus only the lowest-lying glueballs will significantly contribute because the statistical suppression in $e^{-m_g/T}$ is not balanced by the exponentially rising number of states with respect to $m_g$. Moreover, the higher-lying glueballs would contribute if $m_g/T$ was of the order of unity, but $m_g/T_c \gg 1$ even for the $0^{++}$ glueball. We thus take

$$p \approx p_{0^{++}} + p_{2^{++}} \,. \tag{3}$$

Results obtained from this last equation are shown in the right panel of Fig. 1 and compared to the lattice data of [2]. The conclusions are the following. First, using a glueball gas with constant masses fails to reproduce the observed increase of pressure near $T_c$. Second, using the temperature-dependent masses greatly improves the agreement with lattice QCD and is a first argument in favor of the scenario proposed here. Remark that $p(T_c)$ is underestimated; it will be shown in the next section that the contribution coming from the gluon condensate is able to cure this problem. Concerning the higher-lying glueballs, it can be computed that, if the mass reduction mechanism does not cause those glueballs to become lighter than the tensor one, the $0^{-+}$ contribution shifts the pressure of less than 7% while the heavier states contribution is even smaller. Thus only the scalar and tensor glueballs may be considered in a first approximation.

## 3. Gluon condensate and the trace anomaly

The next step is now the computation of the trace anomaly, defined from (3) by

$$\bar{\Delta} = T^5 \partial_T \left( \frac{p}{T^4} \right) . \tag{4}$$

A look at Fig. 2 clearly shows that our model with $m_g(T)$, although satisfactorily reproducing the lattice pressure at low $T$, severely underestimates the trace anomaly near the phase transition. This situation can

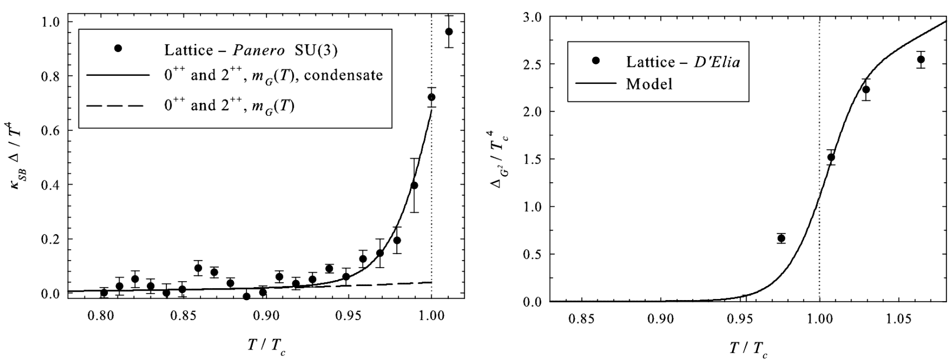


Fig. 2. Left: Same as the right panel of Fig. 1 but for the trace anomaly. The dashed line is the glueball contribution (4), while the solid line comes from Eq. (5). Right: Trace anomaly contribution in which $c_e(T)$ has been taken from the lattice study [9] (full circles). The fitted curve used in our calculations is given for comparison (solid line); its expression can be found in [6].

be clarified by taking into account the nontrivial role of the gluon condensate. It is indeed known that the gluon condensate at temperature $T$, $\langle G^2 \rangle_T = -\langle \frac{\beta}{g} G^a_{\mu\nu} G^{\mu\nu}_a(T) \rangle$, contributes to the QCD trace anomaly as $\Delta_{G^2} = \langle G^2 \rangle_0 - \langle G^2 \rangle_T$ [8]. Thus the total trace anomaly, $\Delta$, should rather be

$$\Delta = \bar{\Delta} + \Delta_{G^2} . \tag{5}$$

Writing the gluon condensate as the sum of a magnetic and an electric part, *i.e.* $\langle G^2 \rangle_T = \langle G^2_e \rangle_T + \langle G^2_m \rangle_T$ in Euclidean space, it appears from lattice QCD simulations that $\langle G^2_m \rangle_T \approx \langle G^2_m \rangle_0$, and that $\langle G^2_e \rangle_T$ is such $\langle G^2_e \rangle_0 \approx \langle G^2 \rangle_0 / 2$

but then falls very quickly near $T_c$ to reach a zero value just after the phase transition [9]. Consequently, one expects $\Delta_{G^2} = \langle G^2\rangle_0[1 - c_e(T)]/2$, where $c_e(T) = \langle G_e^2\rangle_T/\langle G_e^2\rangle_0$ can be known from lattice computations [9].

Since $c_e(T)$ is only known at a few temperatures from the lattice study [9], we have fitted it for convenience, using an ansatz whose explicit form can be found in [6]. A look at the right panel of Fig. 2 shows that the fit is very good, while it can be checked in Fig. 2 that the trace anomaly (5) fits the lattice data very well. Typical values $T_c = 265$ MeV and $\langle G^2\rangle_0 = 0.030$ GeV$^4$ have been used [10]. In our opinion, the agreement between the needed value of the gluon condensate in our model and the one theoretically expected from independent lattice calculations is a relevant check of the mechanism presented here describing the phase transition.

The coherence of our scenario requires the gluon condensate contribution to the pressure to be computed from the thermodynamical relation (4) and then to be added to the glueball pressure. One has thus

$$p_{G^2} = T^4 \int_0^T \frac{\tilde{\Delta}_{G^2}(x)}{x^5}\, dx \,, \tag{6}$$

and

$$p = p_{0^{++}} + p_{2^{++}} + p_{G^2} \,. \tag{7}$$

It is readily observed in the right panel of Fig. 1 that, near $T_c$, the total pressure (7) is no longer underestimated and reaches an excellent agreement with the lattice data.

Since in Ref. [2], to which our model is compared, the energy density as well as the entropy density are obtained as linear combinations of the pressure and trace anomaly following standard thermodynamical relations, it is enough for our purpose to have considered $p$ and $\Delta$.

## 4. Conclusion and large $N_c$ limit

A new way of understanding the pure glue QCD equation of state below $T_c$ has been proposed. The basic idea is that the pure glue hadronic matter consists in both a glueball gas and a gluon condensate part. As a consequence of their increasing thermal decay width, the glueball masses are significantly reduced near $T_c$. This effect has also to be combined with the vanishing of the electric gluon condensate at the critical temperature. In order to illustrate the proposed scenario, lattice data have been used as numerical inputs in the model: The glueball masses and the gluon condensate values have been taken from [4] and [9] respectively. Those data, when incorporated into computations, lead to an equation of state in very good agreement with that computed in Ref. [2], thus providing an *a posteriori* coherent interpretation of various independent existing results in finite-temperature QCD.

Finally, what can be expected in the large $N_c$ limit? Above $T_{\rm c}$, the dominant contribution to the equation of state should come from deconfined gluons, whose $N_c^2$ color degrees of freedom cancel the $\kappa_{\rm SB}$ factor scaling as $1/N_c^2$, leading to globally constant observables with respect to $N_c$ as observed in [2]. Below $T_{\rm c}$, all glueballs are in a color singlet and their masses scale as 1 while the gluon condensate scales as $N_c^2$. The trace anomaly and pressure at large $N_c$ are then expected to read $\Delta \approx \Delta_{G^2}$ and $p \approx p_{G^2}$ respectively in our model. Thus, we predict that the pressure below $T_{\rm c}$ will be strongly suppressed at large $N_c$ while the trace anomaly will decrease only slightly once normalized to $\kappa_{\rm SB}$. It can be hoped that future studies of glueball and glueball condensate properties at finite-temperature and large $N_c$ will allow a more accurate validation of the ideas developed in this work.

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