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ROLE OF MONOPOLES IN A GLUONIC PLASMA*

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The role of color-magnetic monopoles in a pure gauge plasma at high temperature $T > 2T_{\rm c}$ is considered. In this temperature regime, monopoles can be considered heavy, rare objects embedded into matter consisting mostly of the usual "electric" quasiparticles, quarks and gluons. The gluon–monopole scattering is found to hardly influence thermodynamic quantities, yet it produces a large transport cross-section, significantly exceeding that for pQCD gluon–gluon scattering up to quite high T. This mechanism keeps viscosity small enough for hydrodynamics to work at LHC.

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1. Introduction

Creating and studying Quark Gluon Plasma (QGP) in the laboratory has been the goal of experiments at CERN SPS and recently at the Relativistic Heavy Ion Collider (RHIC) facility in Brookhaven National Laboratory, soon to be continued by the ALICE Collaboration at the Large Hadron Collider (LHC). RHIC experiments have revealed robust collective phenomena in the form of radial and elliptic flows, which turned out to be quite accurately described by near-ideal hydrodynamics. QGP thus seems to be the most perfect liquid known, with the smallest viscosity-to-entropy ratio η/s. One of the central questions is how sQGP with "near-perfect fluidity" will change into a weakly coupled wQGP with increasing T. In view of the next round of heavy ion experiments at LHC, a quite urgent question is what transport properties are expected to be observed there, at temperatures reaching about twice those reached at RHIC. To answer this question, one has to understand where the "perfect fluidity" of QGP comes from.

Recently, the *electric-magnetic* duality has been proposed, and used to explain unusual properties of the QGP [1]: in this so-called "magnetic scenario", the near-$T_{\rm c}$ region is dominated by magnetic monopoles. Another line

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of work based on lattice monopoles has led Chernodub and Zakharov [2] at the same time to a very similar conclusion. An important feature of this scenario is the opposite running of the electric coupling e and the magnetic one g, induced by the Dirac condition $eg = \text{const}$. As recently shown in [3], this feature has been dramatically confirmed by the behavior of the lattice correlation functions [4], which indeed display monopole–monopole and antimonopole–monopole correlations *increasing* with T. As shown in [3], those correlations are well described by a picture of classical Coulomb gas. The main input in those calculations is the magnetic Coulomb coupling g *growing* with T. Furthermore, it was shown to be simply the inverse of the gauge coupling e, as Dirac predicted. We consider the correlations observed in [4] to be a decisive confirmation of the existence of the long-distance magnetic Coulomb field of the monopoles. It is natural to investigate the role played by these objects in the QGP [5–10]: we therefore address here the issue of QGP transport properties in the "magnetic scenario" framework. We move away from the phase transition region to higher temperatures, where QGP is still dominated by the usual electric quasiparticles — quarks and gluons — and the coupling is moderately small. Our goal is to study the interaction between electric and magnetic sectors. Our main result is the explicit solution of the problem of quantum gluon–monopole scattering, from which we calculate the corresponding transport cross-sections. This will allow us to get a temperature-dependent estimate of the ratio η/s.

2. Quantum gluon–monopole scattering

The problem of quantum gluon–monopole scattering is solved in the point-like monopole approximation. In this case, and for $j \neq 0$, the equations for the radial functions $T_{j\alpha}(\xi)$ reduce to generalized Bessel-like equations with noninteger index $j' = -1/2\,[-1 + \sqrt{(2j+1)^2 - 4n^2}]$, where $n = eg$ is the product of electric and magnetic couplings and j is the total angular momentum quantum number[1]:

$$T''_{j\alpha}(\xi) - \left[-\omega^2 + 1 + \frac{j(j+1) - n^2}{\xi^2}\right] T_{j\alpha}(\xi) = 0\,. \tag{1}$$

The index α runs from 1 to 9 and indicates all possible combinations of charge and spin polarization for gluons. After gauge fixing, three combinations turn out to be unphysical, and only six survive. The radial solution of the gluon–monopole scattering is easy; the complications reside in the angular functions. In fact, classically the gluon moves on the surface of a cone;

[1] The point-like monopole approximation is justified by the information about the monopole size that we obtain from the lattice, which indicate a monopole radius of ~ 0.15 fm [11].

the angular functions therefore are modified vector spherical harmonics that describe the conical motion in the classical limit of large angular momentum. The scattering phase that we obtain from Eq. (1) is $\delta_{j'} = -j'\frac{\pi}{2}$, independent of energy. This feature is very important, since the contribution of this kind of scattering to thermodynamics is given by the Beth–Uhlenbeck formula

$$\delta M_m = \frac{T}{\pi}\sum_j (2j+1)\int dk \frac{d\delta_j}{dk} f(k,T) \tag{2}$$

which vanishes identically for a constant scattering phase. Therefore, we find that the gluon–monopole scattering does not contribute to thermodynamics. There is an exception to this result, for $j = 0$. In this case, the gluon can penetrate the monopole core and form bound states. We do not discuss this case here, for all details we refer the reader to Ref. [5].

3. Transport cross-section: results and conclusions

The scattering amplitude $f(\theta)$ is given by the following formula:

$$2ikf(\theta)_{n,\nu} = \sum_{j=|\nu|}^{j_{\max}} (2j+1)e^{i\pi(j'-j)} d^{(j)}_{\nu,-\nu}(\theta)\,, \tag{3}$$

where $\nu = n + \sigma = \left(\vec{T}\cdot\hat{r}\right) + \left(\vec{S}\cdot\hat{r}\right) = -J_3$. The sum over j has an upper cutoff $j_{\max}$: in matter there is a finite density of monopoles. A sketch of the setting, assuming strong correlation of monopoles into a crystal-like structure, is shown in the left panel of Fig. 1. A "sphere of influence of one monopole" (the dotted circle) gives the maximal impact parameter to be used. As a result, the impact parameter is limited from above by some $b_{\max}$, which implies that only a finite number of partial waves should be included. The range of partial waves to be included in the scattering amplitude can be estimated as $j_{\max} = \langle p_x\rangle n_m^{-1/3}/2 \sim aT \sim 1/e^2(T) \sim \log(T)$. Since at asymptotically high T the monopole density $n_m \sim (e^2T)^3$ is small compared to the density of quarks and gluons $\sim T^3$, $j_{\max}$ asymptotically grows logarithmically with T. So, only in the academic limit $T \to \infty$ one gets $j_{\max} \to \infty$ and the usual free-space scattering amplitudes calculated in [12] where all partial waves are recovered. However, in reality we have to recalculate the scattering, retaining only several lowest partial waves from the sum. Taking the lattice results on the monopole density as a function of the temperature [4], we estimate $j_{\max} \simeq 6$ in our temperature regime. This dramatically changes the angular distribution, by strongly depleting scattering at small angles and enhancing scattering backwards. This is evident

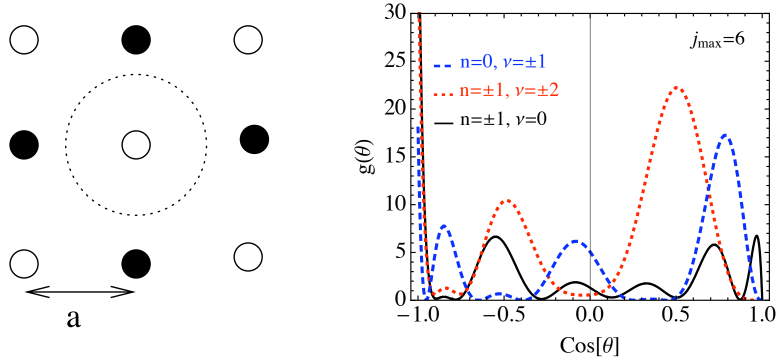

Fig. 1. Left: A charge scattering on a 2-dimensional array of correlated monopoles (open points) and antimonopoles (closed points). The dotted circle indicates a region of impact parameters for which scattering on a single monopole is a reasonable approximation. Right: Integrand of the transport cross-section $g(\theta) = (1-\cos(\theta))|f(\theta)|^2$ with only 6 lowest partial waves included, for a gluon with $n=0,\ \nu=\pm1$, $n=\pm1,\ \nu=0$ and $n=\pm1,\ \nu=\pm2$. The strong peak backwards is due to the presence of the cutoff $j_{\max}$.

in the right panel of Fig. 1, where we show the angular distribution of the integrand of the transport cross-section $\sigma_{\rm t}$:

$$(\sigma_{\rm t})_{n,\nu} = \int_{-1}^{1} d\cos\theta(1-\cos\theta)|f(\theta)_{n,\nu}|^2 \,. \tag{4}$$

The integrand exhibits a strong peak backwards, which would disappear in the absence of $j_{\max}$.

We now proceed to evaluate the scattering rate of gluons on monopoles:

$$\frac{\dot{w}_{gm}}{T} = \frac{\langle n_m(\sigma_{\rm t})_{gm}\rangle}{T}\,, \tag{5}$$

where the $\langle ...\rangle$ indicates an average over the incoming gluon. The gluon density has the following form:

$$\begin{aligned} n_g(T) &= \frac{8\pi}{(2\pi)^3}\int k^2 dk \left[\frac{2}{\exp{(\beta\epsilon_k)}-1} + \frac{2}{\exp{(\beta\epsilon_k)}\exp{(i\beta\mathcal{A}_0^3)}-1}\right. \\ &\quad + \frac{2}{\exp{(\beta\epsilon_k)}\exp{(-i\beta\mathcal{A}_0^3)}-1} + \frac{1}{\exp{(\beta\epsilon_k)}\exp{(2i\beta\mathcal{A}_0^3)}-1} \\ &\quad \left. + \frac{1}{\exp{(\beta\epsilon_k)}\exp{(-2i\beta\mathcal{A}_0^3)}-1}\right] = \frac{4\pi}{(2\pi)^3}\int k^2 dk \rho_g(k,T)\,, \end{aligned} \tag{6}$$

where we have taken into account the suppression of electric particles due to the coupling with the Polyakov loop (see for example [13]). In the average over the incoming gluon, we have to take this suppression into account by integrating over $\vec{k}$ with the weight $\rho_g(k,T)$. We show $\dot{w}_{gm}/T$ in the left panel of Fig. 2 (the red, continuous line). Also shown is the same quantity for the gg scattering process (black, dotted line). The approximate relation of the scattering rate to viscosity/entropy ratio is $(\eta/s) \approx (T/5\dot{w})$. We plot η/s in the right panel of Fig. 2. We observe a qualitative agreement between our results and the experimental value for η/s observed at RHIC, which is indicated in the right panel of Fig. 2 as a box (green on line). Our present results, however, deal with the purely gluonic sector of QCD only. For a more quantitative and meaningful comparison with RHIC results, quarks need to be incorporated in the analysis. Our main finding is that the contribution of gluon–monopole scattering is very important for transport properties. While the monopole density may be small, the gm scattering amplitudes have $e^2g^2 \sim O(1)$ coupling instead of small $e^4 \ll 1$. Furthermore, in our setting (with a limited number of partial waves $j < j_{\max}$ included) there is an additional enhancement for large angle (or even backward) scattering. It follows from this comparison of the gluon–monopole curve with the gluon–gluon one that the former remains the leading effect till very high T, although asymptotically it is expected to get subleading. The maximal T expected at LHC does not exceed $4T_c$, where the total $\eta/s \sim 0.15$. This value is well in the region which would ensure hydrodynamical radial and elliptic flows, although deviations from ideal hydro would be larger than at RHIC (and measurable!).

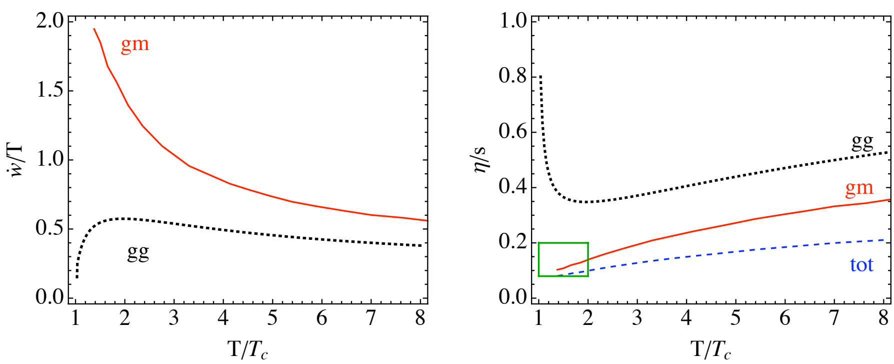

Fig. 2. Left panel: gluon–monopole and gluon–gluon scattering rate. Right panel: gluon–monopole and gluon–gluon viscosity over entropy ratio, η/s. The (blue) dashed curve is the total η/s, which is evaluated from the gg and gm contributions. The box (green) represents the present estimate of η/s in the RHIC temperature regime.

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