PARTONS IN A STRONGLY COUPLED PLASMA FROM AdS/CFT*

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I describe the parton picture at strong coupling emerging from the gauge/gravity duality and some of its phenomenological consequences for high-energy scattering. I focus on the hard probes of a strongly coupled plasma, as potentially relevant for heavy collisions at RHIC and LHC.

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1. Motivation: Jet quenching at RHIC

Some of the experimental discoveries at RHIC, notably the unexpectedly large medium effects known as elliptic flow and jet quenching, led to the suggestion that the deconfined QCD matter produced in the intermediate stages of a heavy ion collision might be strongly interacting.

The phenomenon of jet quenching is particularly intriguing in that sense, as this probes the hadronic matter on a relatively hard scale (a few GeV), where QCD is *a priori* expected to be weakly coupled, by asymptotic freedom. One manifestation of this phenomenon is the 'away jet suppression' in Au + Au collisions, as shown in Fig. 1, left: unlike in $p+p$ or $d+\mathrm{Au}$ collisions, where the hard particles typically emerge from the collision region as pairs of back-to-back jets, in the Au + Au collisions one sees 'mono-jet' events in which the second jet is missing. This has the following natural interpretation (see Fig. 1, right): the hard scattering producing the jets has occurred near the edge of the interaction region, so that one of the jets has escaped and triggered a detector, while the other one has been deflected, or absorbed, via interactions in the surrounding medium.

If this medium is composed of (weakly interacting) quasiparticles (quarks and gluons), then the deflection of the hard jet is due to its successive scattering off these quasiparticles, as illustrated in Fig. 2, left. This leads to the

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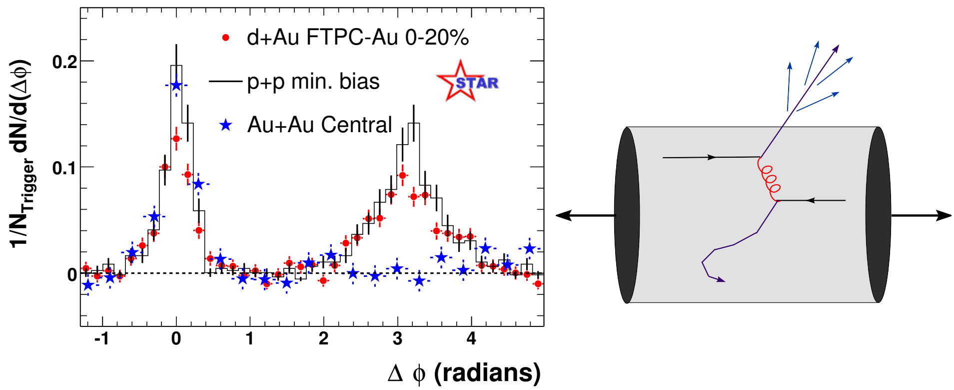

Fig. 1. Left: Azimuthal correlations for jet measurements at RHIC (STAR) in $p+p$, $d+$Au, and Au + Au collisions. Right: Jet production in a nucleus–nucleus collision.

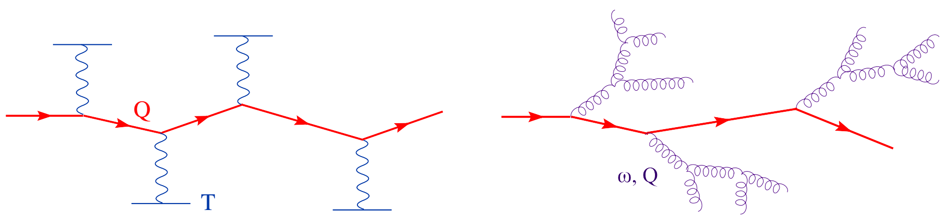

Fig. 2. Transverse momentum broadening for a heavy quark which propagates through a quark–gluon plasma. Left: weak coupling (successive scattering off thermal quasiparticles). Right: strong coupling (medium induced branching).

following estimate for the rate of transverse momentum broadening

$$\hat{q} \equiv \frac{d\left\langle k_\perp^2\right\rangle}{dt} \simeq \alpha_{\rm s} N_c \mathcal{G}\left(x, Q^2\right) , \tag{1}$$

where $\mathcal{G}(x, Q^2)$ is the gluon distribution in the medium on the resolution scale $Q^2 \sim \langle k_\perp^2 \rangle$ of the hard jet, as produced via the quantum evolution of the quasiparticles from their intrinsic energy scale to the hard scale Q. For instance, if we assume the medium to be a finite-temperature plasma with temperature T, then $\mathcal{G} \simeq n_q(T)\,\mathcal{G}_q + n_g(T)\,\mathcal{G}_g$, where $n_{q,g}(T) \propto T^3$ are the quark and gluon densities in thermal equilibrium and $\mathcal{G}_{q,g}(x, Q^2)$ are the gluon distributions produced by a single quark, respectively gluon, on the scale $Q \gg T$. Some typical values are $Q \sim 2 \div 10$ GeV and $T \sim 0.4$ GeV. Assuming weak coupling, it is straightforward to evaluate all these quantities within perturbative QCD. But by doing that, one finds an estimate $\hat{q}_{\rm pQCD} \simeq 0.5 \div 1$ GeV^2/fm, which is one order of magnitude smaller then the value extracted from the RHIC data!

This discrepancy suggests that the actual gluon distribution in the plasma is significantly larger than expected in pQCD. A possible explanation for that is a stronger value for the coupling, which would enhance the quantum evolution from T up to Q. Note that there is not necessarily a conflict with asymptotic freedom: to get an enhanced gluon distribution on the relatively hard scale Q, it is enough to have a stronger coupling at the lower end of the evolution, that is, at the relatively soft scale T. We have indeed $g(T) \sim 2$ for the temperatures T of interest at RHIC and LHC. Actually, it should be possible to study some aspects of this evolution in lattice QCD at finite temperature, and thus verify the hypothesis of strong coupling [2].

2. DIS and parton picture at strong coupling

The previous discussion invites us to a better understanding of parton evolution in deconfined QCD matter at strong coupling, that is, for $\alpha_{\rm s} \equiv g^2/4\pi \simeq 1$. However, even without the complications of confinement, the QCD calculations at strong coupling remain notoriously difficult. (In particular, lattice QCD cannot be used for real-time phenomena so like scattering.) So it has become common practice to look to the $\mathcal{N}=4$ supersymmetric Yang–Mills (SYM) theory, whose strong coupling regime can be addressed within the AdS/CFT correspondence, for guidance as to general properties of strongly coupled plasmas (see the review papers [3–5]).

$\mathcal{N}=4$ SYM has the 'color' gauge symmetry $\mathrm{SU}(N_c)$, so like QCD, but differs from the latter in some other aspects: it is maximally supersymmetric, and hence conformal (the coupling g is fixed), and all the fields in its Lagrangian (gluons, scalars, and fermions) transform in the adjoint representation of $\mathrm{SU}(N_c)$. But these differences are believed not to be essential for a study of the quark–gluon plasma phase of QCD in the temperature range of interest for heavy ion collisions at RHIC and LHC, that is, $2T_{\rm c} \lesssim T \lesssim 5T_{\rm c}$ with $T_{\rm c} \simeq 170$ MeV the critical temperature for deconfinement.

The AdS/CFT correspondence is the statement that the conformal field theory (CFT) $\mathcal{N}=4$ SYM is 'dual' (*i.e.*, equivalent) to a string theory in a $(9+1)$-dimensional space time with $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry. This equivalence is conjectured to hold for arbitrary values of the parameters g and N_c, but in practice this is useful only in the strong 't Hooft coupling limit $\lambda \equiv g^2 N_c \to \infty$ with $g \ll 1$, in which the string theory becomes tractable. This is generally not a good limit for studying scattering, since the amplitude is suppressed as $1/N_c^2$ [6, 7]. Yet, this is meaningful for processes taking place in a deconfined plasma, which involves N_c^2 degrees of freedom per unit volume, thus yielding finite amplitudes when $N_c \to \infty$. In this limit, the $\mathcal{N}=4$ SYM plasma at finite temperature is described as a black-hole (BH) embedded in AdS_5 and the string theory dynamics reduces to classical gravity in this curved space-time [3–5]. The BH is homogeneous in the physical 4 dimensions but has an horizon in the radial dimension of AdS_5 (denoted as χ in Fig. 3), at a distance $\chi = 1/T$ away from the Minkowski boundary ($\chi = 0$) where lives the 4-dimensional gauge theory.

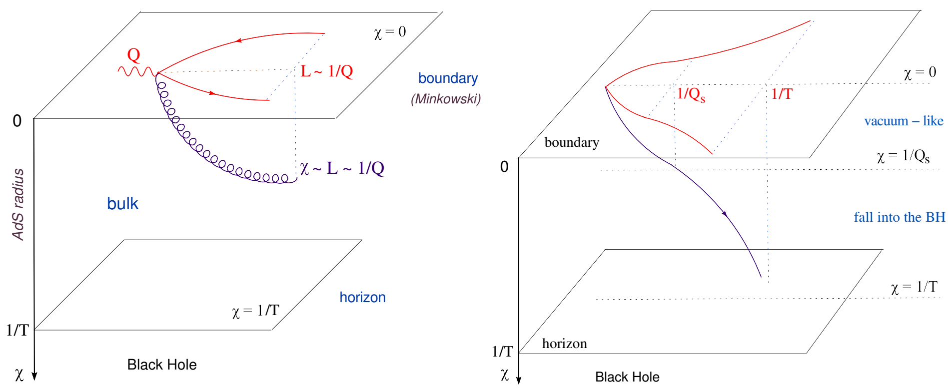

Fig. 3. Space-like current in the plasma: the trajectory of the wave packet in AdS_5 and its 'shadow' on the boundary. Left: low energy — the Maxwell wave gets stuck near the boundary. Right: high energy — the wave falls into the BH.

As explained in Section 1, we would like to study the response of the strongly-coupled plasma to a 'hard probe' with high energy and transverse resolution scale $Q \gg T$. The simplest such a probe is a virtual photon undergoing deep inelastic scattering (DIS) off the plasma. The respective cross-section (the 'structure function' $F_2(x, Q^2)$) is a direct measure of the parton distributions in the plasma. Thus its calculation within AdS/CFT gives us information about parton evolution at strong coupling [6, 7].

The dual, supergravity, picture of DIS is illustrated in Fig. 3: a space-like photon, with 4-momentum $q^\mu = (\omega, 0, 0, q)$ and virtuality $Q^2 \equiv q^2 - \omega^2 \gg T^2$, acts as a perturbation on the Minkowski boundary of AdS_5 ($\chi = 0$), thus inducing a massless, vector, supergravity field A_m (with $m = \mu$ or χ) which propagates towards the bulk of AdS_5 ($\chi > 0$), according to Maxwell equations in curved space-time

$$\partial_m \left(\sqrt{-g} g^{mp} g^{nq} F_{pq}\right) = 0 \,, \qquad \text{where} \qquad F_{mn} = \partial_m A_n - \partial_n A_m \,. \qquad (2)$$

These equations describe the gravitational interaction between the Maxwell field A_m and the BH (implicit in the 5-dimensional metric tensor g^{mn}). Note that there is no explicit coupling constant in the equations: the gravitational scattering is fully controlled by the kinematics. Specifically, there is a competition between a *repulsive* potential $\propto Q^2$ which opposes to the penetration of the Maxwell field in AdS_5 and an *attractive* one $\propto \omega^2 T^4$, representing the gravitational attraction by the BH. For relatively low energy, or high Q^2, such that $\omega T^2 \ll Q^3$, the repulsion wins and the Maxwell wave remains stuck near the Minkowski boundary, within a distance $\chi \lesssim 1/Q \ll 1/T$ (*cf.* Fig. 3, left). For higher energies, or lower Q^2, such that $\omega T^2 \gtrsim Q^3$, the attraction wins and the wave falls into the BH (*cf.* Fig. 3, right).

The physical interpretation of this dynamics back in the original gauge theory can be inferred with the help of the 'UV/IR correspondence', which states that the radial penetration χ of the wave packet in AdS$_5$ is proportional to the transverse size L of the quantum fluctuation of the virtual photon in the dual gauge theory. Then the repulsive potential alluded to above is merely a manifestation of the energy-momentum conservation together with the uncertainty principle: a space-like photon cannot decay into on-shell quanta, but only fluctuate into a virtual partonic system with transverse size $L \sim 1/Q$ and lifetime $\Delta t \sim \omega/Q^2$. The gauge-theory meaning of the gravitational attraction is more subtle: unlike the photon, which is color neutral, its partonic fluctuation has a dipolar color moment and thus it can interact with the plasma. Via such interactions, the partons can acquire the energy and momentum necessary to get (nearly) on-shell; when this happens, the fluctuation decays — or, more precisely, it *thermalizes*: the partons become a part of the thermal bath.

The critical value $Q_{\rm s} \sim (\omega T^2)^{1/3}$ ('saturation momentum') separating between the two regimes can be understood as follows: when $Q \sim (\omega/Q^2)T^2$, the lifetime $\Delta t \sim \omega/Q^2$ of the partonic fluctuation becomes large enough for the mechanical work $W = \Delta t \times F_T$ done by the *plasma force* $F_T \sim T^2$ to compensate the energy deficit $\sim Q$ of the space-like system. This plasma force represents in an average way the effect of the strongly-coupled plasma on the color dipole fluctuations [8].

Introducing the Bjorken-x variable $x \equiv Q^2/(2\omega T)$, which represents the longitudinal momentum fraction of the plasma constituent struck by the virtual photon, one can rewrite the plasma *saturation line* as $Q_{\rm s}(x) = T/x$ or, alternatively, $x_{\rm s}(Q) = T/Q$. (Note that $x_{\rm s}(Q) \ll 1$.) Then the AdS/CFT results can be summarized as follows: For $Q \gg Q_{\rm s}(x)$ (or, equivalently, $x \gg x_{\rm s}(Q)$), the scattering is negligible and the DIS structure function F_2 is essentially zero[1]. For $x \lesssim x_{\rm s}(Q)$, the scattering is strong and the structure function is parametrically large: $F_2(x, Q^2) \sim x N_c^2 Q^2$. This result is consistent with energy-momentum conservation, which requires the integral $\int_0^1 dx\, F_2(x, Q^2)$ to have a finite limit as $Q^2 \to \infty$. We have indeed

$$\int_0^1 dx\, F_2\left(x, Q^2\right) \simeq x_{\rm s} F_2\left(x_{\rm s}, Q^2\right) \sim N_c^2 T^2 \,, \qquad (3)$$

where the integral is dominated by $x \simeq x_{\rm s}(Q)$.

[1] More precisely, it is exponentially small, $F_2(x, Q^2) \sim \exp\{-Q/Q_{\rm s}(x)\}$, since in this high-$Q^2$ regime the scattering proceeds via tunneling through the repulsive potential.

When interpreted in the plasma infinite-momentum frame (IMF), these results suggest a *partonic picture* for the strongly coupled plasma [8]. As usual, the partons are virtual quanta (here, of $\mathcal{N} = 4$ SYM) whose lifetime is enhanced by the Lorentz boost to the IMF. However, unlike what happens at weak coupling, where partons exist on all scales of x and Q^2, at strong coupling they exist only for sufficiently small values of x (for a given Q^2), namely for $x \lesssim x_{\rm s}(Q) \ll 1$. This is a consequence of the *parton branching*, which becomes very efficient at strong coupling: through successive branchings, the partons with large x and high Q^2 dissociate into softer partons, with lower and lower values of x and Q^2, down to the smallest values of x consistent with energy conservation, *cf.* Eq. (3). This branching process is 'quasi-democratic': at each splitting, the energy is almost equally divided among the daughter partons, so that there are no partons surviving at high Q^2 and/or large x (no pointlike constituents). The total energy is rather carried by the small-x partons, in fact, mostly by those along the saturation line. This picture of parton evolution is very different from the one prevailing at weak coupling [9]: the latter proceeds predominantly via the emission of soft gluons, which carry only a small fraction of the longitudinal momentum of their parent partons; this leads to a rapid rise in the number of gluons at small x, and eventually to gluon saturation, but most of the total energy is still carried by the few remaining partons at large x.

The saturation regimes also are quite different at weak and, respectively, strong coupling. At weak coupling, the gluon occupation numbers at saturation are large, of $\mathcal{O}(1/\alpha_{\rm s})$, and the respective saturation momentum grows rather slowly with $1/x$: $Q_{\rm s}^2 \sim 1/x^{\omega}$ where $\omega \sim \mathcal{O}(\alpha_{\rm s})$ is determined by the BFKL dynamics; its numerical value, $\omega \simeq 0.2 \div 0.3$, agrees quite well with the experimental data for DIS at HERA [9]. By contrast, at strong coupling the parton occupation numbers at saturation are of $\mathcal{O}(1)$ and the saturation momentum grows much faster with $1/x$: $Q_{\rm s}^2(x) \propto 1/x^2$ for the infinite plasma and $Q_{\rm s}^2(x) \propto 1/x$ for a finite-size 'hadron' (a glueball or a 'nuclear' shockwave) [7,11]. On the supergravity side, this rapid growth is associated with graviton exchanges and the fact that graviton has spin $j = 2$. From the viewpoint of the gauge theory, this can be understood within the operator product expansion for DIS [2,6]: the only leading-twist operator to survive in OPE at strong coupling is the energy-momentum tensor.

3. High-energy scattering at strong coupling

The parton picture previously described has important consequences for the high-energy processes at strong coupling, which should look quite different from what we normally see in QCD. For instance, the absence of large-x partons means that, in a hypothetical scattering between two strongly coupled hadrons, there should be no particle production at forward and backward rapidities. This is in sharp contrast to the situation at RHIC, where the large-x partons from the incoming nuclei are seen to emerge from the collision, as hadronic jets, along their original trajectories.

A related prediction of AdS/CFT is the absence of jets in electron–positron annihilation at strong coupling [8, 10]. Fig. 4 exhibits the typical, 2-jet, final state in e^+e^- annihilation at weak coupling (left) together with what should be the corresponding state at strong coupling (right). In both cases, the final state is produced via the decay of a time-like photon into a pair of partons and the subsequent evolution of this pair. At weak coupling this evolution typically involves the emission of soft and collinear gluons, with the result that the leading partons get dressed into a pair of well-collimated jets of hadrons (*cf.* Fig. 4, left). At strong coupling, parton branching is much more efficient, as previously explained, and rapidly leads to a system of numerous and relatively soft quanta, with energies and momenta of the order of the soft, confinement, scale, which are isotropically distributed in space (*cf.* Fig. 4, right) [10].

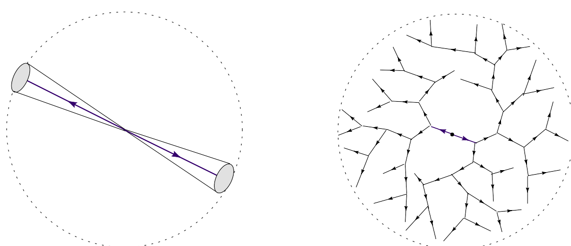

Fig. 4. e^+e^- annihilation. Left: weak coupling. Right: strong coupling.

We finally return to the heavy-ion problem which motivated our excursion through strong coupling: the propagation of a 'hard probe' through a strongly-coupled plasma. Consider *e.g.* the jet quenching of a heavy quark. The respective AdS/CFT calculations have been given in Refs. [12, 13] and the physical interpretation of the results [14] turns out to be quite interesting: unlike in perturbative QCD, the dominant mechanism at work is not thermal rescattering (*cf.* Fig. 2), but rather *medium-induced parton branching* (*cf.* Fig. 2, right). The heavy quark continuously emits and absorbs virtual quanta of $\mathcal{N} = 4$ SYM; some of these quanta — namely those having a virtuality Q lower than the plasma saturation momentum $Q_\mathrm{s}(x)$ on the relevant scale of x — can escape to the plasma, thus providing both energy loss and momentum broadening.

In fact, using the physical picture developed so far, one can easily estimate the rate for energy loss as follows:

$$-\frac{dE}{dt} \simeq \sqrt{\lambda}\,\frac{\omega}{(\omega/Q_\mathrm{s}^2)} \simeq \sqrt{\lambda}\,Q_\mathrm{s}^2 \sim \sqrt{\lambda}\,\gamma T^2\,, \tag{4}$$

where ω and $Q_{\rm s}$ are the energy and virtuality of an emitted quanta which is absorbed in the plasma, $\Delta t \sim \omega/Q_{\rm s}^2$ is the emission time, and γ is the Lorentz factor for the heavy quark. The factor $\sqrt{\lambda}$ expresses the fact that, at strong coupling, the heavy quark radiates a large number of quanta, $\sim \sqrt{\lambda}$, in the time interval Δt. One can similarly estimate the momentum broadening: the $\sqrt{\lambda}$ quanta emitted during Δt are uncorrelated with each other, so they randomly modify the transverse momentum of the heavy quark, thus yielding

$$\frac{d\langle p_\perp^2\rangle}{dt} \sim \frac{\sqrt{\lambda}\,Q_{\rm s}^2}{(\omega/Q_{\rm s}^2)} \sim \sqrt{\lambda}\,\frac{Q_{\rm s}^4}{\gamma Q_{\rm s}} \sim \sqrt{\lambda}\,\sqrt{\gamma}\,T^3\,. \tag{5}$$

Eqs. (4) and (5) are consistent with the respective AdS/CFT results [12, 13]. Note the strong enhancement of the medium effects at high energy, as expressed by the Lorentz γ factor: this is in qualitative agreement with the strong suppression of particle production seen in Au + Au collisions at RHIC. However, one should keep in mind the intrinsic limitations of the AdS/CFT approach whenever comparing its predictions to the QCD phenomenology.

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