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NONLINEAR $k_\perp$-FACTORIZATION AND THE UNINTEGRATED GLUE OF A NUCLEUS*

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In hard processes on heavy nuclei, multiple gluon exchanges are enhanced by the size of the target. As a consequence, the familiar linear $k_\perp$-factorization is broken, and must be replaced by a new, nonlinear $k_\perp$-factorization, where hard scattering observables are in general nonlinear functionals of a properly defined nuclear unintegrated glue. We discuss the small-x evolution properties of the nuclear glue as well as of quasielastic cross-sections. In addition, we discuss how photon-jet correlations depend on the nuclear unintegrated glue.

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1. Dipole scattering amplitude defines the unintegrated glue

Small-x hard processes, like deep inelastic scattering on a heavy, opaque, nucleus differ in an essential way from their well studied counterparts on the free nucleon described in textbooks. Namely, for strongly absorbing targets, diffractive processes, where the target is left intact(!), will make up for 50% of all deep inelastic interactions [1]. Recent calculations [2] for realistic nuclei find a substantial $\sim 30\%$ ratio of diffractive to inclusive DIS.

The presence of diffractive final states is, via unitarity, intimately related to nuclear shadowing of nuclear structure functions, and when it is large it means that we must give up the standard treatment of deep inelastic scattering in terms of parton densities which involve two partons in the t-channel of the forward Compton amplitude.

It is a great advantage of the color-dipole approach to small-x interactions, that it allows us to impose the unitarity constraints, and to account for the related multiple scattering effects, in a simple and direct way.

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In the (interacting) two-gluon exchange approximation, it is well known, that the color-dipole formalism is equivalent to the so-called (linear) $k_\perp$-factorization in momentum space. The main ingredient of the latter is the unintegrated gluon density

$$f(x,\boldsymbol{p}) = \frac{4\pi\alpha_{\rm S}}{N_c}\frac{1}{\boldsymbol{p}^4}\frac{\partial G\left(x,\boldsymbol{p}^2\right)}{\partial\log(\boldsymbol{p})}\,, \tag{1}$$

which encodes, in momentum space, the infromation contained in the color-dipole cross-section. We generalize the concept of an unintegrated gluon distribution of a nucleus, via the Fourier-transform of the scattering amplitude $\Gamma(\boldsymbol{b},x,\boldsymbol{r})$ of a color-dipole of size $\boldsymbol{r}$ at impact parameter $\boldsymbol{b}$

$$\int\frac{d^2\boldsymbol{r}}{(2\pi)^2}\Gamma(\boldsymbol{b},x,\boldsymbol{r})e^{-i\boldsymbol{p}\boldsymbol{r}} \;=\; (1-w_0(\boldsymbol{b},x))\delta^{(2)}(\boldsymbol{p}) - \phi(\boldsymbol{b},x,\boldsymbol{p})\,, \tag{2}$$

for the explicit form of w_0, which has the meaning of Bjorken's gap survival probability, see *e.g.* [3]. It is now important to realize, that after having imposed unitarity constraints by including multiple scattering effects into the unintegrated glue, we have to give up the idea of universality of the unintegrated gluon distribution to a certain extent [3,4]. Namely, we cannot evaluate observables by simply plugging the above defined unintegrated glue into linear $k_\perp$-factorization formulas, but instead a new, nonlinear $k_\perp$-factorization emerges [3,4].

Starting at moderately small $x \sim x_A \sim 0.01$, where only the $q\bar{q}$ state is coherent over the whole nuclear size, one can construct the nuclear glue by recognising the simple Glauber–Gribov form of the dipole amplitude

$$\Gamma(\boldsymbol{b},x_A,\boldsymbol{r}) = 1 - \exp[-\sigma(x_A,\boldsymbol{r})T_A(\boldsymbol{b})/2]\,, \tag{3}$$

where $T_A(\boldsymbol{b})$ is the nuclear matter density. The nuclear unintegrated glue can then be expressed as an expansion over multiple convolutions of the free-nucleon unintegrated glue $f(x,\boldsymbol{p})$ [5]

$$\phi(\boldsymbol{b},x_A,\boldsymbol{p}) = \sum w_j(\nu_A)f^{(j)}(x_A,\boldsymbol{p})\,. \tag{4}$$

Here

$$f^{(j)}(x_A,\boldsymbol{p}) = \int\left[\prod^{j} d^2\boldsymbol{\kappa}_i f(x_A,\boldsymbol{\kappa}_i)\right]\delta^{(2)}\left(\boldsymbol{p}-\sum\boldsymbol{\kappa}_i\right)$$

is the collective glue of j overlapping nucleons, and

$$\nu_A = \nu_A(\boldsymbol{b},x_A) = \tfrac{1}{2}\alpha_{\rm S}\left(q^2\right)\,\sigma_0(x_A)T_A(\boldsymbol{b})\,, \tag{5}$$

is an effective opacity; the explicit form of the weights w_j will again be found *e.g.* in [5], and $\sigma_0(x) = \int d^2\boldsymbol{p} f(x,\boldsymbol{p})$ is a nonperturbative parameter, which has the meaning of the cross-section of a large color-dipole.

At very small $x \ll x_A$ one has to include the effect of higher $q\bar{q}g\ldots g$ Fock states, and following [7], they can be absorbed into the small-x evolution of the dipole amplitude/unintegrated glue. For the nuclear target the emerging evolution equation is the nonlinear Balitsky–Kovchegov (BK) equation [8], which in the form of [9] reads

$$\frac{\partial \phi(\nu_A, x, \boldsymbol{p})}{\partial \log(1/x)} = \mathcal{K}_{\rm BFKL} \otimes \phi\,(\nu_A, x, \boldsymbol{p}) + \mathcal{Q}\,[\phi]\,(\nu_A, x, \boldsymbol{p})\,, \tag{6}$$

with a linear, BFKL, piece, and a quadratic "gluon fusion" term $\mathcal{Q}[\phi]$.

When solving the evolution equation, Eq. (6), one integrates over all transverse momenta, some regularization in the infrared domain is therefore inevitable. We use a running coupling $\alpha_{\rm S}$ which freezes at small momenta at a value of 0.8. Furthermore, a finite gluon correlation radius μ_G^{-1}, is introduced to remove unphysical long-range gluon exchange contributions. In distinction to sharp cutoffs in momentum space found elsewhere in the literature, our way of introducing the correlation radius (see the first Ref. of [7]) does not upset gauge cancellations in the kernel. In Fig. 1, we show numerical results from a solution of the regularized BK equation. The boundary condition at $x_A = 0.01$ was constructed in terms of a free-nucleon glue fitted to HERA data. We used $\mu_G^2 = 0.5\,\mathrm{GeV}^2$. The plotted quantity $\boldsymbol{p}^2\phi(\nu_A, x, \boldsymbol{p}) \propto \partial G/\partial \boldsymbol{p}^2$ is propotional to the phase space density of gluons. The position of the maximum in Fig. 1 defines the saturation scale. The latter is a function of the opacity as well as of x. It increases for more central collisions (where absorption is stronger) as well as for smaller x.

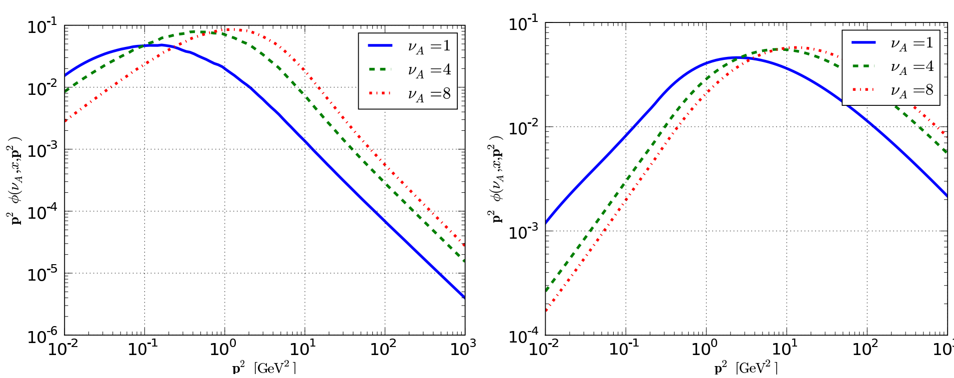

Fig. 1. Unintegrated gluon distribution $\boldsymbol{p}^2\phi(\nu_A, x, \boldsymbol{p})$ for different opacities ν_A. Left: $x = 0.01$; Right: after evolution at $x = 10^{-6}$.

2. Incoherent diffraction off nuclei and its small-x evolution

We now turn to the small-x evolution of incoherent diffractive production of vector mesons on a nucleus. The usual, coherent diffraction on a nucleus is so sharply forward peaked, that its measurement is experimentally quite challenging. At larger momentum transfers $\Delta^2 \gg R_A^{-2}$, an important final state is the so-called incoherent diffractive production, where the final state in the nuclear consist of excited nuclear states and/or fragments from the nuclear breakup, but no new particles (pions *etc.*) are produced (see Fig. 2). The differential cross-section for the $\gamma^* \to V$ transition via the $q\bar{q}$-loop takes the form

$$16\pi \frac{d\sigma_{\rm inc}}{d\Delta^2} = \sum_n \frac{1}{n!} \int d^2\boldsymbol{b} T_A^n(\boldsymbol{b}) p_n(\Delta) \left| \langle V | \Omega^{(n)}(\boldsymbol{b},\boldsymbol{r}) | \gamma^* \rangle \right|^2 . \tag{7}$$

Here the $p_n(\Delta)$ are calculable in terms of the Δ-dependence of diffraction on the free nucleon, their precise form is not important here. Now, upon taking account of higher $q\bar{q}g\dots g$ Fock states, the functions $\Omega^n = \sigma^n(\boldsymbol{r})\exp[-\sigma(\boldsymbol{r})T_A(\boldsymbol{b})/2]$ become dependent on x. They fulfill a novel set of evolution equations, which are coupled with the BK-equation for the $q\bar{q}$ S-matrix. At intermediate Δ^2 the absorbed single scattering term $\Omega^{(1)}$ dominates; its x-dependence is obtained from solving the equations

$$\begin{aligned}
\frac{\partial S_{q\bar{q}}(x,\boldsymbol{r},\boldsymbol{b})}{\partial \log(1/x)} &= \int d^2\boldsymbol{\rho}\, K(\boldsymbol{\rho},\boldsymbol{\rho}+\boldsymbol{r}) \Big[S_{q\bar{q}}(x,\boldsymbol{\rho}+\boldsymbol{r},\boldsymbol{b}) S_{q\bar{q}}(x,\boldsymbol{\rho},\boldsymbol{b}) - S_{q\bar{q}}(x,\boldsymbol{r},\boldsymbol{b}) \Big] \\
\frac{\partial \Omega^{(1)}(x,\boldsymbol{b},\boldsymbol{r})}{\partial \log(1/x)} &= \int d^2\boldsymbol{\rho}\, K(\boldsymbol{\rho},\boldsymbol{\rho}+\boldsymbol{r}) \Big[S_{q\bar{q}}(x,\boldsymbol{\rho};\boldsymbol{b}) \Omega^{(1)}(x,\boldsymbol{b},\boldsymbol{\rho}+\boldsymbol{r}) \\
&\quad + S_{q\bar{q}}(x,\boldsymbol{\rho}+\boldsymbol{r};\boldsymbol{b}) \Omega^{(1)}(x,\boldsymbol{b},\boldsymbol{\rho}) - \Omega^{(1)}(x,\boldsymbol{b},\boldsymbol{r}) \Big] .
\end{aligned} \tag{8}$$

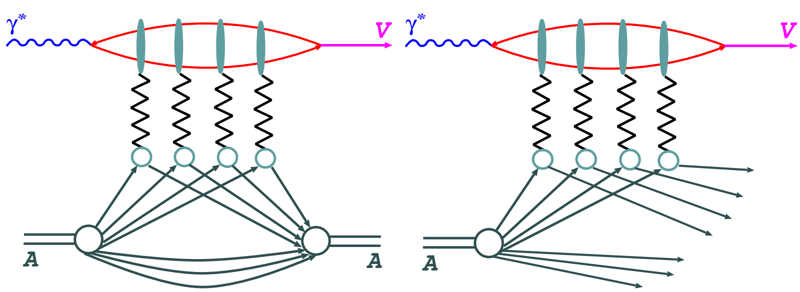

Fig. 2. Left: coherent diffractive production of a vector meson on a nucleus; right: a typical contribution to incoherent diffractive production, with breakup of the nucleus, but without production of new particles.

Here $K(\boldsymbol{\rho}, \boldsymbol{\rho} + \boldsymbol{r})$ is the Kernel of the color-dipole form of the BFKL equation [7]. A particular consequence of these evolution equations is, that $\Omega^{(1)}(x, \boldsymbol{b}, \boldsymbol{r}) \neq \sigma(x, \boldsymbol{r}) \exp[-\sigma(x, \boldsymbol{r}) T_A(\boldsymbol{b})/2]$ at $x \neq x_A$. Incidentally these equations also contradict a fan diagram interpretation of the BK-equation.

3. Photon-jet correlations

Finally, I discuss a process, which displays a rather direct dependence on the nuclear unintegrated glue, the associated production of an isolated high $p_\perp$ photon and a jet in the proton fragmentation region of pA collisions. Here the production of such photon-jet pairs can be viewed, in the nucleus rest frame, as an excitation of the $q\gamma$ Fock state of the incident quark, $q^* \to q\gamma$. The fact that the photon in the $q\gamma$ Fock state does not interact has the important consequence, that the emerging dijet spectrum becomes a *linear* functional of the unintegrated glue.

On the free nucleon target, we obtain for the parton-level cross-section

$$\frac{2(2\pi)^2 d\sigma_N(q \to q\gamma)}{dz d^2\boldsymbol{p} d^2\Delta} = f(x, \Delta)\, P_{\gamma q}(z)\, K(\boldsymbol{p}, \boldsymbol{p} - z\Delta)\,, \tag{9}$$

with

$$K(\boldsymbol{p}_1, \boldsymbol{p}_2) = \left| \frac{\boldsymbol{p}_1}{\boldsymbol{p}_1^2 + \varepsilon^2} - \frac{\boldsymbol{p}_2}{\boldsymbol{p}_2^2 + \varepsilon^2} \right|^2, \qquad P_{\gamma q}(z) = 2e_Q^2 \alpha_{\rm em} \frac{1+(1-z)^2}{z}\,. \tag{10}$$

Here $z, \boldsymbol{p}$ are the photon's light-cone momentum fraction and transverse momentum, $\varepsilon^2 = z m_q^2$. The decorrelation momentum $\Delta = \boldsymbol{p} + \boldsymbol{p}_q =$ measures the deviation of the dijet system from the back-to-back kinematics. Going to the nuclear target, we obtain

$$\frac{(2\pi)^2 d\sigma_A(q \to q\gamma)}{dz d^2\boldsymbol{p} d^2\Delta d^2\boldsymbol{b}} = \Big[\phi(\nu_A, x, \Delta) + w_0(\nu_A)\, \delta^{(2)}(\Delta)\Big] P_{\gamma q}(z) K(\boldsymbol{p}, \boldsymbol{p} - z\Delta)\,.$$

Notice that a potential diffractive contribution $\propto w_0(\nu_A)$, which would violate the linear $k_\perp$-factorization, vanishes on a heavy nucleus, where the momentum transfer distribution is δ-function-like.

Nuclear effects are best visualized by the ratio (we stay at the parton-level throughout)

$$R_{pA}(\nu_A, \boldsymbol{p}, \Delta) = \frac{d\sigma_A}{T_A(\boldsymbol{b}) d\sigma_N} = \frac{\phi(\nu_A, x, \Delta)}{\nu_A\, f(x, \Delta)}\,. \tag{11}$$

And similarly the central-to-peripheral ratio, which involves only nuclear quantities

$$R_{\rm cp}(\nu_>, \nu_<, \boldsymbol{p}, \Delta) = \frac{\nu_<\, \phi(\nu_>, x, \Delta)}{\nu_>\, \phi(\nu_<, x, \Delta)}\,. \tag{12}$$

Both these ratios do not depend on the photon's transverse momentum $\boldsymbol{p}$. We show the ratio R_{pA} at $x = x_A$ for different opacities in the top panel of Fig. 3. We observe a shadowing at small values of Δ and a Cronin-type peak which position reflects the ν_A-dependent saturation scale. In the lower panel we show $R_{\rm cp}$ and its evolution with x. While it displays the same Cronin-peak as R_{pA} for $x = x_A$, the latter is entirely quenched at small x.

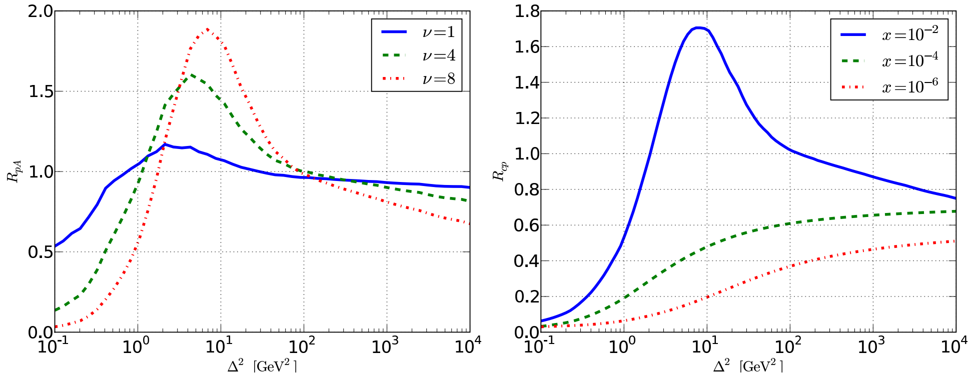

Fig. 3. Left: R_{pA} at $x = 0.01$, Right: $R_{\rm cp}$ for $\nu_> = 8, \nu_< = 1$ for different x.

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