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PINCH TECHNIQUE GLUON PROPAGATOR AND LIMITS ON A DYNAMICALLY GENERATED GLUON MASS*

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Within a simple Ansatz for renormalized gluon propagator and using gauge invariant pinch-technique for Schwinger–Dyson equation, the limits on the effective gluon mass is derived. We calculated scheme invariant running coupling, which in order to be well defined, gives the lower limit on the gluon mass. We conclude that mass, m, should be larger than $0.4\,\Lambda$ in order to avoid Landau ghost. The upper limit is estimated from assumed quark mass generation which requires gauge coupling must be large enough to trigger chiral symmetry breaking. It allows only small range of m, which lead to a reasonably large infrared coupling. Already for $m \simeq \Lambda$ we get no chiral symmetry breaking at all. Further, we observe that sometimes assumed or postulated Khallen–Lehmann representation for running coupling is not achieved for any value of m.

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1. Introduction

Pinch Technique (PT) [1–3] rearranges the original gauge scheme dependent Greens Functions GFs in a unique way such that unphysical degrees of freedom are eliminated. It has been proved to all orders of perturbation theory that the PT GFs satisfy Ward identities and that extracted effective charges are process independent. Using Schwinger–Dyson equations it typically leads to the so-called decoupling solution wherein the massless pole of gluon propagator disappear and gluon propagator is finite (but nonzero) in the infrared (for the topical review see [6]). In the meantime, as the lattice

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calculations [7, 8] in conventional Landau gauge start to support the decoupling solution reaching recently quite infrared momenta down to 75 MeV [9], it attracts new attention again.

In the paper [10] the PT propagator based on the WTI improved vertex [5] was considered. Making a simple parametrization of the solution the SDE has been solved analytically. We adopt here the form of the solution proposed in [10] and obtain the running coupling for all q^2.

The product of the gauge coupling g^2 and PT gluon propagator $\hat{d}$ defines renormalization invariants. It is certainly allowed to rewrite this product by a new one, where one function can represent invariant running charge while the second, say the function H stays for the rest, let us assume the function H shows up a massive pole instead the massless one. Using the same convention as in [10] we can write

$$g^2\hat{d}\left(q^2\right) = \bar{g}^2\left(q^2\right)\hat{H}\left(q^2\right) . \tag{1}$$

Clearly the functions on r.h.s. of Eq. (1) are obviously not uniquely defined if one does not say more. This problem is simply avoided if one assume the form of H explicitly since then only one $\bar{g}^2(q^2)$ function needs to be identified.

The simplest parametrization we can use is the following hard mass approximation

$$g^2\hat{d}\left(q^2\right) = \bar{g}^2\left(q^2\right)\frac{1}{q^2 - m^2 + i\varepsilon} . \tag{2}$$

The PT SDE is represented by a non-linear integral equation derived in [4] and solved first time in [10], it reads

$$\left[\bar{g}^2\hat{d}\left(q^2\right)\right]^{-1} = q^2 bZ - \frac{ib}{\pi^2}\int d^{4k}\hat{H}(k)\hat{H}(k+q)\left[q^2 + \frac{m^2}{11}\right] + C , \tag{3}$$

where C is momentum independent constant and where the hard mass approximation has been employed. The one loop beta function coefficient is $b = \frac{11N_c - 2N_f}{48\pi^2}$.

After the renormalization, which was made by on shell-subtraction (note, renormalization is not multiplicative here, for details see [10]) we get for Eq. (3)

$$\left[\bar{g}^2\hat{d}\left(q^2\right)\right]^{-1} = b\left[J(q)\left(q^2 + \frac{m^2}{11}\right) - J(m)\frac{12m^2}{11}\right] , \tag{4}$$

where the function J is renormalized in accordance with the correct one loop ultraviolet asymptotic and it reads

$$J(q) = -\int\limits_{4m^2}^{\infty} d\omega\frac{q^2}{\omega}\frac{\rho(\omega;m)}{q^2 - \omega + i\varepsilon} + 2 + 2\ln\left(m/\Lambda\right) , \tag{5}$$

where Λ is usual QCD scale valued few hundred MeV for $N_f = 2$ and $\rho(\omega; m) = \sqrt{1 - \frac{4m^2}{\omega}}$. The integral is a textbook scalar 1-loop integral and can be easily evaluated.

In what follows we are looking for a physically admissible solutions. Inverting the SDE (4), the pinch technique gauge invariant running charge can be straightforwardly evaluated

$$b 4\pi \alpha_{\text{massive}}\left(q^2\right) = b\bar{g}^2\left(q^2\right) = \frac{q^2 - m^2}{J(q)\left(q^2 + \frac{m^2}{11}\right) - J(m)\frac{12m^2}{11}} \tag{6}$$

with the nonzero imaginary part starting at $4m^2$.

As our approximated PT gluon propagator has a single real pole, it does not respect confinement issues. Nevertheless, we expect that this is a good approximation of the true full (exact) PT solution for the gluon propagator which could posses a large enhancement at vicinity of $q^2 \simeq m^2$ instead of the real pole we use. The singularity of exact PT propagator are very likely situated away of the real axis and need not be a simple pole but a branch point(s).

For high $s = q^2$ the absorptive part exactly corresponds with known analyticized one loop QCD coupling [11], *i.e.* for $s \gg m^2, \Lambda^2$ we can get

$$\operatorname{Im} \alpha(s) \to \frac{4\pi/b}{\pi^2 + \ln^2(s/\Lambda^2)}\,, \tag{7}$$

while the appearance of nontrivial threshold which crucially changes real part of the running coupling behaviour when m is nonzero and $\alpha(s)$ itself does not satisfy Khallen–Lehmann representation at all.

By construction, the correct one loop perturbation theory behaviour is reproduced for any m, *i.e.* $\alpha \simeq 1/\ln(q^2/\Lambda^2)$ for $q^2 \gg \Lambda, m$. According to observation made in [10], there exists certain critical mass, say $m_{\rm c}^{\rm I}$, bellow which the coupling α is more singular as it posses nonsimple pole, this critical mass is approximately given by the ratio $m_{\rm c}^{\rm I}/\Lambda = 1.2$ here. Decreasing the mass parameter m we can find second critical point, say $m_{\rm c}^{\rm II}$,where this pole crosses Minkowski light cone $q^2 = 0$ and becomes well known unphysical spacelike Landau pole. This Landau ghost is really unacceptable and it gives severe boundary on the gluon mass, in our case we get approximately $m_{\rm c}^{\rm II} = 0.4\,\Lambda$. Examples of running coupling are shown in Fig. 1 for various m. Singularities move to the left as m decrease. For large m enough, $m > 1.2\,\Lambda$, the only standard two particle branch point singularity located at $4m^2$ remains and the pole has gone. The singularity can appear only under the threshold, wherein the zero of α^{-1} is not protected by a nonzero absorptive part.

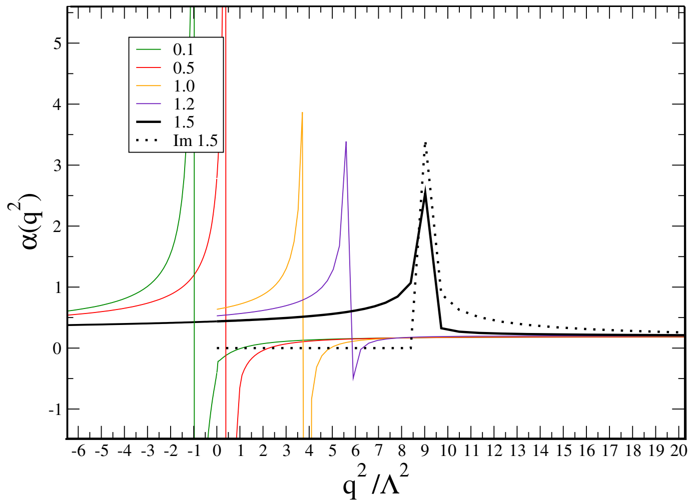

Fig. 1. Pinch technique coupling α for a various ratio m/Λ. For better identification Im part is displayed for regular solution with $m/\Lambda = 1.2$ only.

As we have discussed, the mass m is severely constrained from bellow by requirement of absence of the spacelike Landau pole. For an indication of what m should be we do not choose the criterion of KLR, which appear as a quite obscured requirement in confining theory, but we require that pinch technique running coupling must be large enough in order to trigger correct chiral symmetry breaking in QCD. In the real QCD the dynamical chiral symmetry breaking is phenomenon responsible for the most of nucleon mass (*i.e.* for the u, d quarks dynamical mass generation) while it simultaneously explains the lightness of the pions. To describe all these observable in selfcontained way, the chiral symmetry breaking must be correctly incorporated into the formalism. Such requirement very naturally gives upper boundary on the pinch technique gluon mass since for gluon heavy enough, the pinch technique running coupling is too weak and it does not trigger chiral symmetry breaking.

In principle, employing the formalism of PT SDEs for gluons and quarks simultaneously solved with the Bethe–Salpeter equations one should be able to fit the mass in the PT gluon propagator from meson spectra. Unhappily, the recent calculations are still far from this stage even in the more conventional gauge fixed schemes and the form of propagators entering the calculation is still dubious. First, we will describe arising obstacles of such a treatment and we suggest simplified way to make an reliable estimate of m, which is solely based on the solution of quark gap equation in the ladder approximation.

In lattice QCD, a static quark–antiquark potential can be computed with the Wilson loop technique. This gives us confining linear potential V_{L} between infinitely heavy quarks. For a correct description of excited mesons this could be principally involved covariantly in the quark–antiquark kernel of BSE. From the other side the various hadronic observables were calculated in the framework of Schwinger–Dyson equations during last two decades, it includes the meson spectra and decays [12, 13]. Most of them use the ladder approximation of the quark SDE (and meson BSE) in Landau gauge, while first steps beyond the ladder approximation have been considered only quite recently. Recall, the ladder approximation means that "very effective" one gluon exchange is considered only, while a more gluon exchanges are needed and more topologically more complicated diagrams must contribute to get linear potential in non-relativistic limit. Such higher skeletons quite naturally generates important scalar part of the quark–antiquark potential [14]. We conjecture that if the quark–antiquark BSE kernel analogue of V_{L} is not included then this is the main source of discrepancy when one compare GFs used in meson ladder calculations and the GFs actually obtained from SDEs. It is more then obvious that the effective gluon propagator used in a typical ladder approximated BSE more or less models unconsidered higher order skeletons. Without reasonable matching of SDEs calculations on gluon propagator and the one entering kernels of the meson BSE, the infrared behaviour of gluon propagator is not obvious.

On the other hand, our knowledge of Wilson lattice results combined with the knowledge of typical quark SDE solutions offers economical way, which we argue is efficient enough in order to estimate the PT gluon propagator in the infrared. For this purpose, let us consider the solutions of the quark SDE when one gluon and one gluon plus infrared enhanced effective interaction V_{L} is added. The difference of these two solutions has been studied in the paper [15] and it gives approximative double enhancement of the quark dynamical mass in deep infrared Euclidean Q^2 when V_{L} is taken into account. Using these arguments, the main issue is that infrared quark mass $M(0)$ should be already as large as $M(0) \simeq \Lambda = 250$ MeV when one uses the ladder quark gap equation alone but now with the PT running coupling implemented in. The additional unconsidered term V_{L} could be then responsible for an additional grow of the quark mass in the same order. It automatically gives the limit on the running coupling, its value must be significantly larger then the critical coupling, below which there is no symmetry breaking at all. As the dynamical quark mass function obtained in the ladder approximation is quite universal, we did need to perform a detailed numerical analyzes of the quark gap equation with PT running coupling and we can estimate the solution from the infrared value of the coupling, which must be $\alpha(0) \simeq 2.0$ or larger. To get such value we can see that we must use the solution with $m/\Lambda = 0.4$–0.7. Since required interval lies between $m_{\rm c}^{\rm I}$ and $m_{\rm c}^{\rm II}$ we always have the running coupling singularity at the timelike regime. In this way the pinch technique offers possible scenario for Infrared Slavery again, albeit with coupling enhancement in the timelike region simply due to the massiveness of the gluon.

To conclude, we have used recently obtained pinch technique gluon propagator [10] and the limits on the effective gluon mass have been reconsidered. It is confirmed that in order to avoid unphysically singular running coupling, the gluon mass must be bounded from bellow. We argue that the requirement of KLR for the running coupling is not a good guide for this purpose and we assume that running coupling can be enhanced, or even singular in the timelike region of the momenta. It is suggested that the upper boundary on the gluon mass m stems from chiral symmetry breaking when quarks are considered. As the infrared enhancement of the interaction is necessary to get correct triggering of symmetry breaking and since the running coupling crucially depends on mass m, the upper boundary stems from the minimal required pinch technique running coupling. It gives the acceptable region of the gluon mass $m \simeq 0.4$–$0.7\,\Lambda$, or so. This is in reasonable agreement with the recent lattice results and simultaneously it does not contradict the existence of chiral symmetry breaking in QCD.

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