

CONFINEMENT AND CHIRAL SYMMETRY IN A DENSE MATTER*

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We discuss a possibility for existence of confining but chirally symmetric phase at large baryon densities and low temperatures.

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1. General discussion

How does the strongly interacting matter look like at large (moderately large) baryon density and how does QCD work in this regime? It is one of the most interesting questions for the next decade. At the same time lattice QCD will not help us — it intrinsically relies on the Monte Carlo sampling in Euclidean space-time, but at finite chemical potential the Boltzmann weight factor becomes complex, so one cannot decide which specific amplitude (path) is important and which is not. It is a conceptual problem and unlikely could be overcome. We will have to rely on empirical results and on our qualitative understanding of QCD.

The most important and interesting question is related to interconnections of confinement and chiral symmetry breaking and how they both influence possible phases at finite density and determine a nature of mass of a dense matter. What will be a phase at not high temperatures and at densities higher than the nuclear matter density? We know a bit about the subject at the very large densities. At $N_c = 3$ and asymptotically large chemical potential at low temperatures the confining color-electric part of the gluonic field is totally Debye screened by the quark–quarkhole loops and one obtains a color-superconducting phase [1]. In this phase a matter is a liquid and physics is dictated by the diquark Cooper pairs near the Fermi surface. At small densities, around the nuclear matter density, the strongly interacting matter is also a liquid. Here quarks and gluons are confined

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inside nucleons, chiral symmetry is strongly broken and physics is mostly driven by interactions between nucleons induced by spontaneous breaking of chiral symmetry. Whether the QCD matter at $N_c = 3$ will be a liquid or a crystal at densities above the nuclear matter densities but below the very high densities of the color-superconductor depends on many fine details of dynamics that is not under our control. However, it is hard to imagine that it is a crystal, because then a phase diagram at low temperatures with increasing density would be a sequence of the following type: liquid–crystal–liquid. We do not know any example of this type in Nature. A natural assumption would be that at low temperatures we always have a liquid, all the way up to very high density. Obviously, properties of this liquid should be quite different at different densities.

This picture is drastically different from a crystalline structure that persists in the Skyrmionic description of a dense matter and even of a few-nucleon systems [2]. The crystalline structure in this case is entirely because of a topological nature of the nucleon within the Skyrme model. Crystal breaks both translational and rotational invariances. A crystalline type structure (chiral spiral) is also observed in solvable 1+1 dimensional models (large N_c QCD in 1+1 dimensions and Gross–Neveu model) [3]. In this case a translational invariance is broken even for the one-nucleon solution, which is certainly not the case in the real 4-dimensional world at $N_c = 3$.

If the QCD matter is a liquid at intermediate and high densities, then, by assumption, both translational and rotational invariances are not broken. Recent lattice results for $N_c = 2$ QCD [4] support such assumption.

Another key question is what degrees of freedom determine thermodynamical properties of the system. In the confining mode such degrees of freedom are color-singlet excitations of the system. Hence, in the confining mode relevant degrees of freedom are the color-singlet baryons or baryon-like systems near the Fermi surface. This is certainly the case at the nuclear matter density. A key question is then at which density (chemical potential) a deconfining transition occurs in the $N_c = 3$ world?

In the large N_c world with quarks in the fundamental representation there are neither dynamical quark–antiquark nor quark–quarkhole loops. Consequently there is not Debye screening of the confining gluon propagator and a gluodynamics at low temperatures is the same as in vacuum. Confinement persists up to arbitrary large density. In such case it is possible to define a quarkyonic matter [5]. In short, it is a strongly interacting matter with confinement and with a well-defined Fermi sea of baryons or quarks. At smaller densities it should be a Fermi sea of nucleons (so it matches with standard nuclear matter), while at higher densities, when nucleons are in a strong overlap, a quark Fermi surface should be formed. While a quark Fermi sea is formed, the system is still with confinement and excitation modes are of the color-singlet hadronic type.

In the large N_c 't Hooft limit such a matter persists at low temperatures up to arbitrary large densities. At which densities in the real $N_c = 3$ world will we have a deconfining transition (which could be a very smooth crossover) to a quark matter with uncorrelated single quark excitations? Lattice results for the $N_c = 2$ suggest that such a transition could occur at densities of the order of $100 \times$ nuclear matter density [4]. If correct, then it would imply that at all densities relevant to future experiments and astrophysics we will have a dense quarkyonic matter with confinement.

A very interesting question is what happens with chiral symmetry dynamical breaking in such a dense matter with confinement. If it is a liquid with unbroken translational and rotational invariances one expects that due to Pauli blocking of the quark levels (required for the very existence of the quark–antiquark condensate of the vacuum) there will happen a chiral restoration phase transition and we will obtain a confining but chirally symmetric subphase within the quarkyonic matter [6–8]. In such a phase a bulk mass of the system is generated via the chirally-invariant dynamics.

We cannot solve QCD and such a conclusion cannot be obtained from QCD itself. At best, what we can do at the present stage is to answer a question whether it is possible or not in principle. If yes, then a key question is: *How* could it happen and which physics is behind such unusual situation? Consequently we need a confining and chirally symmetric model. We cannot expect any realistic numbers from such a model, because this model would be at best a great oversimplification of real QCD. However, still such illustrative model is important because it gives us the insight into physics and because it was believed for many years that a confining phase with vanishing quark–antiquark condensate of the vacuum is impossible. Definitive requirements for the model are that it must be confining, chirally symmetric, provide dynamical chiral symmetry breaking in the vacuum. Obviously, the issue of a matter with confinement and restored chiral symmetry cannot be formulated and studied with the Nambu and Jona-Lasinio model and its variants which has been used sofar for study of the phase diagram of the strongly interacting matter.

2. The model

The model which we will use as a laboratory to get the insight is a generalization of the 't Hooft model, that is QCD in large N_c limit in $1+1$ dimensions. In the latter case the only gluonic interaction is a linear confining instantaneous Coulomb potential between quarks. Solving a gap equation one obtains a quark Green function with dynamical mass. Given this quark Green function it is possible to calculate exactly a meson spectrum, a quark condensate, *etc.* Consider a straightforward generalization, *i.e.*, a model with a linear instantaneous Coulomb-like potential in $3+1$ dimensions [9, 10]. All other possible gluonic interactions are neglected. A very

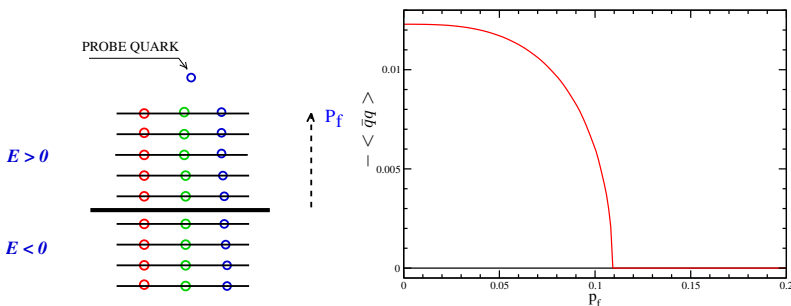
important aspect of this model is that it exhibits the effective chiral restoration in hadrons with large J [11]. This means that their mass comes not from the quark condensate of the vacuum, but mostly from the manifestly chiral-invariant dynamics. A chiral symmetry breaking in the vacuum is only a tiny perturbation to the chiral-invariant mass of these high-spin hadrons. It explicitly demonstrates that it is possible to construct hadrons such that their mass origin is not the quark condensate of the vacuum. If so, it is clear a priori that there are good chances to obtain confining but chirally symmetric matter at low temperatures within this model [6–8].

A key point is that the quark Green function (that is a solution of a gap equation in a vacuum) contains not only a chiral symmetry breaking part A_p , but also a manifestly chirally symmetric part B_p

$$\Sigma(\vec{p}) = A_p + \left(\vec{\gamma} \cdot \vec{p} \right) [B_p - p]. \quad (1)$$

A linear potential requires the infrared regularization. Otherwise all loop integrals are infrared divergent. All observable color-singlet quantities are finite and well defined in the infrared limit (*i.e.*, when the infrared cutoff approaches zero). These are hadron masses, the quark condensate, *etc.* In contrast, all color-nonsinglet quantities are divergent. *E.g.*, single quarks have infinite energy and consequently are removed from the spectrum. This is a manifestation of confinement within this simple model.

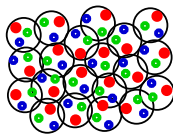
Assume now that we have a dense baryon matter with a well defined quark Fermi sea and the quark Fermi momentum is P_f . At the same time the interquark linear potential is not yet screened (in the large N_c limit it is not screened at any density). In order to understand what happens with chiral symmetry we have to solve a gap equation for a probe quark, see the left figure below. It is also assumed that the translational and rotational invariances are not broken according to arguments of the previous section. All intermediate quark levels below P_f are Pauli blocked and do not contribute to the gap equation. Consequently, at sufficiently large P_f a chiral restoration phase transition happens, see the right figure.



The chiral symmetry is restored like in the NJL model, because there is not available phase space in the gap equation to create a nontrivial solution with broken chiral symmetry. This required phase space is removed by the Pauli blocking of levels with positive energy. The standard quark–antiquark condensate of the vacuum vanishes.

As a consequence, above the chiral restoration phase transition the chiral symmetry breaking part of the quark self-energy identically vanishes, $A_p = 0$. However, there is also a chirally symmetric part of the quark self-energy, B_p . It does not vanish and is still infrared-divergent, like in vacuum. This means that even in the chirally restored regime a single quark energy is infinite and a single quark is removed from the spectrum. This infrared divergence is exactly canceled, however, in any color-singlet excitations of baryonic or mesonic type. Consequently, a spectrum of excitations consists of a complete set of all possible chiral multiplets. Energy of these excitations is a finite and well-defined quantity. The mass of this confining matter is chirally symmetric and comes from the chiral-invariant dynamics.

Such a confined phase with restored chiral symmetry can be viewed as a system of chirally symmetric baryons that are in a strong overlap, see a cartoon below. Confining gluonic fields are not screened, but quarks can move not only within each individual baryon, but also within the matter by hopping from one baryon to another. Such a motion is a simple consequence of Pauli principle. So one cannot say to which specific baryon a given quark belongs. (Actually even in a deuteron, which is certainly in a confining mode, one cannot say to which specific nucleon a given quark belongs: The valence quarks are transported from one nucleon to another by the meson-exchange force.) However, it would be a mistake to consider these quarks as free particles. They are colored and are subject to strong gluonic confining fields. Their dispersion law is a complicated one, by far not as for free particles. This picture is different from the naive perlocation picture, where one thinks that due to perlocation of baryons the quarks are free.

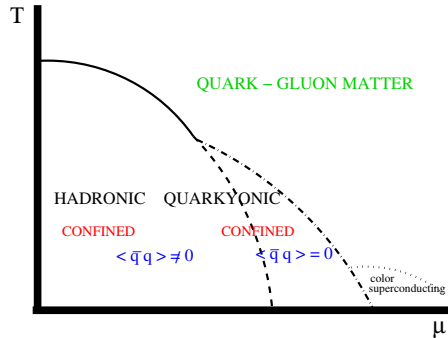


It should be emphasized that existence of chirally symmetric hadrons at large density is not prohibited and the Casher argument [12] can be easily bypassed in this situation [8].

An interesting question is what happens near the Fermi surface of such a dense confining matter with vanishing quark–antiquark condensate. There could be some surface phenomena like chiral density waves [13]. These chiral density waves have been derived so far as an instability of the quark Fermi

sea (with free unconfined quarks) due to a gluon-exchange force between quarks and quarkholes with large momenta near the Fermi surface. It is by far not clear whether it will happen or not in a system with confinement.

Below we show a possible phase diagram that incorporates a phase with confinement (or its remnants) but with vanishing quark–antiquark condensate as a phase at low temperatures just between a dense nuclear matter and a color-superconducting phase.



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