

ON CONFINEMENT AND CHIRAL SYMMETRY  
CRITICAL POINTS\* \*\*

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We study the QCD phase diagram, in particular we study the critical points of the two main QCD phase transitions, confinement and chiral symmetry breaking. Confinement drives chiral symmetry breaking and, due to the finite quark mass, at small density both transitions are a crossover, while they are a first or second order phase transition in large density. We study the QCD phase diagram with a quark potential model including both confinement and chiral symmetry. This formalism, in the Coulomb gauge Hamiltonian formalism of QCD, is presently the only one able to microscopically include both a quark–antiquark confining potential and a vacuum condensate of quark–antiquark pairs. This model is able to address all the excited hadrons, and chiral symmetry breaking, at the same token. Our order parameters are the Polyakov loop and the quark mass gap. The confining potential is extracted from the Lattice QCD data of the Bielefeld group. We order to address how the quark masses affect the critical point location in the phase diagram.

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**1. Introduction**

Our main motivation is to contribute to understand the QCD phase diagram [1], for finite  $T$  and  $\mu$ , to be studied at LHC, RHIC and FAIR. Moreover, our formalism, in the Coulomb gauge Hamiltonian formalism of QCD, is presently the only one able to microscopically include both a quark–antiquark confining potential and a vacuum condensate of quark–antiquark pairs. This model is able to address excited hadrons and chiral symmetry at the same token, and we recently suggested that the infrared enhancement of the quark mass can be observed in the excited baryon spectrum at CBELSA

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and at JLAB [2, 3]. Thus the present work, not only addresses the QCD phase diagram, but it also constitutes the first step to allow us in the future to compute the spectrum of any hadron, extending, say the Fig. 1 (left) computed in reference [2] to finite  $T$ .

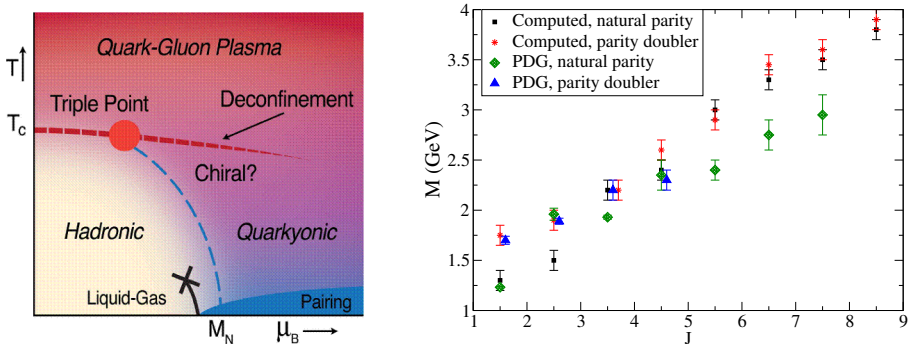


Fig. 1. Right: A sketch of the QCD Phase Diagram, according to the CBM Collaboration at FAIR [1]. It is usually assumed that the critical point for deconfinement coincides with the critical point for chiral symmetry restoration. Left: First calculation of excited baryons with a chiral invariant quark model.

Here we address the finite temperature string tension, the quark mass gap for a finite current quark mass and temperature, and the deconfinement and chiral restoration crossovers. We conclude on the separation of the critical point for chiral symmetry restoration from the critical point for deconfinement.

## 2. Fits for the finite $T$ string tension from the Lattice QCD energy $F_1$

At vanishing temperature  $T = 0$ , the confinement, modelled by a string, is dominant at moderate distances

$$V(r) \simeq \frac{\pi}{12r} + V_0 + \sigma r. \quad (1)$$

At short distances we have the Luscher or Nambu–Gotto Coulomb due to the string vibration + the OGE Coulomb, however the Coulomb is not important for chiral symmetry breaking. At finite temperature the string tension  $\sigma(T)$  should also dominate chiral symmetry breaking, and thus one of our crucial steps here is the fit of the string tension  $\sigma(T)$  from the Lattice QCD data of the Bielefeld Lattice QCD group [4–8].

The Polyakov loop is a gluonic path, closed in the imaginary time  $t_4$  (proportional to the inverse temperature  $T^{-1}$ ) direction in QCD discretized in a periodic boundary euclidian Lattice. It measures the free energy  $F$  of

one or more static quarks

$$P(0) = Ne^{-F_q/T}, \quad P^a(0)\bar{P}^{\bar{a}}(r) = Ne^{-F_{q\bar{q}}(r)/T}. \quad (2)$$

If we consider a single solitary quark in the Universe, in the confining phase, his string will travel as far as needed to connect the quark to an antiquark, resulting in an infinite energy  $F$ . Thus the 1 quark Polyakov loop  $P$  is a frequently used order parameter for deconfinement. With the string tension  $\sigma(T)$  extracted from the  $q\bar{q}$  pair of Polyakov loops we can also estimate the 1 quark Polyakov loop  $P(0)$ . At finite  $T$ , we use as thermodynamic potentials the free energy  $F_1$  and the internal energy  $U_1$ , computed in Lattice QCD with the Polyakov loop [4–8]. They are related to the static potential  $V(r) = -fdr$  with  $F_1(r) = -fdr - SdT$  adequate for isothermic transformations. In Fig. 2 we extract the string tensions  $\sigma(T)$  from the free energy  $F_1(T)$  computed by the Bielefeld group, and we also include string tensions previously computed by the Bielefeld group [9].

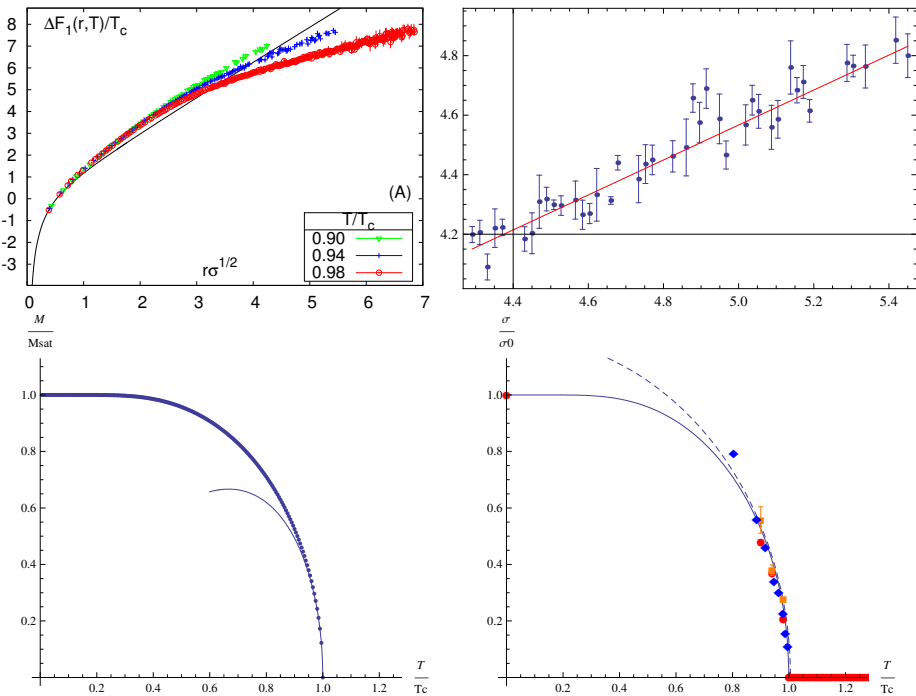


Fig. 2. Top left: The Bielefeld free  $F_1$  energy at  $T < T_c$ . Top right: Detail of the string tension fit in the case of  $T = 0.94T_c$ . Bottom left: The critical curve for  $M/M_{\text{sat}}$  as a function of  $T/T_c$ , for  $T \simeq T_c$  it behaves like a square root. Bottom right: Comparing the magnetization critical curve with the string tension  $\sigma/\sigma_0$ , fitted from the long distance part of  $F_1$ , they are quite close.

We also find an ansatz for the string tension curve, among the order parameter curves of other physical systems related to confinement, *i.e.* in ferromagnetic materials, in the Ising model, in superconductors either in the BCS model or in the Ginzburg–Landau model, or in string models, to suggest ansatz for the string tension curve. We find that the order parameter curve that best fits our string tension curve is the spontaneous magnetization of a ferromagnet [10], solution of the algebraic equation

$$\frac{M}{M_{\text{sat}}} = \tanh \left( \frac{T_c}{T} \frac{M}{M_{\text{sat}}} \right). \quad (3)$$

In Fig. 2 we show the solution of Eq. 3 obtained with the fixed point expansion, and compare it with the string tensions computed from lattice QCD data.

### 3. The mass gap equation with finite $T$ and finite current quark mass $m_0$

Now, the critical point occurs when the phase transition changes to a crossover, and the crossover in QCD is produced by the finite current quark mass  $m_0$ , since it affects the order parameters  $P$  or  $\sigma$ , and the mass gap  $m(0)$  or the quark condensate  $\langle \bar{q}q \rangle$ . The mass gap equation at the ladder/rainbow truncation of Coulomb Gauge QCD in equal time reads,

$$m(p) = m_0 + \frac{\sigma}{p^3} \int_0^\infty \frac{dk}{2\pi} \frac{I_A(p, k, \mu) m(k) p - I_B(p, k, \mu) m(p) k}{\sqrt{k^2 + m(k)^2}}, \quad (4)$$

$$I_A(p, k, \mu) = \left[ \frac{pk}{(p-k)^2 + \mu^2} - \frac{pk}{(p+k)^2 + \mu^2} \right],$$

$$I_B(p, k, \mu) = \left[ \frac{pk}{(p-k)^2 + \mu^2} + \frac{pk}{(p+k)^2 + \mu^2} + \frac{1}{2} \log \frac{(p-k)^2 + \mu^2}{(p+k)^2 + \mu^2} \right].$$

The mass gap equation (4) for the running mass  $m(p)$  is a non-linear integral equation with a nasty cancellation of Infrared divergences [11–13]. We devise a new method with a rational ansatz, and with relaxation, to get a maximum precision in the IR where the equation is nearly almost unstable. The solution  $m(p)$  is shown in Fig. 3 for a vanishing momentum  $p = 0$ .

At finite  $T$ , one has only to change the string tension to the finite  $T$  string tension  $\sigma(T)$  [14], and to replace an integral in  $\omega$  by a sum in Matsubara Frequencies. Both are equivalent to a reduction in the string tension,  $\sigma \rightarrow \sigma^*$  and thus, all we have to do is to solve the mass gap equation in units of  $\sigma^*$ . The results are depicted in Fig. 4. Thus at vanishing  $m_0$  we have a chiral symmetry phase transition, and at finite  $m_0$  we have a crossover, that gets weaker and weaker when  $m_0$  increases. This is also sketched in Fig. 3.

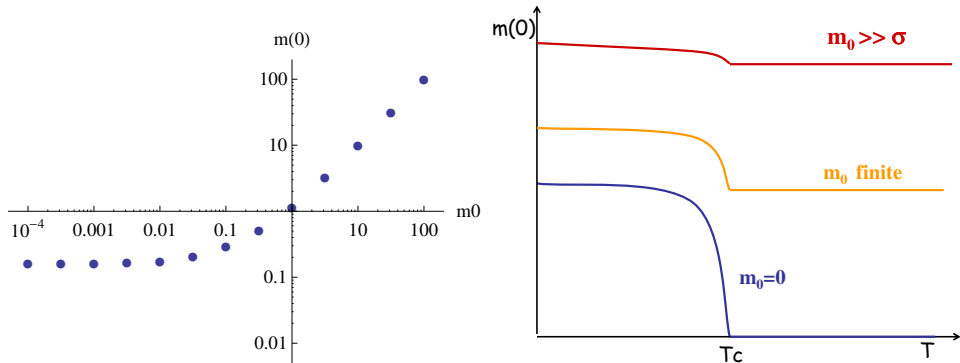


Fig. 3. Left: The mass gap  $m(0)$  solution of as a function of the quark current mass  $m_0$ , in units of  $\sigma = 1$ . Right: Sketch of the effect of  $m_0$  on the crossover *versus* phase transition of chiral restoration at finite  $T$ .

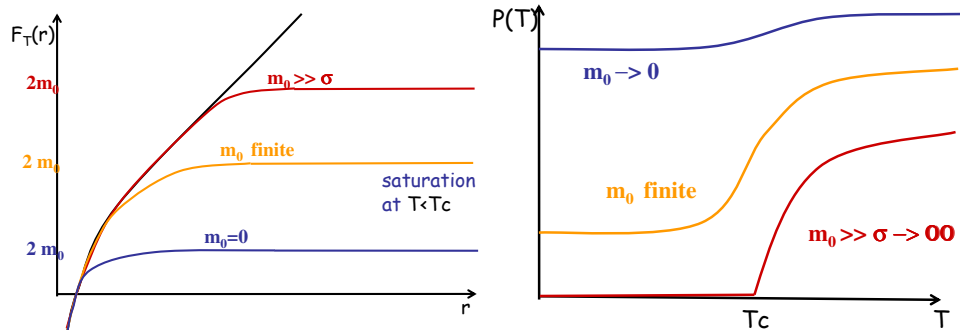


Fig. 4. Sketches of the saturation of confinement (left), and of the corresponding crossover in the order parameter  $P$  Polyakov loop (right).

#### 4. Chiral symmetry and confinement crossovers with a finite current quark mass

In what concerns confinement, the linear confining quark–antiquark potential saturates when the string breaks at the threshold for the creation of a quark–antiquark pair. Thus the free energy  $F(0)$  of a single static quark is not infinite, but is the energy of the string saturation, of the order of the mass of a meson *i.e.* of  $2m_0$ . For the Polyakov loop we get

$$P(0) \simeq N e^{-2m_0/T} . \quad (5)$$

Thus at infinite  $m_0$  we have a confining phase transition, while at finite  $m_0$  we have a crossover, that gets weaker and weaker when  $m_0$  decreases. This is sketched in Fig. 4.

Since the finite current quark mass affects in opposite ways the crossover for confinement and the one for chiral symmetry, we conjecture that at finite  $T$  and  $\mu$  there are not only one but two critical points (a point where a crossover separates from a phase transition). Since for the light  $u$  and  $d$  quarks the current mass  $m_0$  is small, we expect the crossover for chiral symmetry restoration critical to be closer to the  $\mu = 0$  vertical axis, and the crossover for deconfinement to go deeper into the finite  $\mu$  region of the critical curve in the QCD phase diagram depicted in Fig. 1.

## 5. Outlook

We soon plan to complete the part of this work which is only sketched here, *i.e.* to compute the crossover curves for the chiral symmetry restoration and for the deconfinement at finite  $T$ . This will also require the study of light hadrons at finite temperature  $T$ . Thus we also plan to address the excited hadron spectrum at finite  $T$ , continuing the work initiated with Tim Van Cauteren, Marco Cardoso, Nuno Cardoso and Felipe Llanes-Estrada.

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