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SELECTED ISSUES ON JUSTIFICATION OF HOLOGRAPHIC APPROACH TO QCD*

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Some problems with theoretical foundations of bottom-up holographic models are briefly discussed. It is pointed out that the spectroscopic aspects of these models in principle do not require the AdS/CFT prescriptions and may be interpreted as just an alternative language expressing the phenomenology of QCD sum rules in the large-N_c limit. A general recipe for incorporation of the chiral symmetry breaking scale into the soft-wall holographic models is proposed.

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1. Introduction and some general discussions

At the present time the bottom-up holographic models for QCD (called also AdS/QCD models) have become the very popular approach to description of phenomenology of strong interactions. An interesting question appears if it is possible to understand the 5-dimensional models in terms of traditional theoretical methods, *i.e.* without appealing to the AdS/CFT correspondence or any other ideas from the string theory. The main purpose of the given report is to demonstrate that at least in one important case the answer seems to be positive. The so-called soft-wall models introduced in [1] reproduce correctly the phenomenology of QCD sum rules in the large-N_c limit. One can try to invert the underlying logic of the soft-wall models: start from a Lagrangian describing an infinite number of free stable mesons with fixed quantum numbers which are expected in the large-N_c limit of QCD, fix the Regge form of spectrum without corrections[1], require to reproduce the analytic structure of the Operator Product Expansion for

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[1] Although such corrections are expected on the theoretical grounds [2] we simplify the matter in order to arrive at holographic-like models in the shortest way.

the two-point correlation functions, and use the Kaluza–Klein reduction, *i.e.* the fact that a free 5D field is equivalent to the infinite number of free 4D fields whose masses are determined by 5D background. It turns out that one arrives then uniquely at the soft-wall models, other possibilities are less satisfactory in the phenomenology [3]. In addition, when integrating back the fifth coordinate, a boundary term does not vanish for the vector and (in one particular case) scalar fields and the emerging term may be interpreted as a source for those fields thus justifying the use of AdS/CFT conjecture for the given cases. The soft-wall models in their spectroscopic applications may be consequently viewed as just an alternative language expressing the phenomenology of QCD sum rules in the large-N_c limit with ensuing accuracy and number of input parameters. The details of the corresponding analysis are given in Ref. [3]. In the present short contribution, we will dwell on a couple of comments which were not discussed in [3].

The observation above waives the following objection against the holographic models [4]: QCD has an infinite number of operators with any set of quantum numbers and in general these operators mix. One could, therefore, expect that an arbitrarily large subset of them may contribute to any given process because there is no obvious suppression scale. The approximation of lowest-dimension operators that is typically used for constructing holographic models looks thus *ad hoc*. We note that this argument is equally applicable to the standard QCD sum rules and for some reasons the approximation above works usually very well. As long as the bottom-up holographic models are deeply related to the QCD sum rules, it is not surprising that the same restriction to a minimal set of the simplest operators probing the quantum numbers of interest is successful in the holographic approach.

Unfortunately, the relationship of bottom-up holographic models with the planar sum rules entails the following property: There is no internal criterion indicating whether we may trust the results or not, the goodness of a holographic model is always checked *a posteriori* by comparison with the phenomenology. The original QCD sum rules possess such a criterion — the existence of a "Borel window" (a region of stability of Borel parameter), but this criterion is lost when taking the limit of large-N_c.

The next comment concerns the foundations of AdS/QCD models. The whole approach is often criticized for its speculative nature. Which specific assumptions have no solid ground and which are reasonable? An optimistic point of view that is widespread among practitioners of holographic models for QCD consists in a belief that the AdS/QCD correspondence is a reasonable approximation to holographic duals of QCD because the latter is approximately conformal in the UV limit. Thus the conformal symmetry of AdS metric ensures the right behavior of correlators and what is left to speculations is how to set the confinement scale and how to model chiral symmetry breaking. We find this belief poorly justified. First of all, QCD at large space-like momenta is weakly coupled, this feature is opposite to what is required by the conjecture of AdS/CFT correspondence which, taking at face value, would lead to a strongly coupled holographic dual in the UV limit, hence, the semiclassical approximation fails for the latter. Moreover, the large-N_c limits of QCD and of field theory in Maldacena's example [5] are quite different ($\alpha_{\mathrm{s}} N_c$ is not constant in the latter but grows with N_c), let alone the fact that the operators in QCD have anomalous dimensions[2] contrary to the $N = 4$ supersymmetric Yang–Mills theory considered in [5]. This makes really questionable why the AdS/QCD prescriptions may be used for approximate QCD duals without modifications and even why we may hope at all that such duals exist. Such a belief represents thus the main speculative assumption in the whole enterprise.

One usually hopes, however, that if we are lucky enough in appropriate breaking conformality in the IR region then we may obtain nice 5D effective models for QCD using the AdS/CFT correspondence as a guiding principle. In other words, we may try to use the conformal symmetry of QCD at high energies as the first approximation and account for breaking of this symmetry by means of some successful parametrization despite the fact that the breaking of conformal symmetry shapes drastically the physics of strong interactions. Is this hope achievable? We are inclined to think that rather no than yes. The matter is that within QCD there is an explicit counterexample to such a programme. QCD possesses an approximate chiral symmetry in the sector of light flavors which is also broken. Nevertheless, it is well known that the use of the manifest chiral symmetry as the first approximation for building low-energy effective models is not satisfactory whatever parametrization accounting for the breaking of chiral symmetry is applied. The reason is known as well — the chiral symmetry of QCD is realized in the Nambu–Goldstone mode at low energies and this very realization must be used for model building as the first approximation. The chiral symmetry breaking effect shapes drastically the physics of strong interactions at low energies and, therefore, by no means may be regarded as some correction to another approximation. As to the highly excited states, the linear chiral symmetry, most likely, is not restored (see, *e.g.*, discussions in reviews [6]). The lesson with the chiral symmetry seems to suggest that a search for "another realization" of conformal symmetry in hadron physics would be promising, the modern AdS/QCD models are not in this line.

Although objections concerning the foundations of AdS/QCD models are quite serious, the holographic methods are certainly interesting as a language that allows to discuss within a uniform framework various approaches

[2] We note incidentally that if the bottom-up models are regarded as a 5D reformulation of QCD sum rules then this problem does not look dangerous.

to modeling the interactions and spectral characteristics of light hadrons, heavy-light systems, hadron form factors, QCD phase diagram, and other phenomenological aspects which were previously the subjects of investigations for different communities. This unifying property is remarkable indeed. As was shown in Ref. [3], at least in some important applications one can, in principle, dispense with the prescriptions dictated by the AdS/CFT correspondence in constructing 5-dimensional effective models for QCD with the same final results. This is, however, possible at the cost of alternative assumptions, the use of AdS/CFT prescriptions as a guide turns out to be more compact and more elegant. In addition, the holographic approach proposes an interesting new way for description of the chiral symmetry breaking. We discuss briefly this subject in the next section.

2. Chiral Symmetry Breaking

The incorporation of the Chiral Symmetry Breaking (CSB) into 5-dimensional effective hadron models has a lot of freedom for speculations because there are no quarks, hence no microscopic model of CSB related directly to QCD may be constructed, the best we may do is to describe the consequences of CSB on the hadron level. The first such consequence is the phenomenological fact that the masses of parity partners which seemingly belong to the same chiral multiplet are quite different. To reflect this effect one should introduce a mechanism for this mass splitting. Within the 5D framework, the chiral symmetry does not exist at all because there is no analogue for the matrix γ_5 in five dimensions, the CSB can be, therefore, simulated only indirectly by means of somewhat different description of states with equal spin but opposite parity.

The second consequence of CSB consists in the appearance of massless (in the chiral limit) pseudoscalar mesons due to the Goldstone theorem. The both consequences should be described with the help of a mechanism that explains, *e.g.*, why the relation $m_\pi = 0$ is naturally related with $m_{a_1}^2 \gtrsim 2m_\rho^2$ (instead of $m_{a_1} = m_\rho$ as naively expected from the linear chiral symmetry). The simplest such a mechanism that is realized in the AdS/QCD models is borrowed from the low-energy effective field theories: One introduces a scalar field X that acquires a non-zero vacuum expectation value (v.e.v.) $X_0(z)$ and is coupled to the axial-vector field A_M through the covariant derivative, $D_M X = \partial_M X - ig_5 A_M X$. The minimal part of 5D action describing CSB is

$$S_{\mathrm{CSB}} = \int d^4x \int dz \sqrt{|G_{MN}|} e^{\Phi} \left(|D_M X|^2 - m_X^2 |X|^2\right) . \tag{1}$$

It is sufficient to retain the quadratic in fields part because the equations of motion can provide a non-zero v.e.v. already in this case if the bulk space is curved. The term (1) must be added to a basic action of a holographic model in question, for instance such an action for the soft-wall model may be written in the form [3]

$$S_{5\mathrm{D}} = (-1)^J \int d^4x\, dz e^{\Phi} a^{-2J+3} \Big\{ (\partial_\mu \varphi_J)^2 - (\partial_z \varphi_J)^2 - m_J^2 a^2 \varphi_J^2 \Big\} , \qquad (2)$$

where the gauge $\varphi_{z\ldots} = 0$ is accepted for the gauge fields φ_J of arbitrary spin J, the function $a(z)$ parametrizes the metric, $ds^2 = a^2(z)(dx_\mu dx^\mu - dz^2)$, and the function $\Phi(z)$ represents a dilaton background. The equation of motion for action (2) yielding the mass spectrum can be cast into the form of Schrödinger equation

$$-\psi_n'' + U\psi_n = m_n^2 \psi_n , \qquad (3)$$

$$U = \frac{\Phi''}{2} + \left(\frac{\Phi'}{2}\right)^2 + \left(\frac{3}{2} - J\right) \frac{\Phi' a' + a'' + \left(\frac{1}{2} - J\right) \frac{(a')^2}{a}}{a} + a^2 m_J^2 , \qquad (4)$$

where $\varphi_n = e^{-\Phi/2} a^{J-3/2} \psi_n$ and the prime denotes the derivative with respect to z.

The equation of motion determining the v.e.v. X_0 is nothing but the equation (3) for a massless scalar particle. According to a recipe based on the AdS/CFT correspondence [7], it must behave at $z = 0$ as $X_0(z)|_{z\to 0} = C_1 z + C_2 z^3$, where C_1 is associated with the current quark mass, $C_1 \sim m_q$, and C_2 with the quark condensate, $C_2 \sim \langle \bar{q}q \rangle$. This interpretation implies a somehow well established correspondence of the model to QCD. As we do not have such a rigorous correspondence, it would be more honest to say that the incorporation of the 4D massless scalar particle can be related to the spontaneous appearance of two order parameters with mass dimension one and three and this property may be exploited to mimic the CSB.

In fact, there is the third consequence of the CSB in QCD: The effective emergence of the scale $\Lambda_{\mathrm{CSB}} \simeq 4\pi f_\pi \approx 1 \div 1.2$ GeV. The physics of strong interactions is known to be substantially different below Λ_{CSB} and above Λ_{CSB}. This consequence of the CSB is not incorporated into the existing AdS/QCD models. In what follows we propose a possible scheme for incorporation of Λ_{CSB} into the holographic models [3].

The equation of motion for X represents a second order linear differential equation that has two independent solutions

$$X_0(z) = C_1 X_1(z) + C_2 X_2(z) . \qquad (5)$$

In general $X_0(z)$ is not normalizable, namely the solution $X_1(z)$ spoils the normalizability at $z \to 0$ while (in the soft wall models) $X_2(z)$ does at

$z \to \infty$. We can require the normalizability for $X_0(z)$ and construct the following normalizable solution

$$X_0(z) = C_2 X_2(z)|_{z \le z_0} + C_1 X_1(z)|_{z > z_0}\,. \tag{6}$$

As long as the fifth coordinate z is associated with the inverse energy scale in the holographic approach, it looks natural to interpret z_0^{-1} as the scale $\Lambda_{\rm CSB}$ because the physics below and above this scale will be different.

In the case of the soft-wall model with positive-sign dilaton background the corresponding solutions are

$$X_1(z) = z e^{-z^2} U\left(-1/2, 0; z^2\right)\,, \qquad X_2(z) = z^3 e^{-z^2} M\left(1/2, 2; z^2\right)\,, \tag{7}$$

where U and M stay for the U and M Kummer functions. The "potential" (4) has the following asymptotics for the axial-vector mesons

$$U|_{z\gg z_0} = z^2\left(1 + g_5^2 C_1^2 e^{-2z^2}\right) + \frac{3}{4z^2}\,, \qquad U|_{z\ll z_0} = z^2\left(1 + g_5^2 C_2^2 z^2\right) + \frac{3}{4z^2}\,, \tag{8}$$

while for the vector case $C_1 = C_2 = 0$. This behavior demonstrates that the spectrum of axial-vector states approaches rapidly to the spectrum of vector mesons. The rate of such a "Chiral Symmetry Restoration" is exponential that is in agreement with the analysis [2] based on the QCD sum rules. This qualitative feature differs from the typical predictions within the soft-wall models where the shift between the vector and axial-vector masses square tends to a constant.

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