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IMPORTANCE OF MIXING FOR EXOTIC BARYONS*

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Exotic antidecuplet baryons are predicted to be not only surprisingly light but also very narrow. First, we explain how small decay width arises in the quark soliton model. Next, we study possible mixing of exotic antidecuplet with Roper octet and discuss its phenomenological consequences.

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1. Introduction

Despite recent skepticism concerning early announcements of the discovery of exotic strange baryon $\Theta^+(1540)$ two collaborations DIANA and LEPS confirmed their original results [1–3]. We refer the reader to recent experimental reviews [4–6]. In this article following [7] we assume that Θ^+ exists with a mass equal 1540 MeV and total width $\Gamma < 1$ MeV. An immediate consequences is the existence of the whole exotic SU(3) multiplet: $\overline{10}$. Apart from truly exotic states antidecuplet contains cryptoexotic nucleon- ($N_{\overline{10}}$) and Σ-like states ($\Sigma_{\overline{10}}$). The interpretation of these states is not well understood: one may try to associate them with some known resonances, or one may postulate the existence of new resonances with nucleon or Σ quantum numbers. This is the approach which we adopt here [7]. Following Refs. [8–12] we assume that there exists new, narrow nucleon resonance $N(1685)$ which we will interpret as a member of $\overline{10}$. If so, $N_{\overline{10}}$ decays have to satisfy the following constraints [7] originally discussed in [8]

$$\Gamma_{N_{\overline{10}}\to\pi N} < 0.5\,\mathrm{MeV}\,, \qquad \mathrm{Br}_{N_{\overline{10}}\to\eta N} > 0.2\,, \qquad 5\,\mathrm{MeV} < \Gamma^{\mathrm{tot}}_{N_{\overline{10}}} < 25\,\mathrm{MeV}\,. \quad (1)$$

Unfortunately, the small partial width of $N(1685)$ to πN contradicts SU(3) symmetry relations between the decay constants of $N_{\overline{10}}$.

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In this short note we would like to emphasize the importance of mixing both for decays and mass spectra of the putative exotic antidecuplet baryons. The exotic states have been already anticipated by the founders of the quark model although they did not elaborate on them. Later the arguments have been raised that they should be heavy and wide. In contrast, chiral soliton models predicted that the pentaquark masses were generically small (*i.e.* in the range of 1.5–1.6 GeV) [13–15]. It was much more difficult to accommodate the small decay width of Θ^+ [16]. In Sec. 2 we explain how small decay width arises naturally in the Chiral Soliton Quark Model (χQSM). Next, in Sec. 3 we argue that the decay coupling $g_{\Theta NK}$ is further reduced due to Gell-Mann–Okubo (GMO) mixing caused by the nonzero m_s. In Sec. 4 we show how additional mixing of $\overline{10}$ with Roper octet can change decay patterns of $N_{\overline{10}}$. We estimate allowed range of mixing angles and present predictions for remaining members of $\overline{10}$: $\Sigma_{\overline{10}}$ and $\Xi_{\overline{10}}$. Conclusions are presented in Sec. 5.

2. Decay widths in chiral soliton quark model

In in Ref. [15] the following nonrelativistic formula for the decay width has been used

$$\Gamma_{B_1\to B_2\varphi} = \frac{g^2_{B_1B_2\varphi}}{2\pi(M_1+M_2)^2}\,p^3_\varphi\,. \tag{2}$$

It follows from the Goldberger–Treimann relation between axial and strong decay constants. Here M_1 is the mass of the decaying baryon, M_2 the mass of the decay product and p_φ is the outgoing meson momentum. Generically $\Gamma_{\Theta^+\to NK}$ given by (2) would be still in the range of a few hundreds of MeV [16] if not for the terms non-leading in $1/N_c$ expansion. Indeed, even without mixing the decay constant $g_{B_1B_2\varphi}$, which stands for the matrix element of the tensor decay operator $O^{(8)}_\varphi$ between the physical baryon wave functions $\left|B^{\rm phys}\right\rangle$

$$g_{B_1B_2\varphi} = \left\langle B_2^{\rm phys}\right| O^{(8)}_\varphi \left|B_1^{\rm phys}\right\rangle \tag{3}$$

is the sum of three different contributions that are formally of different order in $1/N_c$. However, they are multiplied by the SU(3) Clebsch–Gordan coefficients that also depend on N_c [17]. For example,

$$\begin{aligned} g^2_{\Theta NK} &= \frac{9(N_c+1)}{(N_c+3)(N_c+7)}\,G^2_{\overline{10}} \\ \text{with}\qquad G_{\overline{10}} &= G_0 - \frac{N_c+1}{4}\,G_1 - \tfrac{1}{2}\,G_2\,, \end{aligned} \tag{4}$$

where $G_0 \sim N_c^{3/2}$, $G_{1,2} \sim N_c^{1/2}$. Similarly

$$g^2_{\Delta N\pi} = \frac{3(N_c-1)(N_c+5)}{2(N_c+1)(N_c+7)}\, G_{10}^2\,, \qquad \text{with} \qquad G_{10} = G_0 + \tfrac{1}{2}\, G_2\,. \tag{5}$$

Chiral soliton models provide us with specific predictions for constants $G_{0,1,2}$ [18]. Had we neglected G_1 and G_2 in Eqs. (4,5) (which would be inconsistent for $g_{\Theta NK}$ because of N_c enhancement of G_1) we would have obtained for $N_c = 3$

$$g_{\Delta N\pi} = g_{\Theta NK} \sim 17.6$$

estimating G_0 from the experimental value of Δ decay width, and consequently

$$\Gamma_{\Theta NK} \sim 150\,\text{MeV}\,. \tag{6}$$

We see that small decay width of Θ^+ results from the cancellation in (4). Indeed, the authors of Ref. [15] have shown that in the nonrelativistic limit of χQSM one obtains that $G_0 = -(N_c+2)G$, $G_1 = -4G$, $G_2 = -2G$, with $G \sim N_c^{1/2}$ and consequently $\Gamma_{\Theta NK} = 0$! In the same limit χQSM predicts that $g_A = 3/5$ and $\mu_p/\mu_n = -3/2$. It follows that antidecuplet decay constants are small.

3. Gell-Mann–Okubo mixing

Treating m_s corrections as perturbation introduces mixing [7]

$$\begin{aligned}
\left|B_8^{\text{phys}}\right\rangle &= \left|8_{1/2},B\right\rangle + c_{\overline{10}}^B \left|\overline{10}_{1/2},B\right\rangle + c_{27}^B \left|27_{1/2},B\right\rangle\,,\\
\left|B_{\overline{10}}^{\text{phys}}\right\rangle &= \left|\overline{10}_{1/2},B\right\rangle + d_8^B \left|8_{1/2},B\right\rangle + d_{27}^B \left|27_{1/2},B\right\rangle + d_{\overline{35}}^B \left|\overline{35}_{1/2},B\right\rangle\,,
\end{aligned} \tag{7}$$

where subscripts refer to spin. For some specific states mixing constants c_R^B, $d_R^B \sim m_s$ may be equal zero due to the isospin. For example Θ^+ mixes only with $\overline{35}$, but this component of the wave function does not contribute to the decays to octet. Therefore only mixing of the final nucleon with $\overline{10}$ and 27 modifies the decay constant

$$g^2_{\Theta NK} = \tfrac{3}{5}\left[G_{\overline{10}} + \tfrac{5}{4} c_{\overline{10}}\, H_{\overline{10}} - \tfrac{7}{4} c_{27} H'_{27}\right]^2. \tag{8}$$

Since $G_{\overline{10}}$ is small the admixtures proportional to the reduced matrix elements

$$\begin{aligned}
H_{\overline{10}} &\sim \left\langle \overline{10}_{1/2},B'\right|\hat{O}_\varphi^{(8)}\left|\overline{10}_{1/2},B\right\rangle\,,\\
H'_{27} &\sim \left\langle 27_{1/2},B'\right|\hat{O}_\varphi^{(8)}\left|\overline{10}_{1/2},B\right\rangle
\end{aligned} \tag{9}$$

are important even if mixing parameters $c_{\overline{10}}$ and c_{27} are not large (for definitions see Ref. [19]). Neither $H_{\overline{10}}$ nor H'_{27} vanish in the nonrelativistic limit. Therefore in this limit Θ^+ decay occurs entirely due to the mixing. For realistic model parameters (when $G_{\overline{10}} > 0$) there is a cancellation (note that $H_{\overline{10}} < 0$ and $H'_{27} > 0$) between different terms in (8) and $g_{\Theta NK}$ is further suppressed. Mixing affects decay patterns of pentaquarks violating SU(3) relations between the decay constants $g_{B_1 B_2 \varphi}$ [19].

On somewhat more phenomenological ground let us consider only $8 \leftrightarrow \overline{10}$ mixing [7] defining

$$g_{\theta NK} = \cos\alpha\, g_{\overline{10}} + \sin\alpha\, h_{\overline{10}}\,, \tag{10}$$

which can be directly extracted from (2) if $\Gamma_{\Theta NK}$ is known. Throughout this paper we assume that $\Gamma_{\Theta NK} \simeq 1$ MeV, hence $g_{\theta NK} \simeq 1.4$. It is then possible to express decay constants of other members of antidecuplet in terms of measurable physical parameters such as $g_{\pi NN} \simeq 13.2$, $\varepsilon = F/D \simeq 0.56$, mixing angle α and one *a priori* unknown parameter $h_{\overline{10}}$ which can be estimated from χQSM calculations. Here we take $h_{\overline{10}} = -7$ [7]. Then we have for example,

$$\begin{aligned} g_{N_{\overline{10}}N\pi} &= \tfrac{1}{2}\cos\alpha\, g_{\theta NK} - \tan\alpha\sqrt{3}g_{\pi NN}\,, \\ g_{N_{\overline{10}}N\eta} &= \tfrac{1}{2}\cos\alpha\, g_{\theta NK} - \tfrac{1}{2}\sin 2\alpha\, h_{\overline{10}} + \tan\alpha\frac{3\varepsilon-1}{1+\varepsilon}\,\frac{g_{\pi NN}}{\sqrt{3}}\,. \end{aligned} \tag{11}$$

If we want to interpret $N(1685)$ as $N_{\overline{10}}$ we need to satisfy bounds (1). This is quite difficult within one angle scenario. It is possible to nullify $g_{N_{\overline{10}}N\pi}$ by a suitable choice of mixing angle $\alpha \sim 0.03$, but then the mean octet mass (*i.e.* the nucleon mass before mixing) which is experimentally 1151 MeV comes out wrong [7]

$$M_8 = M_N^{\rm phys}\cos^2\alpha + M^2_{N_{\overline{10}}}\sin^2\alpha \simeq M_N^{\rm phys}\,. \tag{12}$$

The mixing angle that satisfies (1) is an order of magnitude too small to account for baryon masses. For realistic mixing angles $g_{N_{\overline{10}}N\pi}$ is dominated by $g_{\pi NN}$ and $\Gamma_{N_{\overline{10}}N\pi} > \Gamma_{N_{\overline{10}}N\eta}$ in contradiction with experimental data for $N(1685)$.

4. Mixing with Roper

In order to satisfy conditions (1) with more realistic mixing angles we have considered in Ref. [7] scenario in which exotic antidecuplet can mix with Roper resonance octet (mixing angle ϕ). For Roper octet GMO mass formulae work with much worse accuracy than for the ground state octet [20, 21], so there is a need for additional mixing. Since $\varepsilon_{\rm Roper} \sim 1/3$ [21]

Roper decay to $N\eta$ is negligible. However Roper admixture contributes to $N\pi$ and other decay modes

$$\begin{aligned} g_{N_{\overline{10}}N\pi} &= \tfrac{1}{2}\cos\phi\cos\alpha\, g_{\theta NK} - \cos\phi\tan\alpha\sqrt{3}g_{\pi NN} - \tan\phi\, g_{RN\pi}\,, \\ g_{N_{\overline{10}}N\eta} &= \tfrac{1}{2}\cos\phi\cos\alpha\, g_{\theta NK} - \tfrac{1}{2}\cos\phi\sin 2\alpha\, h_{\overline{10}} \\ &\quad + \cos\phi\tan\alpha\frac{3\varepsilon-1}{1+\varepsilon}\,\frac{g_{\pi NN}}{\sqrt{3}}\,. \end{aligned} \tag{13}$$

Since $g_{RN\pi}\sim 12$ is comparable with $g_{\pi NN}$ one may suppress $g_{N_{\overline{10}}N\pi}$ without changing much $g_{N_{\overline{10}}N\eta}$. In Ref. [7] we have found that conditions (1) are satisfied in the vicinity of the line

$$\phi(\alpha) = 0.0508 - 2.207\alpha\,, \qquad 0.079 < \alpha < 0.159\,. \tag{14}$$

We see that mixing angles are reasonable. The decay widths and branching ratio to $N\eta$ along (14) are plotted in Fig. 1.

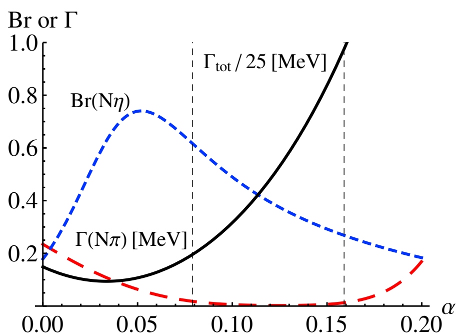

Fig. 1. Total, solid black line, decay width (divided by 25) and partial decay width of $N_{\overline{10}}$ to πN, long dashed (red) line in MeV together with branching ratio, short dashed (blue) line of $N_{\overline{10}}\to\eta N$ as functions of mixing angle α along the line (14). Thin vertical lines correspond to the limits on the mixing angle α. The plot is made for $h_{\overline{10}} = -7$.

In Fig. 2 we plot $g_{N_{\overline{10}}N\pi}$ and $g_{N_{\overline{10}}N\eta}$ together with their different components along the line (14). We see that indeed $g_{N_{\overline{10}}N\pi}$ is small due to the cancellation between $g_{\pi NN}$ and $g_{RN\pi}$, while $g_{N_{\overline{10}}N\eta}$ rises moderately when mixing increases.

Having established the range of mixing angles we can predict masses of the remaining antidecuplet members [7]

$$1795\,\text{MeV} < M_{\Sigma_{\overline{10}}} < 1830\ \text{MeV}\,, \tag{15}$$

$$1900\,\text{MeV} < M_{\Xi_{\overline{10}}} < 1970\ \text{MeV}\,. \tag{16}$$

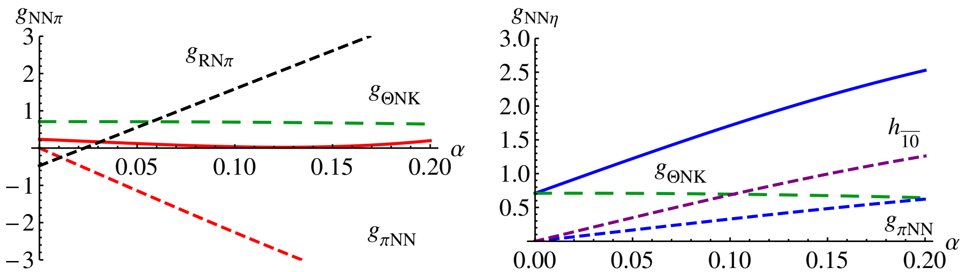

Fig. 2. Decay constants of $N_{\overline{10}}$ (solid lines) with different components defined in Eq. (13) shown by dashed lines. The plot is made for $h_{\overline{10}} = -7$.

Note that bound (16) contradicts the result of NA49 $M_{\Xi_{\overline{10}}} \sim 1860$ MeV [22]. From our analysis it follows that total decay width of $\Xi_{\overline{10}}$ to $K\Sigma$ and $\pi\Xi$ is of the order of 10 MeV. Total width of $\Sigma_{\overline{10}}$ does not exceed 30 MeV but is also constrained from below to be larger that 10 MeV. Most prominent decay channels are KN and $\pi\Lambda$ with branching ratios approximately 60% and 20%, respectively. Due to the mixing SU(3) forbidden decays to decuplet are possible, but small, at the level of 5 to 9%.

5. Summary and conclusions

Mixing induced by m_s was first studied within the χQSM already in Ref. [15] but only in the leading order in N_c. It was extended to nonleading terms in Ref. [8] and [19]. Mixing appears also in other approaches to pentaquarks. For example, in a diquark model [23] antidecuplet mixes with an accompanying cryptoexotic octet. Diagonalization of strangeness induces *ideal* (large) mixing between these two representations. The resulting physical states of nucleon quantum numbers have been interpreted as Roper and $N^*(1710)$. However, due to the ideal mixing these two states should have comparable widths [24], while experimentally they differ substantially. The discussion of masses and decay widths of the N^* states under assumption that they correspond to the Roper and $N^*(1710)$ done in Ref. [25] still indicates that it is impossible to match the mass splittings with the observed branching ratios for these two resonances even for arbitrary mixing. Whether any different assignment of the diquark N^* states would be compatible with the decay patterns deserves a separate study.

In this short note we have argued that due to the smallness of the reduced matrix elements of $\overline{10} \to 8$, which is natural in chiral soliton models, mixing with other SU(3) representations has to be taken into account. Unfortunately, for the time being we can only speculate which mixing scenario is phenomelogically impossible, allowed or desired. Here we have examined a possibility that antidecuplet mixes with the Roper octet. Mixing of Roper and the ground state octets is presumably very small. Indeed, the first order GMO mass formulae work very well for the ground state octet so there is almost no space for additional mixing.

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