Vol. 3 (2010) *Acta Physica Polonica B Proceedings Supplement* No 4

LIGHT TETRAQUARK STATE AT NONZERO TEMPERATURE*

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(Received August 9, 2010)

We study the implications of a light tetraquark on the chiral phase transition at nonzero temperature T: the behavior of the chiral and four-quark condensates and the meson masses are studied in the scenario in which the resonance $f_0(600)$ is described as a predominantly tetraquark state. It is shown that the critical temperature is lowered and the transition softened. Interesting mixing effects between tetraquark and quarkonium configurations take place.

PACS numbers: 11.30.Rd, 11.30.Qc, 11.10.Wx, 12.39.Mk

1. Introduction

In the last decades theoretical and experimental work on light scalar mesons with masses below $\sim 1.8\,\mathrm{GeV}$ [1] initiated an intense debate about their nature. Quarkonia, tetraquark and mesonic molecular assignments, together with the inclusion of a scalar glueball state around $1.5\,\mathrm{GeV}$ as suggested by lattice simulations, have been proposed and investigated in a variety of combinations and mixing patterns [2].

Nowadays evidence toward a full nonet of scalars below $1\,\mathrm{GeV}$ is mounting: $f_0(600)$, $f_0(980)$, $a_0(980)$, and $K_0^*(800)$. An elegant way to explain such resonances is the tetraquark assignment proposed long ago by Jaffe [3]. The reversed mass ordering is naturally explained in this way and also decays can be successfully reproduced [4]. Within this context the lightest scalar resonance $f_0(600)$ is interpreted as a predominantly tetraquark state $1/2[u,d][\bar{u},\bar{d}]$, where the commutator indicates an antisymmetric flavor configuration of the diquark.

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* Presented by A. Heinz at the Workshop "Excited QCD 2010", Tatranská Lomnica/ Stará Lesná, Tatra National Park, Slovakia, January 31–February 6, 2010.

The lightest quark–antiquark state, *i.e.* the chiral partner of the pion with flavor wave function $\bar{n}n = \sqrt{1/2}(\bar{u}u + \bar{d}d)$, is then predominately identified with the broad resonance $f_0(1370)$. The fact that scalar quarkonia are p-wave states supports this choice. According to this picture quarkonia states, together with the scalar glueball, lie above 1 GeV, see Ref. [5] and references therein.

It is natural to ask how the here outlined scenario affects the physics at nonzero temperature T. It is in fact different form the usual assumptions made in hadronic models at $T > 0$, where the chiral partner of the pion has a mass of about 600 MeV. Moreover, besides the chiral condensate, new quantities emerge: a tetraquark condensate and the mixing of tetraquark and quarkonium states in the vacuum and at nonzero T. Remarkably, the mixing angle increases for increasing T and the behavior of the chiral condensate is affected by the presence of the tetraquark field. Details can be found in Ref. [6], on which this proceeding is based.

2. The model

We work with a simple chiral model with the following fields: the pion triplet $\vec{\pi}$, the bare quarkonium field $\varphi \equiv \bar{n}n$, and bare tetraquark field $\chi \equiv 1/2[u,d][\bar{u},\bar{d}]$. The chiral potential was derived in Ref. [7]

$$V = \frac{\lambda}{4}\left(\varphi^2 + \vec{\pi}^2 - F^2\right)^2 - \varepsilon\varphi + \frac{1}{2}M_\chi^2\chi^2 - g\chi(\varphi^2 + \vec{\pi}^2)\,, \tag{1}$$

where, besides the usual Mexican hat, the parameter g describes the interaction strength between the quark–antiquark fields and the tetraquark field χ. In the limit $g \to 0$ the field χ decouples, and a simple linear sigma model for φ and $\vec{\pi}$ emerges. The minimum of the potential (1) is to order $O(\varepsilon)$

$$\varphi_0 \simeq \frac{F}{\sqrt{1 - 2g^2/(\lambda M_\chi^2)}} + \frac{\varepsilon}{2\lambda F^2}\,, \qquad \chi_0 = \frac{g}{M_\chi^2}\varphi_0^2\,, \tag{2}$$

and $\vec{\pi}_0 = 0$. The condensate φ_0 is identified with the pion decay constant $f_\pi = 92.4$ MeV. Note that the tetraquark condensate χ_0 is proportional to φ_0^2: it is induced by spontaneous symmetry breaking in the quarkonium sector. Shifting the fields by their vacuum expectation values (v.e.v.s) $\varphi \to \varphi + \varphi_0$ and $\chi \to \chi + \chi_0$, and expanding around the minimum, we obtain, up to second order in the fields

$$V = \frac{1}{2}(\chi,\varphi)\begin{pmatrix} M_\chi^2 & -2g\varphi_0 \\ -2g\varphi_0 & M_\varphi^2 \end{pmatrix}\begin{pmatrix} \chi \\ \varphi \end{pmatrix} + \frac{1}{2}M_\pi^2\vec{\pi}^2 + \ldots\,, \tag{3}$$

where

$$M_\varphi^2 = \varphi_0^2 \left(3\lambda - \frac{2g^2}{M_\chi^2}\right) - \lambda F^2 \,, \qquad M_\pi^2 = \frac{\epsilon}{\varphi_0} \,. \tag{4}$$

Since the mass matrix has off-diagonal terms the fields φ and χ are not mass eigenstates. The mass eigenstates H and S, identified with the resonances $f_0(600)$ and $f_0(1370)$, respectively, are obtained upon a SO(2) rotation of the fields φ and χ

$$\begin{pmatrix} H \\ S \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} \chi \\ \varphi \end{pmatrix} , \qquad \theta_0 = \frac{1}{2} \arctan \frac{4g\varphi_0}{M_\varphi^2 - M_\chi^2} \,. \tag{5}$$

The tree-level masses of H and S are

$$M_H^2 \;=\; M_\chi^2 \cos^2\theta_0 + M_\varphi^2 \sin^2\theta_0 - 2g\varphi_0 \sin(2\theta_0) \,, \tag{6}$$

$$M_S^2 \;=\; M_\varphi^2 \cos^2\theta_0 + M_\chi^2 \sin^2\theta_0 + 2g\varphi_0 \sin(2\theta_0) \,. \tag{7}$$

For the reasons discussed in the Introduction, the bare tetraquark is chosen to be lighter than the bare quarkonium, thus: $M_S > M_\varphi > M_\chi > M_H$. The state $H \equiv f_0(600)$ is the predominantly tetraquark state, and the state $S \equiv f_0(1370)$ is the predominantly quarkonium state.

3. Results and discussions

In order to investigate the nonzero T behavior, we employ the CJT-formalism in the Hartree–Fock approximation [8]; for specification of the method in the case of mixing we refer to [9]. The CJT-formalism leads to temperature dependent masses $M_S(T)$, $M_H(T)$, $M_\pi(T)$ and a temperature dependent mixing angle $\theta(T)$. Moreover, both scalar–isoscalar fields have a T-dependent v.e.v., for the quarkonium $\varphi_0 \to \varphi(T)$ and for the tetraquark $\chi_0 \to \chi(T)$. For both fields zero-temperature limits $\varphi(0) = \varphi_0$ and $\chi(0) = \chi_0$ of Eq. (2) hold.

When the tetraquark decouples (limit $g \to 0$), S is a pure quarkonium and H is a pure tetraquark. The transition is crossover for $M_S \leq 0.95\,\mathrm{GeV}$ and first order above this value. This is a well established result, *e.g.* in Ref. [10]. The fact that a heavy chiral partner (*i.e.*, mass larger $1\,\mathrm{GeV}$) of the pion leads to a first order phase transition disagrees with lattice QCD calculations [11].

The inclusion of the tetraquark state changes this conclusion as shown in Fig. 1: In Fig. 1 (a) $M_H = 0.4\,\mathrm{GeV}$ is fixed and the parameters M_S and g are varied. In Fig. 1 (b) the behavior of the quark condensate for fixed $M_S = 1.0\,\mathrm{GeV}$ and $M_H = 0.4\,\mathrm{GeV}$ is shown for different values of the parameter g. One observes that for increasing values of the coupling

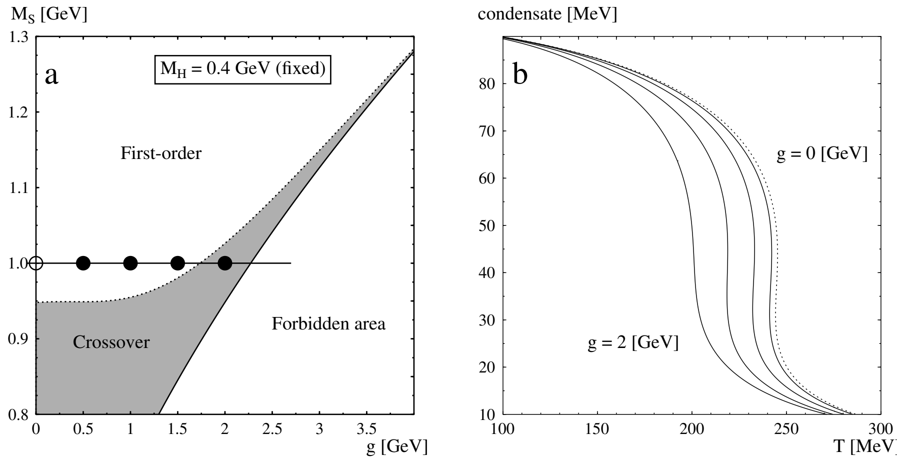

Fig. 1. (a) Order of the phase transition as a function of the parameters of the model. $M_H = 0.4\,\mathrm{GeV}$ and M_S and g are varied. The forbidden area violates the constraint $\left|M_S^2 - M_H^2\right| \geq 4g\varphi_0$ [6]. On the border line between the first-order and the crossover transitions a second-order phase transition is realized. (b) The chiral condensate is shown for $M_H = 0.4$ GeV and $M_S = 1.0\,\mathrm{GeV}$ for different values of g (step of 0.5 GeV). Note, the dots in panel (a) correspond to the curves in panel (b).

g the critical temperature T_c decreases: while $T_\mathrm{c} = 250\,\mathrm{MeV}$ for $g \to 0$, the value $T_\mathrm{c} \simeq 200\,\mathrm{MeV}$ is obtained for $g = 2.0\,\mathrm{GeV}$. Also the order of the phase transition is affected: when increasing the parameter g, the first order transition is softened and, if the coupling is large enough, becomes a crossover.

We now turn to the explicit evaluation of masses, condensates, and the mixing angle at nonzero T. The masses are chosen to be in the range quoted by [1, 12]: $M_S = 1.2\,\mathrm{GeV}$ and $M_H = 0.4\,\mathrm{GeV}$. The coupling strength is set to $g = 3.4\,\mathrm{GeV}$ in order to obtain a crossover phase transition. Together with the pion mass $M_\pi = 139\,\mathrm{MeV}$ and the pion decay constant $\varphi_0 = f_\pi = 92.4\,\mathrm{MeV}$ the parameter are determined as: $\lambda = 52.85$, $M_\chi = 0.96\,\mathrm{GeV}$, and $F = 64.2\,\mathrm{MeV}$.

The behavior of the two condensates is shown in Fig. 2 (a). At $T_\mathrm{c} = 180\,\mathrm{MeV}$ the quark condensate $\varphi(T)$ drops and approaches zero, thus restoring chiral symmetry. Below T_c the tetraquark condensate $\chi(T)$ follows the quark condensate, but above T_c the condensate starts to increase. (This result could be different if additional terms $\sim \chi^4$ in Eq. (1) were included).

By increasing T the function $M_S(T)$ first drops softly, but at a certain temperature $T_s \simeq 160\,\mathrm{MeV}$ a step-like decrease occurs, while the function $M_H(T)$ undergoes a step-like increase. The solid line in Fig. 2 (b) describes

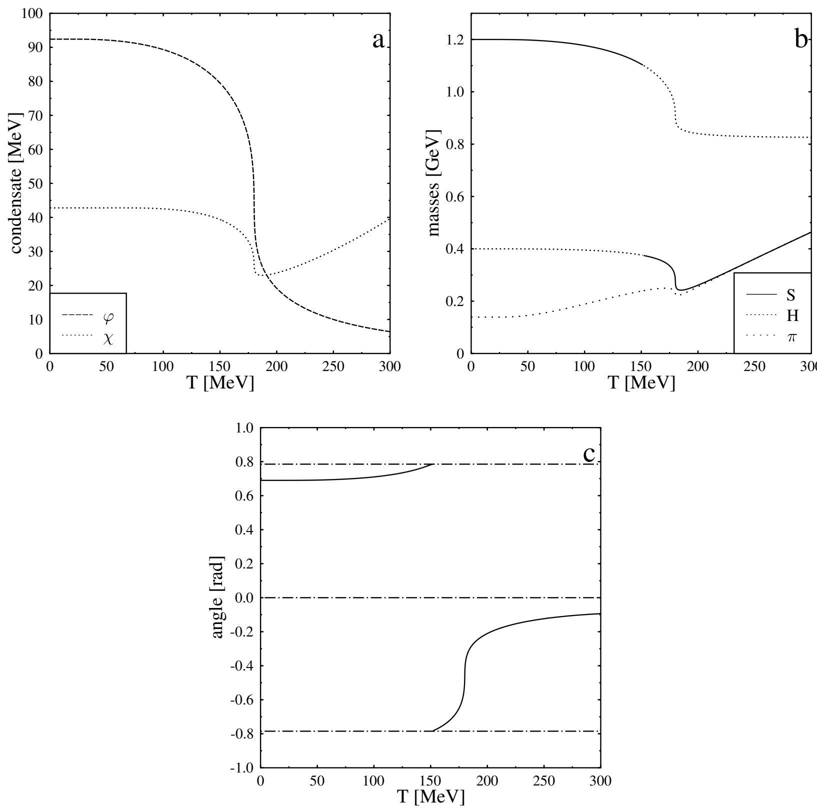

Fig. 2. Condensates (a), masses (b), and mixing angle (c) as function of T. (From Ref. [6]).

the state S according to the following criterion: S is the state containing the largest amount of the bare quarkonium state φ. For $T < T_s$ it corresponds to the heavier state, for $T > T_s$ to the lighter one. A similar analysis holds for the dashed line referring to H as the state with the largest bare tetraquark amount.

The mixing angle $\theta(T)$ shown in Fig. 3 (c). At T_s the mixing becomes maximal and the angle jumps suddenly from $\pi/4$ to $-\pi/4$, $\lim_{T\to T_s} = \mp\pi/4$. At T_s the two physical states H and S have the same amount (50%) of quarkonium and tetraquark.

4. Conclusions

We have shown that the interpretation of $f_0(600)$ as a predominantly tetraquark state sizably affects the thermodynamical properties of the chiral phase transition: the behavior of the quark condensate is softened rendering the order of the phase transition cross-over for large enough tetraquark–quarkonium interaction, and the value of the critical temperature T_c is reduced, in agreement with recent Lattice simulations [11].

In future studies one should include for a complete treatment the other scalar–isoscalar states $f_0(980)$, $f_0(1500)$, and $f_0(1710)$ which appear in a $N_f = 3$ context (together with the inclusion of the scalar glueball). Also, (axial-)vector mesons shall be considered [13]. Nevertheless, the emergence of mixing of tetraquark and quarkonium states is general, and its relevant role at nonzero temperature is expected also in this generalized context.

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