

COMPLEX MASSES IN THE $\mathcal{S}$ -MATRIX*

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Most excited hadrons have multiparticle strong decay modes, which can often be described as resulting from intermediate states containing one or two resonances. In a theoretical approach, such a description in terms of quasi-two-particle initial and final states leads to unitarity violations, because of the complex masses of the involved resonances. In the present paper, an empirical algebraic procedure is presented to restore unitarity of the $\mathcal{S}$ -matrix while preserving its symmetry. Preliminary results are presented in a first application to S -wave $\pi\pi$ scattering, in the framework of the Resonance-Spectrum Expansion.

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1. Introduction

Inspection of the Particle Data Group (PDG) tables [1] reveals that most excited mesons and baryons have strong decay modes involving more than two lighter hadrons. Moreover, one also verifies that many of these decays can be considered “cascades” of decays from intermediate states involving one or more resonances. A few mesonic examples of such processes are [1]

1. $f_0(1370) \rightarrow \rho\rho, 2(\pi\pi)_{S\text{-wave}}, \dots \rightarrow 4\pi;$
2. $K_2(1770) \rightarrow K_2^*(1430)\pi, Kf_2(1270), \dots \rightarrow K\pi\pi;$
3. $\phi(2170) \rightarrow \phi f_0(980) \rightarrow \phi\pi\pi, KKf_0(980) \rightarrow KK\pi\pi.$

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In the first example, $2(\pi\pi)_{S\text{-wave}}$ stands for $\sigma\sigma$, where σ is the very broad $f_0(600)$ [1] scalar resonance. In all three cases, the final state contains 3 or even 4 mesons that are stable with respect to strong interactions. In order to describe such processes in a mathematically rigorous way, respecting unitarity, one would have to resort to (relativistic) Faddeev [2] or Yakubovsky [3] equations, respectively, in the scattering regime. As a matter of fact, such a Faddeev approach was applied [4] to the $\phi(2170)$, as a three-body resonance in ϕKK and $\phi\pi\pi$, though not including the $KKf_0(980)$ component and also not the 4-body $KK\pi\pi$ channel. Note that solving relativistic 3- or 4-body equations with many channels is already a huge task, but it becomes absolutely impracticable if one wants to go beyond an effective description as in Ref. [4], by taking into account the quark substructure of the decaying resonance.

In experimental data analyses, one often falls back upon dispersive or purely phenomenological parametrisations, as *e.g.* in Refs. [5,6] for the description of the 4π channel in S -wave $\pi\pi$ scattering above 1 GeV and the $f_0(1370)$ resonance.

An alternative, theoretical approach is to interpret the intermediate state in a cascade decay as a (quasi-)final state, containing one or two resonances as outgoing particles. The problem then arises how to deal with the, in principle, complex mass(es) of the resonance(s), while preserving two-body unitarity. One way is by discretising the resonance real-mass distribution(s), and accordingly including a large number of channels having the corresponding threshold energies, with relative coupling strengths given by the mass-distribution function. Something in this spirit has been carried out in a coupled-channel description of S -wave meson-meson scattering in a chiral unitary model [7]. A drawback of such a method is the proliferation of channels, besides the problem of properly dealing with resonances not far above threshold.

In the present study, we shall try out a novel, intuitive approach, by simply substituting complex values for the masses of the final-state resonances, according to their listed [1] real (Breit-Wigner) masses and widths. Although this step inexorably destroys unitarity, we can afterwards restore it by a suitable redefinition of the $\mathcal{S}$ -matrix. This quite general procedure is then applied to a concrete test case, in the framework of the Resonance-Spectrum Expansion.

2. Resonance-Spectrum Expansion

The Resonance-Spectrum Expansion (RSE) is a model for the scattering of two mesons in non-exotic channels, via an infinite set of intermediate s -channel $q\bar{q}$ states, *i.e.*, a kind of Regge propagators [8]. The confinement spectrum for these bare $q\bar{q}$ states can, in principle, be chosen freely, but in

all successful phenomenological applications so far we have used a harmonic-oscillator (HO) spectrum with flavour-independent frequency (see Ref. [8] for several references). Because of the separability of the effective meson–meson interaction, the RSE model can be solved in closed form. The relevant Born and one-loop diagrams are depicted in Fig. 1. For N meson–meson channels

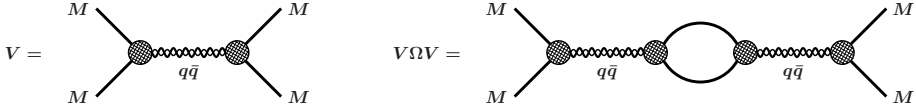


Fig. 1. Born and one-loop term of the RSE effective meson–meson interaction.

and several $q\bar{q}$ channels, the effective potential has the form

$$\begin{aligned} \mathcal{V}_{ij}^{(L_i, L_j)}(p_i, p'_j; E) &= \lambda^2 r_0 j_{L_i}^i(p_i r_0) j_{L_j}^j(p'_j r_0) \sum_{\alpha=1}^{N_{q\bar{q}}} \sum_{n=0}^{\infty} \frac{g_i^{(\alpha)}(n) g_j^{(\alpha)}(n)}{E - E_n^{(\alpha)}} \\ &\equiv \lambda^2 r_0 j_{L_i}^i(p_i r_0) j_{L_j}^j(p'_j r_0) \mathcal{R}_{ij}(E), \end{aligned} \quad (1)$$

where the RSE propagator $\mathcal{R}_{ij}(E)$ contains an infinite tower of s -channel bare $q\bar{q}$ states, corresponding to the spectrum of an, in principle, arbitrary confining potential. Here, $E_n^{(\alpha)}$ is the discrete energy of the n th recurrence in $q\bar{q}$ channel α , $g_i^{(\alpha)}(n)$ [9] is the corresponding coupling to the i th meson–meson channel, $j_{L_i}^i(p_i)$ is the L_i th order spherical Bessel function, p_i is the relativistically defined off-shell relative momentum in meson–meson channel i , r_0 is a distance parameter, and λ is an overall coupling constant. Note that the spherical Bessel function results from assuming 3P_0 $q\bar{q}$ pair creation/annihilation only to take place at a certain distance r_0 [8]. The closed-form off-energy-shell $\mathcal{T}$ -matrix then reads

$$\begin{aligned} \mathcal{T}_{ij}^{(L_i, L_j)}(p_i, p'_j; E) &= \\ -2\lambda^2 r_0 \sqrt{\mu_i p_i \mu'_j p'_j} j_{L_i}^i(p_i r_0) \sum_{m=1}^N \mathcal{R}_{im}(E) \{[\mathbb{1} - \Omega \mathcal{R}]^{-1}\}_{mj} j_{L_j}^j(p'_j r_0), \end{aligned} \quad (2)$$

where

$$\Omega(E) = -2i\lambda^2 r_0 \text{diag} \left(\mu_n k_n j_{L_n}^n(k_n r_0) h_{L_n}^{(1)n}(k_n r_0) \right), \quad (3)$$

with $h_{L_n}^{(1)n}(k_n r_0)$ the spherical Hankel function of the first kind, and k_n the on-shell relative momentum in meson–meson channel n . Finally, the corresponding unitary and symmetric (on-shell) $\mathcal{S}$ -matrix is given by

$$\mathcal{S}_{ij}^{(L_i, L_j)}(k_i, k_j; E) = \delta_{ij} + 2i\mathcal{T}_{ij}^{(L_i, L_j)}(k_i, k_j; E). \quad (4)$$

Note that the $\mathcal{S}$ -matrix is only unitary for real k_i , and so for real meson masses in channel i .

3. Redefining the $\mathcal{S}$ -matrix

As mentioned above, if we take a complex mass in any of the meson–meson channels, the $\mathcal{S}$ -matrix ceases to be unitary, but stays symmetric. This requires a redefinition of the $\mathcal{S}$ -matrix. Now, note that

$$\mathcal{S}^\dagger \mathcal{S} \equiv A \quad (5)$$

is not unity anymore, but it is still a Hermitian matrix, by definition, and so with real eigenvalues, which moreover are all positive in this special case. Thus, A can be diagonalised by a unitary matrix U

$$A_d = U \mathcal{S}^\dagger \mathcal{S} U^\dagger. \quad (6)$$

Define now

$$\tilde{\mathcal{S}} \equiv \mathcal{S} U^\dagger A_d^{-1/2} U, \quad (7)$$

where $A_d^{-1/2}$ is real, because of the positive eigenvalues. Then, it is straightforward to show that

$$\tilde{\mathcal{S}}^\dagger \tilde{\mathcal{S}} = \tilde{\mathcal{S}} \tilde{\mathcal{S}}^\dagger = \mathbb{1}. \quad (8)$$

It is not so easy to prove that $\tilde{\mathcal{S}}$ is also symmetric, but this has been checked numerically with a precision of better than one part in a trillion.

4. Preliminary results

Let us now use the redefined $\mathcal{S}$ -matrix of Eq. (8) in a purely comparative calculation. Starting point is our [10] application of the RSE to S -wave $\pi\pi$ scattering, with pseudoscalar–pseudoscalar, vector–vector, and scalar–scalar channels included.

Then, we substitute complex values for the masses of the broadest resonances in some of these channels, namely for the $f_0(600)$ (alias σ), $K_0^*(800)$ (alias κ), and ρ meson. Note that the precise values we take for the complex masses of the very broad σ and κ resonances are not so important here, but in future model fits to the data one should use the best available, “world-average” values for the corresponding pole positions. In Fig. 2, we plot the curves for the two cases. Quite significant differences become evident, especially above 1 GeV. Nevertheless, the curve obtained with $\tilde{\mathcal{S}}$ in Eq. (7) and complex masses looks reasonable, though no refit has been done.

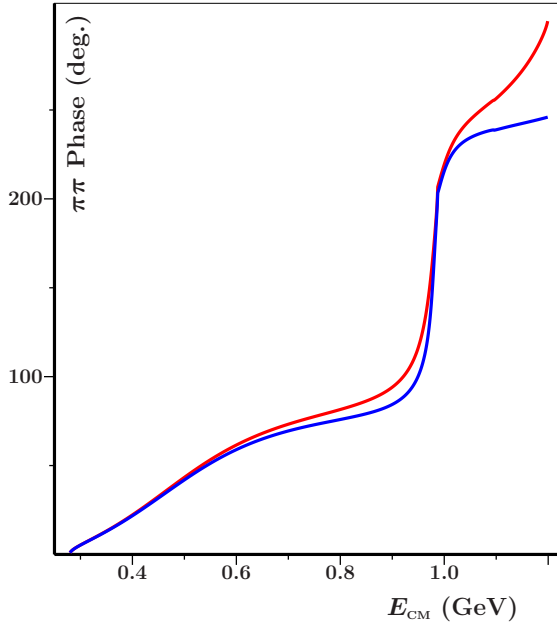


Fig. 2. RSE calculation of S -wave $\pi\pi$ phase shifts. Red (upper) curve: from standard $\mathcal{S}$ -matrix and real σ , κ , and ρ masses; blue (lower) curve: from redefined $\mathcal{S}$ -matrix in Eq. (7) and complex masses.

5. Summary

In the present study, we have carried out an empirical procedure in order to restore unitarity of a coupled-channel $\mathcal{S}$ -matrix with complex masses in the asymptotic states. Although no rigorous mathematical justification is given here for the purely algebraic transformation of the original $\mathcal{S}$ -matrix, preservation of the mandatory symmetry of $\mathcal{S}$ gives us confidence that the method makes sense. A direct comparison of S -wave $\pi\pi$ phase shifts, first calculated with the original $\mathcal{S}$ and using real masses for the σ , ρ , and κ resonances in some of the coupled channels, and then with the $\tilde{\mathcal{S}}$ of Eq. (7) and complex values for the latter masses, shows significant yet reasonable changes. Nevertheless, our procedure will have to be tested in concrete fits to the data so as to find out whether it is really a promising new approach to multiparticle hadronic decays.

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