STRUCTURE OF SCALAR MESONS $f_0(600)$, $a_0(980)$, $f_0(1370)$ AND $a_0(1450)$*

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We calculate low-energy meson decay processes and pion–pion scattering lengths in a globally chirally invariant two-flavour linear sigma model exploring the quark content of the scalar mesons $f_0(600)$ and $a_0(980)$, as well as $f_0(1370)$ and $a_0(1450)$. To this end, we investigate which one of these sets of fields is more likely to contain quark–antiquark states.

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1. Introduction

Mesons and baryons at low energies can be described using various effective field theories and models that contain hadrons as degrees of freedom rather than quarks and gluons. These effective approaches may be based on the linear [1] or the non-linear [2] realisation of the $\mathrm{U}(N_f)_\mathrm{R} \times \mathrm{U}(N_f)_\mathrm{L}$ (the so-called *chiral*) symmetry of the Quantum Chromodynamics (QCD) with N_f flavours. The chiral symmetry is broken in two ways: spontaneously, by the presence of the quark condensate $\langle \bar{q}q \rangle = \langle \bar{q}_\mathrm{R} q_\mathrm{L} + \bar{q}_\mathrm{L} q_\mathrm{R} \rangle \neq 0$ [3], and explicitely, by the non-zero, non-degenerate quark masses as well as the axial anomaly [4]. In this paper, we present a linear sigma model with vector and axial-vectors mesons and global chiral symmetry [5–7].

Our model contains two scalar fields, σ and a_0 [$\sigma = \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$, $a_0^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)$]. As outlined in Refs. [6,7], the fields σ and a_0 can be assigned to physical fields according to two different scenarios: (i) they can be identified with the states $f_0(600)$ and $a_0(980)$ (that form a part of a larger $\bar{q}q$ nonet

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which comprises $f_0(980)$, $a_0(980)$, $\kappa(800)$ and $f_0(600)$ — resonances below 1 GeV); (*ii*) they can be identified with the states $f_0(1370)$ and $a_0(1450)$ forming a part of a $\bar{q}q$ nonet (plus a glueball) that comprises $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $a_0(1450)$, $K_0(1430)$ — resonances above 1 GeV (see also Ref. [8]). Scalar mesons below 1 GeV, whose spectroscopic wave functions possibly contain a dominant tetraquark contribution [9], may be introduced in the second scenario as additional scalar fields or may arise as mesonic-molecular states because of interactions of pseudoscalar mesons [10].

In this paper, we will describe briefly the consequences of the assignments (*i*) and (*ii*). The paper is organised as follows: in Section 2 the model is introduced, in Section 3 the results for both scenarios are presented and in Section 4 we present our conclusions. More detailed results are described in Ref. [7].

2. The model

The Lagrangian of the globally invariant linear sigma model with $\mathrm{U(2)_L} \times \mathrm{U(2)_R}$ symmetry reads [5–7, 11]:

$$\begin{aligned}
\mathcal{L} \;=\; & \mathrm{Tr}\left[(D^\mu \Phi)^\dagger (D^\mu \Phi)\right] - m_0^2 \mathrm{Tr}\left(\Phi^\dagger \Phi\right) - \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \mathrm{Tr}\left(\Phi^\dagger \Phi\right)^2 \\
& - \tfrac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \frac{m_1^2}{2}\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right] \\
& + c\left(\det \Phi + \det \Phi^\dagger\right) - 2ig_2\left(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}\,[R^\mu, R^\nu]\}\right) \\
& - 2g_3\left[\mathrm{Tr}\left(\left\{\partial_\mu L_\nu - ieA_\mu\left[t^3, L_\nu\right] + \partial_\nu L_\mu - ieA_\nu\left[t^3, L_\mu\right]\right\}\{L^\mu, L^\nu\}\right)\right. \\
& \left. + \mathrm{Tr}\left(\left\{\partial_\mu R_\nu - ieA_\mu\left[t^3, R_\nu\right] + \partial_\nu R_\mu - ieA_\nu\left[t^3, R_\mu\right]\right\}\{R^\mu, R^\nu\}\right)\right] \\
& + \frac{h_1}{2}\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\mathrm{Tr}\left[(L^\mu)^2 + (R^\mu)^2\right] + h_2 \mathrm{Tr}\left[|\Phi R^\mu|^2 + |L^\mu \Phi|^2\right] \\
& + 2h_3 \mathrm{Tr}\left(\Phi R_\mu \Phi^\dagger L^\mu\right) + \dots\,, \qquad (1)
\end{aligned}$$

where $\Phi = (\sigma + i\eta_N)\, t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}$ (scalar and pseudoscalar degrees of freedom; our model is currently constructed for $N_f = 2$ — thus, our eta-meson η_N contains only non-strange degrees of freedom), $L^\mu = (\omega^\mu + f_1^\mu)\, t^0 + (\vec{\rho}^\mu + \vec{a}_1^\mu) \cdot \vec{t}$ and $R^\mu = (\omega^\mu - f_1^\mu)\, t^0 + (\vec{\rho}^\mu - \vec{a}_1^\mu) \cdot \vec{t}$ (vector and axial-vector degrees of freedom); t^0, $\vec{t}$ are the generators of U(2); $D^\mu \Phi = \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA^\mu[t^3, \Phi]$ (A^μ is the photon field), $L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu[t^3, L^\nu] - (\partial^\nu L^\mu - ieA^\nu[t^3, L^\mu])$, $R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu[t^3, R^\nu] - (\partial^\nu R^\mu - ieA^\nu[t^3, R^\mu])$. The dots refer to further globally invariant terms of naive scaling dimension four that are not important in the following. The explicit breaking of the global symmetry is described by the term $\mathrm{Tr}[H(\Phi + \Phi^\dagger)] \equiv h\sigma$ ($h = \mathrm{const.}$)

and the chiral anomaly by the term $c(\det\Phi + \det\Phi^\dagger)$ [4]. The reason to restrict the consideration to operators up to fourth order may be found in Ref. [12].

The Lagrangian (1) contains 12 parameters: λ_1, λ_2, c, h_0, h_1, h_2, h_3, m_0^2, g_1, g_2, g_3, m_1. The parameter g_3 is not important for the following. Six parameters can be determined using the masses m_π, m_{η_N} (about 700 MeV, obtained by 'unmixing' the physical η and η' mesons), m_ρ and m_{a_1}, the pion decay constant f_π (via the Eq. $\phi = Zf_\pi$) and the experimentally well-known decay width $\Gamma_{\rho\to\pi\pi} = (149.1 \pm 0.8)$ MeV [13]. One more parameter can be determined from the scenario-dependent value of m_{a_0} set to $m_{a_0(980)} = 980$ MeV in Scenario I and to $m_{a_0(1450)} = 1474$ MeV in Scenario II. All quantities of interest can be then expressed using four parameters, which for convenience are chosen to be m_σ, Z, m_1 and h_2.

3. Results and discussions

3.1. Scenario I: Scalar quarkonia below 1 GeV

The number of independent parameters can be decreased using the decay width $\Gamma_{f_1\to a_0(980)\pi} = (8.748 \pm 2.097)$ MeV which in our model depends on Z and h_2 and thus allows us to express h_2 via Z (the results are virtually independent of the experimental uncertainty of the decay width). We obtain $h_2 \sim 80$. Consequently, we are left with three independent parameters Z, m_1 and $m_\sigma \equiv m_{f_0(600)}$. These parameters can be calculated using the χ^2 method [7] considering the decay width $a_1 \to \pi\gamma$ (the experimental value is $\Gamma_{a_1\to\pi\gamma} = (0.640 \pm 0.246)$ MeV [13]) and the $\pi\pi$ scattering lengths a_0^0 and a_0^2 (with the values $a_0^0 = 0.218 \pm 0.020$ and $a_0^2 = -0.0457 \pm 0.0125$ [14]).

In this way we obtain $Z = 1.67 \pm 0.2$. The parameter m_1 has large uncertainties in this scenario. These can be further constrained using the following consideration: m_1 is a part of the ρ mass term

$$m_\rho^2 = m_1^2 + \frac{\phi^2}{2}[h_1 + h_2(Z) + h_3(Z)] \tag{2}$$

and can be related to the gluon condensate [7]. The term $\phi^2[h_1 + h_2(Z) + h_3(Z)]/2$ in Eq. (2) represents the contribution of the chiral condensate. Note that $h_2 \equiv h_2(Z)$ and $h_3 \equiv h_3(Z)$ are functions of Z [7]. We consider m_1^2 as varying between 0 and m_ρ^2: in fact, a negative m_1^2 would imply that the vacuum is not stable in the limit $\phi \to 0$; $m_1^2 > m_\rho^2$ would imply that the contribution of the chiral condensate is negative. This is counter-intuitive and at odds with various microscopic approaches such as the NJL model. Combining the condition $m_1 \in [0, m_\rho]$ with the results from our χ^2 calculation then yields $m_1 = (652^{+123}_{-652})$ MeV. Finally, the scattering length a_0^0 leads to $m_\sigma = 332^{+145}_{-44}$ MeV [7].

With these values of Z, m_1 and m_σ, we obtain the decay width $\Gamma_{\sigma\to\pi\pi}$, see Fig. 1. The values of $\Gamma_{\sigma\to\pi\pi}$ are too small when compared to the PDG data [13]. Even for the largest value $m_\sigma = 477$ MeV allowed by the scattering length a_0^0 we obtain at most $\Gamma_{\sigma\to\pi\pi} \cong 145$ MeV. In all other cases, the decay width is smaller. Hence, we conclude that the isoscalar meson in our model is most likely not $f_0(600)$, thus disfavouring this resonance (and consequently also $a_0(980)$) as a predominantly $\bar{q}q$ state.

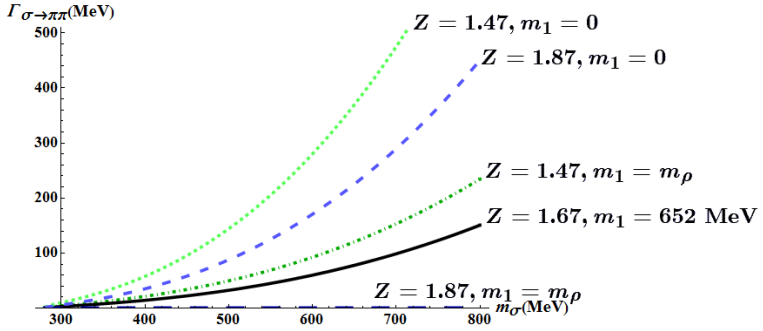


Fig. 1. $\Gamma_{\sigma\to\pi\pi}$ as function of m_σ. The PDG [13] quotes $\Gamma_{\sigma\to\pi\pi} = (600\text{–}1000)$ MeV.

3.2. Scenario II: Scalar quarkonia above 1 GeV

The aforementioned problem of the unphysically small two-pion decay width of the sigma meson may be resolved by identifying the fields σ and a_0 of our model with the resonances $f_0(1370)$ and $a_0(1450)$, respectively.

The number of free parameters (m_σ, Z, m_1 and h_2) can be decreased expressing the parameter h_2 via Z using the total decay width $\Gamma_{a_0\equiv a_0(1450)} = (265 \pm 13)$ MeV [13] (we obtain $h_2 \simeq 5^{+5}_{-30}$). Combining the decay width $\Gamma_{a_1\to\pi\gamma} = (0.640 \pm 0.246)$ MeV [13], that in our model depends only on Z, with the condition $m_1 \leq m_\rho$ in Eq. (2), where the large-N_c suppressed $h_1 \equiv 0$ [7], we obtain $Z = 1.67^{+0.21}_{-0.07}$ and $m_1 = 720^{+55}_{-140}$ MeV. Therefore, the parameter m_1 is much better constrained in Scenario II than in Scenario I; the gluon condensate appears to be dominant in m_ρ. We are thus left with one free parameter, $m_\sigma \equiv m_{f_0(1370)}$, that will be varied within the experimentally known band [13] in order to ascertain whether our result for $\Gamma_{f_0(1370)\to\pi\pi}$ is in agreement with experimental data.

The two-pion decay width of $f_0(1370)$ is represented in Fig. 2. Under the assumption that $f_0(1370)$ predominantly decays into two pions [15], we obtain a good agreement with the experimental values [13] if $m_{f_0(1370)} \leq 1380$ MeV. Other contributions to the decay width may reduce this upper bound on $m_{f_0(1370)}$ but nonetheless the correspondence with the experimental data is apparently a lot better in this scenario. Therefore, our results favour

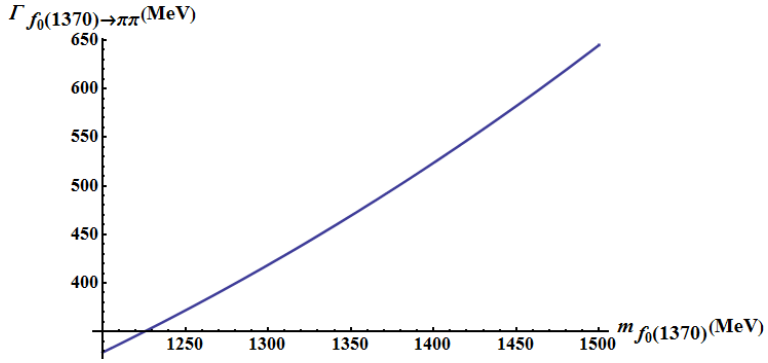


Fig. 2. $\Gamma_{f_0(1370)\to\pi\pi}$ as function of $m_{f_0(1370)}$. The PDG [13] quotes the mass in the range 1200–1500 MeV and the width between 200 and 500 MeV.

$f_0(1370)$ as a (predominantly) scalar quarkonium. Then the $a_0(1450)$ meson (the decay width of which was used to express h_2 via Z) is also favoured to be predominantly a $\bar{q}q$ state. Note that the results for the four-pion decay channel of $f_0(1370)$ which follow from the Lagrangian (1) are also in a good agreement with the experimental data (see Ref. [7]).

4. Conclusions

A linear sigma model with vector and axial-vector mesons and global chiral invariance has been presented. The assignment of the fields in the model to the physical states is straightforward in all cases with the exception of the scalars σ and a_0. They can be assigned either to $f_0(600)$ and $a_0(980)$ (Scenario I) or to $f_0(1370)$ and $a_0(1450)$ (Scenario II).

As shown in the previous section, Scenario I fails to describe the two-pion decay width of the $f_0(600)$ meson thus disfavouring it, and also $a_0(980)$, as (predominantly) $\bar{q}q$ states. The decay modes of other particles that can be considered in this scenario are, however, in agreement with experimental data [7].

If the scalar fields σ and a_0 are assigned to the $f_0(1370)$ and $a_0(1450)$ states, respectively, then the result for the two-pion decay width of the isoscalar meson is considerably improved. We obtain $\Gamma_{f_0(1370)\to\pi\pi} \simeq$ (300–500) MeV for $m_{f_0(1370)} =$ (1200–1400) MeV (see Fig. 2). Thus, the assertion that the scalar mesons above 1 GeV, $f_0(1370)$ and $a_0(1450)$, are (predominantly) $\bar{q}q$ states appears to be favoured by the experimental data.

As outlined in Ref. [7], future work should include the mixing of the quarkonia with a glueball as well as calculations in the $N_f = 3$ sector [16] and at nonzero temperatures and chemical potentials (such as those of Ref. [17]) in order to study the chiral transition.

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