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# LATTICE SEARCHES FOR TETRAQUARKS AND MESONIC MOLECULES: LIGHT SCALAR MESONS AND $XYZ$ STATES*

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Searches for tetraquarks and mesonic molecules in lattice QCD are briefly reviewed. In the light quark sector the most serious candidates are the lightest scalar resonances $\sigma$, $\kappa$, $a_0$ and $f_0$. In the hidden-charm sector I discuss lattice simulations of $X(3872)$, $Y(4260)$, $Y(4140)$ and $Z^+(4430)$. The most serious challenge in all these lattice studies is the presence of the scattering states in addition to possible tetraquark/molecular states. The available methods for distinguishing both are reviewed and the main conclusions of the simulations are presented.

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## 1. Introduction

Some of the observed resonances, *i.e.* light scalars [1] and some hidden-charm resonances [2], are strong candidates for tetraquarks $[qq][\bar{q}\bar{q}]$ or mesonic molecules $(\bar{q}q)(\bar{q}q)$. Current lattice methods do not distinguish between both types, so a common name "tetraquarks" will be often used to denote both types of $\bar{q}\bar{q}qq$ Fock components below.

In order to extract the information about tetraquark states, lattice QCD simulations evaluate correlation functions on $L^3\times T$ lattice with tetraquark interpolators $\mathcal{O}\sim\bar{q}\bar{q}qq$ at the source and the sink

$$C_{ij}(t)=\langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle_{\vec{p}=\vec{0}}\stackrel{T\to\infty}{\longrightarrow}\sum_n Z_i^n Z_j^{n*}\ e^{-E_n\ t}\,,\qquad n=1,2,\dots \quad (1)$$

If the correlation matrix is calculated for a number of interpolators $\mathcal{O}_{i=1,\dots,N}$ with given quantum numbers, the energies of the few lowest physical states

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$E_n$ and the corresponding couplings $Z_i^n \equiv \langle 0|\mathcal{O}_i|n\rangle$ can be extracted from the eigenvalues $\lambda^n(t) = e^{-E_n(t-t_0)}$ and eigenvectors $\vec{u}^n(t)$ of the generalized eigenvalue problem $C(t)\vec{u}^n(t) = \lambda^n(t,t_0)C(t_0)\vec{u}^n(t)$, as discussed in [3].

In addition to possible tetraquarks, also the two-meson scattering states $M_1M_2$ unavoidably contribute to the correlation function and this presents the main obstacle in extracting the information about tetraquarks. The scattering states $M_1(k)M_2(-k)$ at total momentum $\vec{p} = \vec{0}$ have discrete energy levels

$$E_{M_1M_2} \simeq E_{M_1}(k)+E_{M_2}(-k)\,, \qquad E_M(k)=\sqrt{m_M^2+\vec{k}^2}\,, \qquad \vec{k}=\frac{2\pi}{L}\vec{n} \quad (2)$$

in the non-interacting approximation when periodic boundary conditions in space are employed.

The resonance manifests itself on the lattice as a state in addition to the discrete tower of scattering states (2) [4–6] and it is often above the lowest scattering state (at $E \simeq M_1 + M_2$ for S-wave decay). So the extraction of a few states in addition to the ground state may be crucial. However, many simulations extract only the ground state energy $E_1$ using a conventional exponential fit $\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \propto e^{-E_1 t}$ at large $t$.

Once the physical states are obtained, one needs to determine whether a certain state corresponds to a one-particle (tetraquark) or a two-particle (scattering) state and the available methods to distinguish both are reviewed in the next section.

## 2. Methods to distinguish one-particle and scattering states

I am listing the available methods, which may be complementary:

- For a one-particle state $n$ the coupling $Z_i^n$ is expected to be almost independent of the lattice size $L$, *i.e.* $Z_i^n(L_1)/Z_i^n(L_2) \simeq 1$. For a two-particle state $n = M_1M_2$ one expects $Z_i^n(L_1)/Z_i^n(L_2) \simeq (L_2/L_1)^{3/2}$ if the range of interaction between $M_1$ and $M_2$ is much smaller than $L$ [4–8]. But this method leads to a reliable distinction only in presence of long stable plateaus, as cautioned in [9].
- One can distinguish whether the ground state is a one-particle or a two-particle state from the time-dependence of the $C_{ii}(t)$ near $t \simeq T/2$ at finite temporal extent $T$. In the case of (anti)periodic boundary conditions $C_{ii}(t) = |Z_i^1|^2\ [e^{-E_1 t} + \{t \to T-t\}]$ for one-particle ground state and $C_{ii}(t) = |Z_i^1|^2\ e^{-E_1 t} + |\tilde{Z}_i^1|^2\ e^{-m_{M_1} t} e^{-m_{M_2}(T-t)} + \{t \to T-t\}$ ($\tilde{Z}_i^n = \langle M_1^\dagger|\mathcal{O}_i|M_2\rangle$) for two-particle ground state [8]. The criteria for distinguishing the excited states is discussed in [5].

- An attractive interaction between two particles in a scattering state is manifested by a scattering length $a > 0$. A formation of a bound state below certain $m_\pi$ can be identified by the change of sign of $a$ from positive to negative as $m_\pi$ is lowered [10, 11]. The scattering length $a$ for S-wave scattering $M_1 M_2$ can be determined from the energy shift $\Delta E = E_1 - m_{M_1} - m_{M_2}$ on a finite lattice [10, 11].
- Certain non-conventional spatial boundary conditions have specified effects on the one- and two-particle energies, which allow to distinguish both types [14].

## 3. Light scalar resonances

It is still not established whether the lightest scalar mesons $\sigma$, $\kappa$, $a_0(980)$ and $f_0(980)$ are conventional $\bar{q}q$ states or they have important $\bar{q}\bar{q}qq$ Fock component, as strongly supported by some phenomenological studies [1]. The tetraquark interpretation implies that $I = 1$ state ($\bar{u}\bar{s}sd$) is heavier than the $I = 1/2$ state ($\bar{u}\bar{d}ds$) due to $m_s > m_d$, in agreement with experimental ordering $m_{a_0(980)} > m_\kappa$. One the other hand, the conventional $\bar{u}d$ and $\bar{u}s$ states can hardly explain the observed mass ordering. The tetraquarks interpretation also naturally explains the large observed coupling of $a_0(980)$ and $f_0(980)$ to $\bar{K}K$, which is due to the additional valence pair $\bar{s}s$.

All lattice simulations that look for tetraquark Fock component of light scalar mesons are quenched except for [4, 5]. All take tetraquark source/sink and omit the disconnected contractions in order to look for states with four valence quarks. The disconnected diagrams are also omitted since they are expensive for numerical evaluation and since they are often noisy.

- Prelovsek *et al.* [4, 5] performed the $N_f = 2$ dynamical and quenched simulation and extracted three lowest energy states in non-exotic $I = 0, 1/2$ and the exotic $I = 2, 3/2$ channels using variational method and a number of $[\bar{q}\bar{q}][qq]$ and $(\bar{q}q)(\bar{q}q)$ interpolators. The ground state in all the channels is found to be the scattering state $M_1(0)M_2(0)$ ($M_1 M_2 = \pi\pi$ or $K\pi$), as demonstrated using the time dependence of the diagonal correlators. The resulting $Z_i^1(L)$ is also roughly consistent with the expectation for a scattering state $Z_i^1(12)/Z_i^1(16) \simeq (16/12)^{3/2}$ (the ratios for $I = 0, 1/2$ have sizable errors, which do not allow to make a distinction) [5]. One of the states in all the channels is close to $M_1(\frac{2\pi}{L})M_2(-\frac{2\pi}{L})$ state.
  Additional light states are found in $I = 0$ and $I = 1/2$ channels, which may be related to observed $\sigma$ and $\kappa$ resonances with strong tetraquark components. The mass dependence of these candidates for $\sigma/\kappa$ on $m_\pi$ are in qualitative agreement with prediction of unitarized ChPT [13].

A simulation which takes into account also the disconnected diagrams will be needed to verify whether the additional states in $I = 0, 1/2$ channels are not some kind of unknown artifacts related to the omission of disconnected diagrams. In the repulsive $I = 2, 3/2$ channels no light state in addition to the scattering states $M_1(0)M_2(0)$ and $M_1(\frac{2\pi}{L})M_2(-\frac{2\pi}{L})$ is found, which is consistent with no experimentally observed resonances in these two channels.

- Mathur *et al.* [6] extract three lowest states in $I = 0$ channel from a single $\pi\pi$ correlator using the sequential Bayes method and a quenched simulation. The ground state energy is consistent with $\pi(0)\pi(0)$ and its coupling is consistent with the scattering state $Z^1(12)/Z^1(16) \simeq (16/12)^{3/2}$. The energy of the third state is consistent with $\pi(\frac{2\pi}{L})\pi(-\frac{2\pi}{L})$. They find an additional state inbetween, which behaves according to a one-particle expectation $Z^1(12)/Z^1(16) \simeq 1$. This state is a candidate for the observed $\sigma$ resonance with a strong tetraquark component. The presence of an additional state needs to be confirmed by a simulation that takes into account the disconnected contractions.
- Suganuma *et al.* [14] extract the ground state from a single $[\bar{q}\bar{q}][qq]$ correlator in $I = 0$ channel. They employ the conventional and hybrid boundary conditions which indicate that their ground state is a $\pi\pi$ scattering state.
- Alford and Jaffe [15] extract the $I = 0, 2$ ground state energy $E_1(L)$ from a $\pi\pi$ correlator for a number of lattice sizes $L$. They argue that $E_1^{I=2}(L)$ is in accordance with a scattering state, while $E_1^{I=0}(L)$ departs from the expected behavior for scattering states and may be an indication for $\sigma$.

## 4. Hidden charm resonances

The most prominent tetraquark candidate is charged $Z^+(4430)$ resonance, discovered by Belle [16]: it decays to $\pi^+\psi'$, so it must have a minimal quark content $\bar{d}u\bar{c}c$, but it has not been confirmed by Babar [17]. I will also discuss the observed neutral hidden charmonium resonances $X(3872)$, $Y(4260)$ and $Y(4140)$ [2,12], which are candidates for tetraquarks or mesonic molecules, although here the charmonium $\bar{c}c$ Fock component cannot be straightforwardly excluded based on the charge alone.

- Chiu and Hsieh [7] simulated states $\bar{c}\bar{q}cq$, $\bar{c}\bar{s}cs$, $\bar{c}\bar{c}cc$ and $\bar{c}\bar{q}cs$ ($q = u, d$) with $J^{\rm PC} = 1^{++}$ and $J^{\rm PC} = 1^{--}$. They used quenched simulation with overlap valence quarks and omit the disconnected diagrams. They extracted only the ground state energy $E_1$ and coupling $Z^1(L)$ from diagonal correlator $C_{ii}(t)$ for at two different $L = 20, 24$.

The ground $\bar{c}\bar{q}cq$ state with $J^{\rm PC} = 1^{++}$ is found at $3890 \pm 30$ MeV, which is indeed close to mass of the $X(3872)$. They find $Z^1(20)/Z^1(24) \simeq 1$, indicating a one-particle (tetraquark/molecular) state. Note however, that the lowest $DD^*$ S-wave scattering state with $E = m_D + m_{D*} \simeq 3879$ MeV is extremely close and that it should be found in addition to the one-particle state before the indication for the tetraquarks/molecules can be fully trusted.

The $\bar{c}\bar{s}cs$ state with $J^{\rm PC} = 1^{++}$ was found (predicted) at $4100 \pm 50$ MeV and $Z^1(20)/Z^1(24) \simeq 1$. A state with similar properties $Y(4140)$ was indeed later observed by CDF [18]. The state is again very close to the scattering threshold $m_\phi + m_{J/\psi} \simeq 4117$ MeV, so the scattering state has to be found also in order to trust the existence of tetraquark/molecule.

The ground $\bar{c}\bar{q}cq$ state with $J^{\rm PC} = 1^{--}$ is found at $4238 \pm 31$ MeV, which is indeed close to mass of the $Y(4260)$. They find $Z^1(20)/Z^1(24) \simeq 1$, indicating a one-particle (tetraquark/molecular) state.

In all three cases a one-particle nature was deduced from $Z^1(20)/Z^1(24) \simeq 1$. However, the cautionary remarks concerning $Z(L)$ [9] have to be kept in mind before concluding that tetraquark/molecule really exist.

- The quenched simulation [11] was to my knowledge the only one aimed at the very interesting state $Z^+(4430)$. The quantum numbers of this state are not established experimentally, but since it is very close to the $D_1D^*$ threshold, the simulation [11] is carried out in $J^{\rm P} = 0^-,\ 1^-,\ 2^-$ channels. The scattering lengths $a$ are extracted with the help of asymmetric box $L_1 \times L_2 \times L_3 \times T$, which allows for a variety of spatial momenta $k_i = \frac{2\pi}{L_i}$. The most reliable results are obtained for $J^{\rm P} = 0^-$, where the attractive interaction between $D_1$ and $D^*$ is found and $a > 0$. But the $a$ does not change the sign with falling $m_\pi$ and the authors conclude that the attraction is probably to weak to form a loosely bound state.

- Liu [10] determined a number of S-wave scattering lengths $a$ for scattering between heavy–light, heavy–heavy and light–light mesons using $2+1$ dynamical simulation. She studies only channels where no disconnected diagrams are present. The scattering lengths are determined from energy shifts $\Delta E = E_1 - m_{M_1} - m_{M_2}$ and the ground state energies $E_1$ are obtained using the $M_1M_2$ interpolators. As far as tetraquarks/molecules are considered, the most interesting result comes from the $D^+\bar{D}^{0*}$ channel with $I = 1$, where $a$ changes the sign at $m_\pi \simeq 280$ MeV. This may be an indication for an existence of a loosely bound state at $m_\pi < 280$ MeV.

- The $2+1$ dynamical simulation of Ehmann and Bali [19] was actually not aimed at searching for tetraquarks/molecules but to study the mixing between charmonia ($J/\psi$, $\eta_c$, $\chi_c$) and $D\bar{D}$ ($D$ stands for $D$, $D^*$, $D_1$) states. They compute the a full correlation matrix with $\bar{c}\Gamma c$ as well as $(\bar{c}\Gamma q)(\bar{q}\Gamma c)$ interpolators in $J^{\mathrm{PC}} = 0^{-+}, 1^{--}, 2^{++}$ channels, taking into account also all disconnected contractions. Using the variational method they determine the spectrum and also the components $\bar{c}\Gamma c$ and $(\bar{c}\Gamma q)(\bar{q}\Gamma c)$ of the charmonia and $D\bar{D}$ physical eigenstates. They do not find any states in addition to the expected charmonia and scattering states and they do not attempt to establish whether their resulting states are one-particle or scattering states. Let me note that a nonzero coupling $\langle 0|(\bar{c}\Gamma q)(\bar{q}\Gamma c)|J/\psi\rangle$, for example, does not mean that $J/\psi$ has a sizable tetraquark component, since $(\bar{c}\Gamma q)(\bar{q}\Gamma c)$ and $\bar{c}\Gamma c$ Fock components mix via singly disconnected contractions in [19].

## 5. Conclusions

Proving a sizable tetraquark or molecular Fock component in a hadronic resonance using lattice QCD simulation is not an easy task. A resonance appears as a state in addition to the discrete tower of scattering states. So the extraction of few states in addition to the ground state is expected to be crucial. Given the resulting physical eigenstates, one needs to determine whether a certain state corresponds to a one-particle (tetraquark/molecular) or a two-particle (scattering) state, and the available methods to distinguish both to are reviewed.

There are some indications for an additional state in $I = 0,\ 1/2$ light scalar channels, which might correspond to observed $\sigma$ and $\kappa$ with strong tetraquark components [4–6]. The corresponding simulations omitted the disconnected contractions when calculating correlators with tetraquark interpolators in order to study genuine tetraquark states with four valence quarks. It would be valuable to verify in the future whether the additional states are present also in a simulation which takes into account the disconnected contractions.

There have been surprisingly few lattice simulation of very interesting exotic $XYZ$ resonances, discovered recently in $B$ factories. Most of simulations extract only the ground state in a given channel and then try to determine whether it corresponds to a one-particle (tetraquark/molecular) state or to a scattering state. There is some indication that $X(3872)$, $Y(4260)$ and $Y(4140)$ are tetraquark/molecular states, but future simulation are needed to verify that.

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