COUPLED-CHANNEL ANALYSIS OF THE $X(3872)$*

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The $X(3872)$ is studied as an axial-vector charmonium state in the multichannel framework of the Resonance-Spectrum Expansion quark–meson model, previously applied to a variety of other puzzling mesonic resonances. Included are the open-charm pseudoscalar–vector and vector–vector channels, the most important of which is the S-wave $\bar{D}^{*0}D^0 + D^{*0}\bar{D}^0$ channel, which practically coincides with the $X(3872)$ structure. The two free parameters of the model are tuned so as to roughly reproduce the $\chi_{c1}(3511)$ mass as well as the enhancement just above the $\bar{D}^{*0}D^0/D^{*0}\bar{D}^0$ threshold. The present model is able to describe the shape of the latter data quite well. However, as no dynamical resonance pole is found, the $X(3872)$ and $X(3940)$ cannot be reproduced simultaneously, at this stage. A possible further improvement is discussed.

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1. Introduction

The $X(3872)$ charmonium-like state was discovered in 2003 by Belle [1], as a $\pi^+\pi^- J/\psi$ enhancement in the process $B^\pm \to K^\pm \pi^+\pi^- J/\psi$. The same enhancement was then seen in several other experiments as well. Finally, the $X(3872)$ was also observed in the $\bar{D}^0 D^0 \pi^0$ [2] and $\bar{D}^{*0}D^0$ [3] channels. It is now listed in the PDG tables [4], with a mass of $3872.3 \pm 0.8\,\text{MeV}$ and a width of $3.0^{+1.9}_{-1.4} \pm 0.9\,\text{MeV}$. The observed decay modes and helicity analysis indicate positive C-parity and flavour 1^{++} or 2^{-+} quantum numbers [4].

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Theoretical work on the $X(3872)$ comprises a variety of model calculations and interpretations, because of its seeming incompatibility with the standard charmonium spectrum. Let us briefly summarise what we consider the most significant approaches.

> Törnqvist [5]: Calls it a deuson, a deuteron-like $D\bar{D}^*$ bound state. Binding gets provided by pion exchange. Large isospin breaking is expected, and so an important $J/\psi\,\rho$ decay mode.

> Bugg [6]: Examines a mechanism for resonance capture at thresholds that involves a threshold cusp interfering constructively with a resonance from confinement and/or attractive t and u-channel exchanges. Conclusion: the $X(3872)$ requires a virtual state or a resonance.

> Gamermann and Oset [7]: Dynamical generation with isospin breaking, in an effective chiral approach. Preference for a slightly unbound, virtual $\bar{D}^{*0}D^0 + D^{*0}\bar{D}^0$ state.

> Lee *et al.* [8]: Possible $D\bar{D}^*$ molecular bound state in a potential model from heavy-hadron chiral perturbation theory.

> Kalashnikova and Nefediev [9]: Analysis from data concludes preference for a dynamically generated virtual $D\bar{D}^*$ state, with a sizable $2\,^3P_1\ c\bar{c}$ component.

> Danilkin and Simonov [10]: Employs the Weinberg eigenvalue method for a model with 1 confinement and 1 meson–meson channel, coupled via a $^3P_0$ pair-creation vertex. Two $^3P_1$ peaks are found, *i.e.*, a narrow one at $3.872\,\mathrm{GeV}$ and a much broader one at $3.935\,\mathrm{GeV}$, a candidate for the $X(3940)$. The $X(3872)$ is of a dynamical nature here.

> Bugg [11]: Discusses several approaches, including the covalent bond. Proposes resonances as linear combinations of $q\bar{q}$ and meson–meson.

Besides these works, there exists a plethora of exotic models, which nevertheless are incapable of describing the $X(3872)$ production data.

2. Resonance-Spectrum Expansion

The Resonance-Spectrum Expansion (RSE) [12] coupled-channel model is designed to describe scattering processes of the form $AB \to CD$. Although $A, B, C,$ and D can in principle be any structures, we usually consider only non-exotic mesons. Between the initial and final meson–meson phase, there is an intermediate one that we call the confinement phase, and it is built up from an infinite series of $q\bar{q}$ states carrying the same — again non-exotic — quantum numbers.

To describe this picture, we define the effective potential

$$V_{ij}(p_i, p'_j; E) = \lambda^2 j^i_{L_i}(p_i a)\, j^j_{L_j}(p'_j a) \sum_{n=0}^{\infty} \frac{g_i(n)g_j(n)}{E - E_n} \,, \tag{1}$$

where p_i and p'_j are the relativistically defined relative momenta of initial channel i and final channel j, and $j^i_{L_i}$ is the spherical Bessel function. As free parameters we have λ, which is an overall coupling constant, and a, a parameter mimicking the average string-breaking distance. The relative couplings $g_i(n)$ between meson–meson channels i and $q\bar{q}$ recurrences n, with discrete energies E_n, are computed with the formalism developed in Ref. [14]. The transition mechanism is the creation/annihilation of $q\bar{q}$ pairs, according to the OZI rule, using a spherical Dirac-delta potential, which Fourier transforms into a spherical Bessel function in momentum space. Due to the separable form of potential (1), the corresponding T-matrix is obtained in closed form, via the Lippmann–Schwinger equation with relativistic kinematics [12]. Note that the resulting S-matrix is manifestly unitary and analytic.

In Eq. (1), we take a harmonic-oscillator (HO) potential with constant frequency, not only because of its simplicity but also its success in many phenomenological applications (see Ref. [12] for references). The corresponding energy levels are

$$E_n = m_q + m_{\bar{q}} + \omega(2n + 3/2 + \ell) \,. \tag{2}$$

3. $X(3872)$ as an axial-vector charmonium state

In the present study we assume the $X(3872)$ to be the $2\,^3P_1$ $c\bar{c}$ state, *i.e.*, the first radial excitation of the $\chi_{c1}(3511)$ [4], with axial-vector (1^{++}) quantum numbers. Then, the most important OZI-allowed decay channels are pseudoscalar–vector and vector–vector, shown in Table I. Note that, for notational simplicity, we have omitted the bars over the anti-D mesons.

TABLE I

Included meson–meson channels.

Channel	Relative L	Threshold [MeV]
$D^0 D^{*0}$	0	3872
$D^{\pm} D^{*\mp}$	0	3880
$D_s D_s^*$	0	4080
$D^0 D^{*0}$	2	3872
$D^{\pm} D^{*\mp}$	2	3880
$D_s D_s^*$	2	4080
$D^* D^*$	2	4018

4. Results and conclusions

Before studying resonance poles in the vicinity of the $X(3872)$, we must first fix the model parameters. For the charm quark mass m_c and the universal HO frequency ω, we take the values $1.562\,\mathrm{GeV}$ and $0.19\,\mathrm{GeV}$, respectively, which have been kept unaltered in all previous work, starting in Ref. [13]. As for a, one of the two free parameters, we take $2.0\,\mathrm{GeV}^{-1}$ (about 0.4 fm), which is roughly in agreement with the value chosen in Ref. [15] for $s\bar{s}$ vector resonances, scaled with an expected factor of $\sqrt{m_s/m_c}$. The overall coupling λ we use here as a tunable parameter in order to study the behaviour of complex-energy poles, which can be found numerically with ease, since the T-matrix is known analytically. An indication for the approximate value of λ may come from the mass of the $\chi_{c1}(1P)$ [4], *viz.* $3511\,\mathrm{MeV}$.

Table II shows some pole positions for different values of λ. The poles labelled 1 and 2 originate from the HO confinement states at $E_0 = 3599$ MeV and $E_1 = 3979\,\mathrm{MeV}$ (*cf.* Eq. (2) with $\ell = 1$), respectively, for $\lambda \to 0$. These we call confinement poles. We also searched for dynamical — or continuum — poles, which cannot be simply linked to a confinement state and, moreover, disappear in the continuum for $\lambda \to 0$, with an ever increasing negative imaginary part. Such poles we observed *e.g.* in Ref. [15], though with a large width. Here, if one showed up close to the lowest threshold, just as in Ref. [10], it might explain the $X(3872)$ structure as a dynamical resonance, but none was found. We shall come back to this point below.

TABLE II

Pole positions as a function of λ.

λ	Pole 1 [MeV]	Pole 2 [MeV]
3.2	3551	3871
3.0	3555	$3873 - i1$
2.2	3572	$3884 - i6$
1.2	3590	$3928 - i16$

Focusing now on Pole 2, which comes out close to the $X(3872)$ structure for $\lambda \sim 3.0$, we plot in Fig. 1 its trajectory around the $\bar{D}^{*0}D^0/D^{*0}\bar{D}^0$ threshold, through two different Riemann sheets (RSs). The trajectory can be split into four branches according to the classification in Table III, where Th stands for the threshold energy. We change from one RS to another by choosing a specific root of the relative momentum k. The first three branches lie on one RS, and the fourth on another RS. We denote branch 2 by *"virtual resonance"*, as it concerns a resonance with negative phase space. This is a phenomenon unique to S-wave thresholds. For other waves, a resonance pole always turns into a virtual bound state directly, without the virtual-resonance branch.
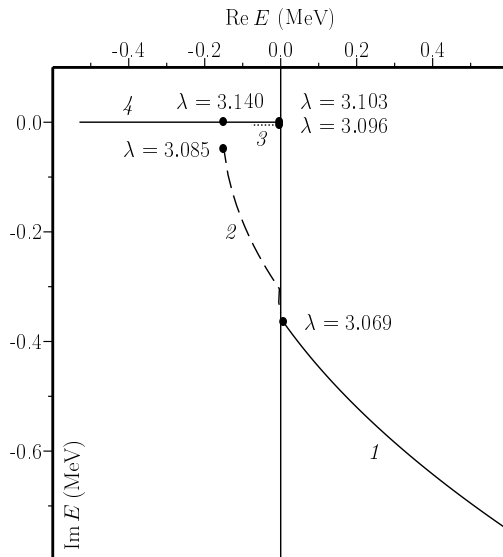
Fig. 1. Trajectory of Pole 2: from a resonance to a bound state.

In Fig. 2, left-hand graph, we depict the elastic $D^0 D^{*0}$ amplitude just above threshold, for 4 values of λ corresponding to the 4 situations described in Table III and in Fig. 1. As a matter of fact, we plot the quantity $k|T|^2$, which is the one to be compared [7] with experiment. The scarce data, not shown here, cannot really distinguish between the 4 scenarios described above, though we conclude that a bound state is less likely [16].

TABLE III

Pole 2 in different Riemann sheets (see text and Fig. 1).

	Re E	Im k	Re k	Type of pole
1	> Th	< 0	> 0	resonance
2	< Th	< 0	> 0	virtual resonance
3	< Th	< 0	= 0	virtual bound state
4	< Th	> 0	= 0	bound state

Finally, in Fig. 2, right-hand graph, we generate a cusp structure around the $D^{\pm} D^{*\mp}$ threshold, by reducing λ and so moving Pole 2 upwards. While quite illustrative, such a scenario is now excluded by the data.

In conclusion, we have studied the $X(3872)$ enhancement as the $2\,^3P_1$ $c\bar{c}$ state in the RSE coupled-channel model. The included OZI-allowed channels are capable of reproducing the observed structure in $D^0 D^{*0}$, via a nearby (virtual) resonance or virtual-bound-state pole. However, the present model
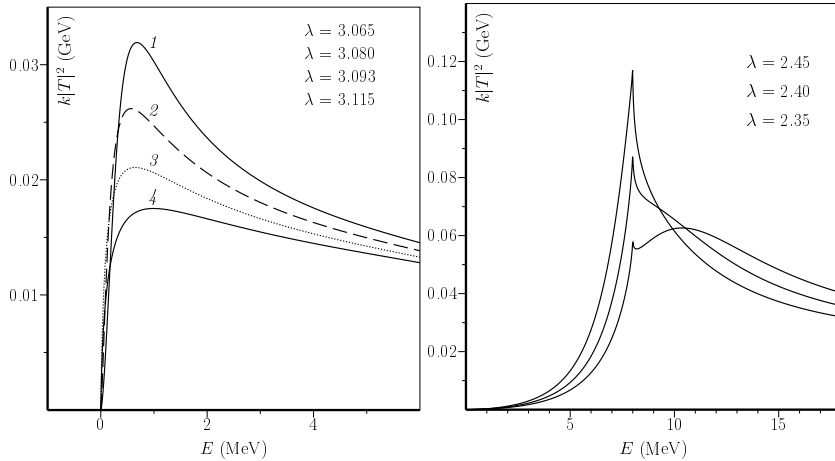
Fig. 2. $k|T|^2$ for elastic $D^0 D^{*0}$ scattering (0 MeV corresponds to 3872 MeV).

does not generate any dynamical pole close to the real axis, so that the $X(3940)$ [4] — if indeed a 1^{++} state as the $X(3872)$ — remains unexplained. We are now working [16] on model extensions, *e.g.* by including the probably observed [17], OZI-violating $J/\psi\,\rho$ decay mode.

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