MULTIBIN CORRELATIONS: A SUMMARY*

A. Bialas†, K. Zalewski

The H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland
and
The M. Smoluchowski Institute of Physics, Jagiellonian University
Reymonta 4, 30-059 Kraków, Poland

(Received January 31, 2011)

A recently proposed method of studying the long-range correlations in multiparticle production is described. It is explained how it can be used in practice to uncover the mechanisms of particle production in high energy collisions.

DOI:10.5506/APhysPolBSupp.4.7
PACS numbers: 25.75.Dw, 25.75.Gz

1. Introduction

Long range correlations among particles produced in high-energy multiparticle production processes have been studied for many years [1]. The usual approach was to look at the forward–backward correlations, *i.e.* on the correlation between the populations of a near-forward bin and a near-backward bin in rapidity. Recently, also analyses using three bins have been published [2,3]. Here we summarize the results of our two recent papers [4,5] where we discuss the general case of correlations between the populations of $B \geq 2$ bins. As was to be expected, the number of predictions, which can be obtained and used to test specific models, as well as the general mechanisms of particle production, increases rapidly with B.

We consider a generic scheme, where the particles are produced by N independent groups of sources. The sources in each group are independent and identical. The number of sources in a group i will be denoted w_i. The numbers w_i can be either fixed, or random governed by a probability distribution $W(w_1, \ldots, w_N)$.

* Presented at the Workshop on Timing Detectors, Kraków, Poland, November 29–December 1, 2010.

† bialas@th.if.uj.edu.pl

Finally, we assume that, for each source, emitted particles fall randomly in various bins and thus they follow the Bernoulli distribution

$$P(n_1, \ldots, n_N) = \frac{n!}{\prod_{j=1}^{B} n_i!} \prod_{k=1}^{B} p_k^{n_k} \,. \tag{1}$$

Here $P(n_1, \ldots, n_B)$ is the probability that n_i particles get into bin i for $i = 1, \ldots, B$, $n = \sum_{j=1}^{B} n_j$, and p_k are probabilities normalized by the condition $\sum_{k=1}^{B} p_k = 1$. The probabilities p_k are the same for identical sources and, in general, different for sources belonging to different groups. This assumption is widely accepted [1, 6, 7], but it ignores *e.g.* short range correlations among the final particles. Therefore (1) is expected to be valid only if the bins are separated by a sufficient distance.

Many specific models are special cases of the general scheme considered here [1]. For Landau's model [8] $N = 1$ and $w_1 = 1$. For deep inelastic scattering it is natural to assume $N = 2$ and $w_1 = w_2 = 1$, because there are the proton remnants and the photon remnants. For the wounded constituent model [9] $N = 2$, and w_i are random integers with a probability distribution $W(w_1, w_2)$. In this case the right moving sources are mirror reflections of the left moving sources, this reduces the complexity of the model to that of models with $N = 1$. In the dual parton model [10] $N = 3$, because besides the left moving sources and the right moving sources there are also the central sources. In this model all the numbers w_i fluctuate.

2. Number of observables and number of parameters

A specific model to be compared with experiment is defined by the number of different sources N, the probability distribution $W(w_1, \ldots, w_N)$ and, sometimes, by some constraints put on the sources. In order to get meaningful tests one should arrange things so that the number of parameters of the model is smaller than the number of the measured observables. The basic observables are the (factorial) moments

$$F_{i_1 \ldots i_B} = \left\langle \prod_{j=1}^{B} \frac{n_j!}{(n_j - i_j)!} \right\rangle , \tag{2}$$

where the averaging is over all the events included. Denoting $r = \sum_{j=1}^{B} i_j$, we propose to measure all the moments with $r < r_{\max}$, where $r_{\max}$ remains to be chosen. If there are no constraints, like forward–backward symmetry in collisions of identical particles, the number of such moments is [5]

$$m(r_{\max}, B) = \frac{(B + r_{\max})!}{B! r_{\max}!} - 1 \,. \tag{3}$$

Corresponding formulae for the forward–backward symmetric case are given in the Appendix of [5]. It is seen that this quantity increases rapidly with both B and $r_{\max}$. For practical reasons it seems reasonable, however, to keep $r_{\max}$ equal two or at most three.

The number of parameters is [5]

$$P(B,N,r_{\max}) = N(B-1) + \frac{(N+r_{\max})!}{r_{\max}!N!} - 1\,. \tag{4}$$

In the special case, when the numbers w_i are fixed and known, this formula is replaced by

$$P(B,N,r_{\max}) = N(B+r_{\max}-1)\,. \tag{5}$$

The terms $N(B-1)$ in (4) and (5) are the numbers of the independent probabilities p_k. The remainder is related to the moments of multiplicity distribution within a source and of the distribution $W(w_1,\ldots,w_N)$.

Substituting numbers one finds that for $N=1$ already for $r_{\max}=2$ and two bins there are predictions to be tested, even in the general case when the number of sources w fluctuates. For $N=2$ to get predictions at fixed w_i one has to study at least three bins for $r_{\max}=2$, or two bins for $r_{\max}=3$. If the numbers of sources fluctuate, the corresponding numbers of bins are four and three. For $N=3$ and fixed numbers of sources w_i the minimal numbers of bins corresponding to $r_{\max}=2$ and $r_{\max}=3$ are five and three. When the numbers of sources fluctuate these numbers increase respectively to six and four. These examples show that the problem becomes more complicated when N increases, but one can cope with it, if the number of bins can be sufficiently increased.

3. How to test a model

Given a model, one should first choose B and $r_{\max}$ sufficiently large to yield predictions. Then the formulae relating the measured moments to the parameters of the model should be written down. It is possible to get general formulae [5], but in practice it is more convenient to use the simpler ones, adapted to the model being considered. In particular, when the numbers of sources w_i are fixed, it is advisable to use cumulants instead of the moments (2). The brute force method is to minimize the χ^2 for these equations. If the resulting χ^2 is acceptable, one can extract and interpret the parameters of the model. If not, one concludes that the model does not work. In many cases, however, it is better to use analytic methods in order to make the analysis simpler and more transparent. Let us consider some examples.

For Landau's model $N = 1$ and one finds

$$F_{i_1 \ldots i_B} = F^{(r)} \prod_{j=1}^{B} p_j^{i_j} \,, \tag{6}$$

where the $B-1$ independent probabilities p_j and the $r_{\max}$ numbers $F^{(r)}$, further called reduced moments, are parameters of the model. It is seen that all the probabilities p_j can be obtained from the first order moments and then there is one new parameter $F^{(r)}$ for each successive $r \le r_{\max}$. This parameter can be determined either by minimizing χ^2, or from the sum rule

$$F^{(r)} = \sum \frac{r!}{i_1! \ldots i_B!} F_{i_1 \ldots i_B} \,, \tag{7}$$

where the summation is over all the sets $i_1, \ldots, i_B$ satisfying the constraint $\sum_{j=1}^{B} i_j = r$. For this analysis it is irrelevant whether or not the multiplicity of sources w fluctuates[1].

For the wounded constituent model $N = 2$ and consequently there are two sets of probabilities p_{iL} and p_{iR} for $i = 1, \ldots, B$. However, in the nucleon–nucleon center of mass system, when the bins in rapidity are chosen symmetrically, we have for each i: $p_{iL} = p_{(B-i)R}$. Therefore the number of independent probabilities is still $B-1$. Since in this model the multiplicities of sources, w_L and w_R, fluctuate, the resulting moments are averages over the probability distribution $W(w_L, w_R)$. We will denote this averaging by the Dirac brackets. The formulae for the moments of the first two orders are

$$\begin{aligned} F_i^{(1)} &= p_{iL} \left\langle \tilde{F}_L^{(1)} \right\rangle + p_{iR} \left\langle \tilde{F}_R^{(1)} \right\rangle \\ F_{ij}^{(2)} &= p_{iL}p_{jL} \left\langle \tilde{F}_L^{(2)} \right\rangle + p_{iR}p_{jR} \left\langle \tilde{F}_R^{(2)} \right\rangle + (p_{iL}p_{jR} + p_{iR}p_{jL}) \left\langle \tilde{F}_L^{(1)} \tilde{F}_R^{(1)} \right\rangle \,, \end{aligned} \tag{8}$$

where $F_i^{(1)} = \langle n_i \rangle$, for $i \neq j$ $F_{ij}^{(2)} = \langle n_i n_j \rangle$ and $F_{ii}^{(2)} = \langle n_i(n_i - 1) \rangle$. The formulae for $F_{ijk}^{(3)}$ can be found in [5].

Let us consider first the asymmetric case. As seen from (8), for $r_{\max} = 2$ there are five parameters besides the probabilities[2]. The $r = 1$ moments yield, after eliminating $\tilde{F}_R^{(1)}$ and $\tilde{F}_R^{(1)}$, $B-2$ probabilities. We assume $B \ge 3$. As seen from (8), the $\frac{1}{2}B(B+1)$ second order moments have to be fitted with four free parameters . Already for $B = 3$ there are two predictions.

[1] A more detailed discussion of the Landau model, adapted to the conditions of the ALICE experiment can be found in [11].

[2] The physical meaning of these parameters is explained in [5].

One easily sees that for each higher value of r the number of new parameters is equal $r+1$ and thus one obtains more constraints. *E.g.* for $r_{\max}=3$ there are eight predictions.

For symmetric collisions the number of independent measurable moments and the number of parameters get reduced. The number of moments to be measured for $r_{\max}=1$ equals K; for $r_{\max}>1$ it is $\frac{(B+r_{\max})!}{2r_{\max}!B!}+Q$ where $Q=(K-1)/2$ for B even. For B odd Q equals $K/2$ and K for $r_{\max}=2$ and $r_{\max}=3$, respectively. Here $K=B/2$ for B even and $K=(B+1)/2$ for B odd. The number of independent probabilities p_{iL},p_{iR} remains equal $(B-1)$, but the number of other parameters for $r_{\max}=1,2,3$ is, respectively, $1,3$ and 5. Substituting numbers, one finds that *e.g.* for three bins there is one prediction for $r_{\max}=2$ and five predictions for $r_{\max}=3$.

The discussion of the dual parton model is similar, except that a new central source has to be included. Thus $N=3$, with two kinds of sources related like for the wounded nucleon model, and the multiplicity distribution for sources is $W(w_L,w_C,w_R)$. This makes the formulae longer, but their discussion is very similar [5] to that in the preceding example. One finds that for $r_{\max}=2$ it is necessary to take $B\geq 5$ to get predictions. For $B=5$ there are two predictions. For $r_{\max}=3$, it is enough to have $B\geq 3$. For $B=3$ there are three predictions and for $B=4$ eight.

4. Conclusions

Modern, high energy experiments make it possible to study the correlations among the particles produced in more than two, well separated bins. We presented a general analysis of such correlations [4,5] (see also [7]). This analysis shows that by increasing the number of bins one can exhibit much better the predictive power of the models and consequently test them more thoroughly. Even more important, such analysis allows to test the very mechanism defining the model since it does not depend on details of formulation, *e.g.* a particular parametrization. On the other hand, if a model agrees with data, the results allow to obtain information about its specific features and ingredients.

For some models predictions appear only when the number of bins exceeds, sometimes significantly, two or three. Then our more general approach is necessary to get any tests at all.

The method is remarkably flexible. One can handle symmetric collisions, which are simpler, as well as asymmetric collisions, which give more information. We have presented in some detail the analysis for Landau's model, which is the simplest, and of the wounded constituent model, which nicely illustrates the modifications necessary, when going from asymmetric to symmetric collisions. Going to more complicated models, like the dual parton model, makes the formulae and the necessary fitting more cumbersome, but does not introduce any new principle.

We described our method as a generalization of the forward–backward correlations, corresponding to bins in rapidity, but exactly the same analysis is applicable to correlations in other variables, *e.g.* in the azimuthal angle.

This investigation was supported in part by the grant N N202 125437 of the Polish Ministry of Science and Higher Education (2009–2012).

REFERENCES

[1] For an extensive review and references, see W. Kittel, E.A. De Wolf, *Soft Multihadron Dynamics*, World Scientific, 2005.

[2] B.I. Abelev *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **103**, 172301 (2009); T.J. Tarnowsky, `arXiv:0807.1941 [nucl-ex]`.

[3] T. Lappi, L. McLerran, *Nucl. Phys.* **A832**, 330 (2010).

[4] A. Bialas, K. Zalewski, *Phys. Rev.* **C82**, 034911 (2010).

[5] A. Bialas, K. Zalewski, `arXiv:1101.1907 [hep-ph]`.

[6] T.T. Chou, C.N. Yang, *Phys. Lett.* **B135**, 175 (1984); P. Carruthers, C.C. Shih, *Phys. Lett.* **B165**, 209 (1985); W.A. Zajc, *Phys. Lett.* **B175**, 219 (1986); J. Benecke, A. Bialas, S. Pokorski, *Nucl. Phys.* **B135**, 488 (1976); *Erratum ibid.* **B115**, 547 (1976).

[7] A. Bzdak, *Acta Phys. Pol. B* **41**, 151 (2010); *Acta Phys. Pol. B* **41**, 2471 (2010); *Acta Phys. Pol. B* **40**, 2029 (2009).

[8] L. Landau, *Izv. Akad. Nauk Ser. Fiz.* **17**, 51 (1953).

[9] A. Bialas, M. Bleszyński, W. Czyż, *Nucl. Phys.* **B111**, 461 (1976); A. Bialas, W. Czyż, W. Furmański, *Acta Phys. Pol. B* **8**, 585 (1977); A. Bialas, W. Czyż, *Acta Phys. Pol. B* **36**, 905 (2005); A. Bialas, A. Bzdak, *Acta Phys. Pol. B* **38**, 159 (2007); *Phys. Lett.* **B649**, 263 (2007); *Phys. Rev.* **C77**, 034908 (2008); For a review, see A. Bialas, *J. Phys. G* **35**, 044053 (2008).

[10] For a review, see A. Capella, U. Sukhatme, C.-I. Tan, J. Tran Thanh Van, *Phys. Rep.* **236**, 225 (1994).

[11] A. Bialas, K. Zalewski, `arXiv:1101.5706 [hep-ph]`.