DYNAMICS OF NUMBER OF PACKETS IN TRANSIT IN FREE FLOW STATE OF DATA NETWORK*

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(Received April 6, 2011)

We study how the dynamics of Number of Packets in Transit (NPT) is affected by the coupling of a routing type with a volume of incoming packet traffic in a data network model of packet switching type. The NPT is a network performance indicator of an aggregate type that measures in "real time", how many packets are in the network on their routes to their destinations. We conduct our investigation using a time-discrete simulation model that is an abstraction of the Network Layer of the ISO OSI Seven Layer Reference Model. This model focuses on packets and their routing. We consider a static routing and two different types of dynamic routings coupled with different volumes of incoming packet traffic in the network free flow state. Our study shows that the order of the values of the NPT *mean value* time series depends on the coupling of a routing type with a volume of incoming packet traffic and changes when the volume of incoming packet traffic increases and is closed to the *critical source load* values, *i.e.* when it is closed to the phase transition points from the network free flow state to its congested states.

DOI:10.5506/APhysPolBSupp.4.237
PACS numbers: 89.20.Ff, 89.20.Hh, 89.75.–k, 89.75.Kd

1. Introduction

The dominant technology of data communication networks is the Packet Switching Network (PSN) [1]. This technology is complex and is organized as various hierarchical layers according to the International Standard Organization (ISO) Open Systems Interconnect (OSI) Reference Model. The

* Presented at the 2nd Summer Solstice International Conference on Discrete Models of Complex Systems, Nancy, France, June 16–18, 2010.

Network Layer of the ISO OSI Reference Model is responsible for delivering packets from their sources to their destinations and for dealing with congestion when it arises in a network. The dynamics of packet traffic in data communication networks is complex, not well understood and depends on many factors and couplings among these factors. Examples of the factors are a network connection topology type, a routing type, a routing table update, a volume of incoming packet traffic, *etc.* Understanding of the dynamics of packet traffic is important for the efficient management of performance of data communication networks, so that they can provide good quality of service (QoS) and for future design of the data communication networks. The performance of data communication networks is measured by various network performance indicators, *e.g.* Number of Packets in Transit (NPT), *critical source load* value, *etc.* The NPT is a network performance indicator of an aggregate type that measures in "real time", how many packets are in the network on their routes to their destinations. The *critical source load* value is the phase transition point from the network free flow state to its congested state.

Our work in [2] has focused on the study of time variability of NPT time series in which the NPT time series were analyzed using the functional fixed effect model at each time point for the different types of *edge cost function* (e.c.f.) at the specific *source loads.* In this paper, we study dynamics of the aggregate NPT time series with respect to different types of e.c.f., *i.e.* we study how the dynamics of NPT *mean value* time series is affected by the coupling of a routing type with a volume of incoming packet traffic in a data network model that is an abstraction of the Network Layer of the ISO OSI Reference Model. For details of this model see [3, 4]. Our model is a time-discrete simulation model that focuses on packets and their routing. In our investigations we consider a static routing and two different types of dynamic routings coupled with different volumes of incoming packet traffic in the network free flow state. Our study shows that the order of the values of the NPT *mean value* time series depends on the coupling of a routing type with a volume of incoming packet traffic and changes when the volume of incoming packet traffic increases and is closed to the *critical source load* values. For small volumes of incoming packet traffic the values of the NPT *mean value* time series are the lowest for the static routing but they are the highest when the volumes of incoming packet traffic are closed to the *critical source load* values. For these values of incoming packet traffic the values of the NPT *mean value* time series are the lowest for the adaptive routing using dynamic e.c.f. taking under consideration not only the number of hops that packets must perform to reach their destinations but also the queue sizes of the routers located along the packets routes. The reason that the values of the NPT *mean value* time series are lower for the adaptive routings than for the static one is that the adaptive routings have the ability to route packets avoiding local congestions that arise from the increased fluctuations in number of packets in transit near phase transition point from free flow to congested network state. The static routing does not have this ability.

The paper is organized as follows. First, we briefly describe the abstraction of the PSN model that we use for our research [3, 4] and its C++ simulator, Netzwerk, [5, 6] that we used to conduct our simulation experiments. Next, we introduce the experimental simulation setups of the PSN model and the definitions of some network performance indicators, *e.g.*, *critical source load*, NPT. Finally, we present selected simulation results and our conclusions.

2. PSN description

For the reader convenience we briefly describe the PSN model, developed in [3, 4], and its C++ simulator, called Netzwerk-1 [5, 6] that we use in our study. The PSN model is an abstraction of the Network Layer of the 7-Layer ISO OSI Reference Model [1]. Our PSN model is concerned primarily with packets and their routings; it is scalable, distributed in space, and time discrete. It avoids the overhead of protocol details present in many PSN simulators designed with different aims in mind than study of macroscopic network-wide dynamics of packet traffic and aggregate measures of network performance.

A PSN connection topology is represented by a weighted directed multigraph $\boldsymbol{L}$ where each node/router corresponds to a vertex and each communication link is represented by a pair of parallel edges oriented in opposite directions. In the PSN model each node performs the functions of *host* and *router* and maintains one incoming and one outgoing queue which is of unlimited length and operates according to a first-in, first-out policy. At each node, independently of the other nodes, packets are created randomly with probability λ called *source load* of *incoming packet traffic*. In the PSN model all messages are restricted to one packet carrying only the following information: time of creation, destination address, and number of hops taken.

In each PSN model setup each cost of transmission of a packet along a link (an edge) is computed using the same type of e.c.f. that is either the e.c.f. called ONE (ONE), or QueueSize (QS), or QueueSizePlusOne (QSPO). The e.c.f. ONE assigns a value of "one", to all edges in the lattice $\boldsymbol{L}$. This results in a static routing since this value does not change during the course of a simulation. The e.c.f. QS assigns to each edge in the lattice $\boldsymbol{L}$ a value equal to the length of the outgoing queue at the node from which the edge originates. The e.c.f. QSPO assigns a value that is the sum of a constant "one", plus the length of the outgoing queue at the node from which the edge originates. The routing decisions made using e.c.f. QS or QSPO result in *adaptive* or *dynamic routing* because they rely on the current state of the network simulation and the packets are routed avoiding congested nodes during the PSN model simulation. In our PSN model, each packet is transmitted via routers from its source to its destination according to the routing decisions made independently at each router and based on a *minimum least-cost criterion* of selecting a *shortest path* from a packet current node to its destination. Thus, if the PSN model is setup with e.c.f. ONE then the routing is the *minimum hop routing* (*minimum route distance*) and if it is setup with e.c.f. QS or QSPO then it is the *minimum length routing*. It is important to notice that, in the case of PSN model setup with e.c.f. QS or QSPO, because these costs are dynamic, each packet is forwarded from its current node to the next one that belongs to a least cost shortest path from the packet current node to its destination at this time. The PSN model uses *full-table routing*, that is, each node maintains a routing table of least path cost estimates from itself to every other node in the network. The routing tables are updated at each time step when the e.c.f. QS or QSPO is used; see [3, 4]. Since the values of the e.c.f. ONE do not change over time the routing tables do not need to be updated for the static e.c.f. ONE; see [3, 4]. We update the routing tables using distributed routing table update algorithm [4].

In the PSN model the time is discrete and we observe its state at the discrete times $k = 0, 1, 2, \ldots, T$, where T is the final simulation time. At time $k = 0$, the setup of the PSN model is initialized with empty queues and the routing tables are computed. The time discrete, synchronous and spatially distributed PSN model algorithm consists of the sequence of five operations advancing the simulation time from k to $k+1$. These operations are: (1) Update routing tables, (2) Create and route packets, (3) Process incoming queue, (4) Evaluate network state, (5) Update simulation time. The detailed description of this algorithm is provided in [3, 4].

A PSN model setup is defined by a selection of: a type of network connection topology, a type of e.c.f., a type of routing table and its update algorithm, a value of *source load*, seeds of two pseudo-random number generators and a final simulation time T. The first pseudo-random number generator provides the sequence of numbers required for packets generation and routing. The second one is used for specifying the network connection topology. The details of PSN model setup are provided in [4].

3. Experimental setups of PSN model and network performance indicators

The simulation experiments were conducted for the PSN model setup with a network connection topology that is isomorphic to $\mathcal{L}^{p}_{\square}(16)$ (*i.e.*, a two-dimensional periodic square lattice with 16 nodes in the horizontal and vertical directions), full-table routing and distributed routing table update and we used the default value for the second pseudo-random number generator. During each simulation run the incoming packet traffic was generated, at each network node independently of the other nodes and times by Bernoulli random variables with expected value λ, *i.e. source load* value. In our simulation experiments we varied the values of the following setup variables: e.c.f., *source load* and seed of the first pseudo-random number generator.

We use, respectively, the following conventions $\mathcal{L}^{p}_{\square}(16, \text{e.c.f.})$ and $\mathcal{L}^{p}_{\square}(16, \text{e.c.f.}, \lambda)$, where e.c.f. = ONE, or QS, or QSPO, when we want to specify with what type of e.c.f. and additionally with what λ value of *source load* the PSN model is setup.

In the PSN model, for each family of network setups, which differ only in the value of the *source load* λ, values of $\lambda_{\text{sub}-\text{c}}$ for which packet traffic is congestion-free are called *sub-critical source loads*, while values $\lambda_{\text{sup}-\text{c}}$ for which traffic is congested are called *super-critical source loads*. The *critical source load* λ_{c} is the largest *sub-critical source load*. Thus, λ_{c} is a very important network performance indicator because it is the phase transition point from free flow to congested state of a network. Details about how we estimate the *critical source load* are provided in [4].

For the PSN model setups considered here the estimated *critical source load* (CSL) values are, respectively, $\lambda_{\text{c}} = 0.115$ for $\mathcal{L}^{p}_{\square}(16, \text{ONE})$, $\lambda_{\text{c}} = 0.120$ for $\mathcal{L}^{p}_{\square}(16, \text{QS})$ and $\lambda_{\text{c}} = 0.120$ for $\mathcal{L}^{p}_{\square}(16, \text{QSPO})$ [7,8].

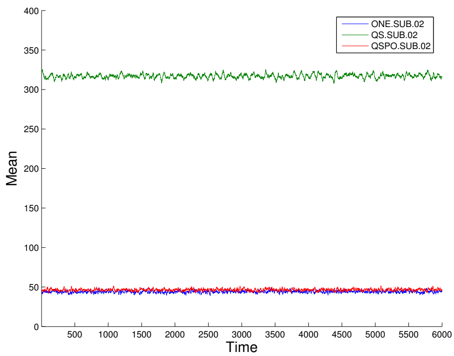
Another very important "real time", network performance indicator is an indicator called Number of Packets in Transit (NPT). This indicator, $N_l(\text{e.c.f.}, \lambda, k)$, for a given PSN model with $\mathcal{L}^{p}_{\square}(16, \text{e.c.f.}, \lambda)$ setup is given by the total number of packets in the network at time k, *i.e.* by the sum over all network nodes of the number of packets in each out-going queue at time k. In $N_l(\text{e.c.f.}, \lambda, k)$ l stands for a seed value of the first pseudo-random number generator used for incoming packet traffic generation and routing decisions that are random. The NPT time series, *i.e.* $N_l(\text{e.c.f.}, \lambda, k)$, for $k = 0, \ldots, T$, is an important time dependent, *i.e.* dynamic, aggregate measure of network performance providing information on how many packets are in the network on their routes to their destinations at time k for a given PSN model setup $\mathcal{L}^{p}_{\square}(16, \text{e.c.f.}, \lambda)$. Thus, $N_l(\text{e.c.f.}, \lambda, k)$ is a "real time", network performance indicator. However, in order to study the network dynamics and their comparison among various PSN model setups it is more appropriate to look at time dependent averages, *i.e.* *mean value functions* of the NPT time series that are calculated over all simulation runs of the PSN model with the same setup but different seed values of the first pseudo-random number generator. In the case when the consecutive natural numbers from $m = 1, \dots, M$, are taken as the seed values of the first pseudo-random number generator, then these *mean value functions* are given by $\hat{N}(\text{e.c.f.}, \lambda, k) = M^{-1} \sum_{m=1}^{M} N_m(\text{e.c.f.}, \lambda, k)$.

Our study focuses on the dynamic behaviors of NPT time series when *source load* values of the incoming packet traffic are in the free flow state for each PSN model setup. We analyze the dynamics of NPT time series of PSN model with $\mathcal{L}^p_\square(16, \text{e.c.f.}, \lambda)$ setups, respectively, for e.c.f. $=$ ONE, QS, QSPO, and *source load* values from very small ones to those close to the phase transition point, *i.e.* for sub-critical source load values $\lambda = 0.020, 0.040, 0.050, 0.060, 0.080, 0.090, 0.095, 0.100, 0.105$ and 0.110. We call the considered set of λ values FreeFlow set. For each PSN model with the setup $\mathcal{L}^p_\square(16, \text{e.c.f.}, \lambda)$, where e.c.f. $=$ ONE, QS, QSPO, respectively, and λ belongs to FreeFlow set we run simulations with 24 different seed values l, where $l = 1, \dots, 24$, of the first pseudo-random number generator. Each simulation is run until the final simulation time $T = 8000$. Even though the final simulation time is $T = 8000$ only the data from $k = 2001$ is accounted for in our analysis in order to remove the initial transient effects caused by the setups of the PSN model always with empty queues. Thus, notice that, in all the presented graphs, time axis scale goes always from 0 to 6000 to account for the discarded data.

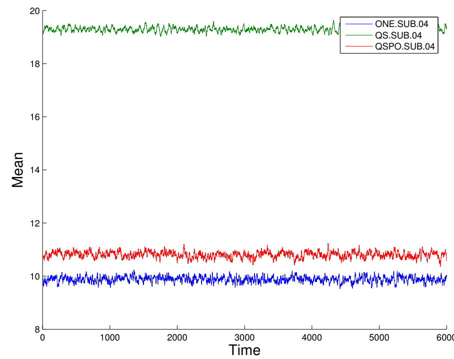
4. Effects of coupling of routing with source load on NPT dynamics

This paper provides an insight into complex dynamics of PSN model emerging near phase transition point from the free flow state to the congested network state. It illustrates that the dynamics of packet traffic is affected by the coupling of a routing type with a *source load* value.

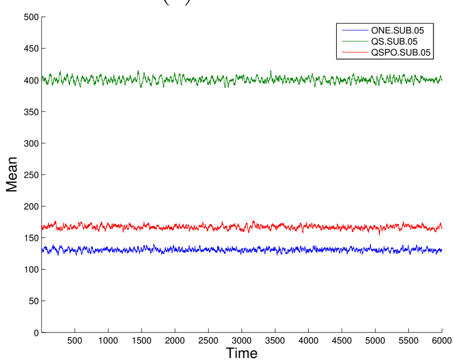
In Fig. 1 and Fig. 2 are displayed graphs of e.c.f., source load and time dependent *mean value functions*, $\hat{N}(\text{e.c.f.}, \lambda, k)$, of NPT time series corresponding to 24 different seeds of the first pseudo-random number generator for each PSN model with $\mathcal{L}^p_\square(16, \text{e.c.f.}, \lambda)$ setup, where e.c.f. $=$ ONE, QS, QSPO, respectively. The *sub-critical* values λ of the *source load* of incoming packet traffic are listed under each plot. The blue graphs correspond to the PSN model with $\mathcal{L}^p_\square(16, \text{ONE}, \lambda)$ setup, the green graphs to the one with $\mathcal{L}^p_\square(16, \text{QS}, \lambda)$ setup and the red graphs to the one with $\mathcal{L}^p_\square(16, \text{QSPO}, \lambda)$ setup. Readers of the hard copy version of this paper should view its online version for the colour figures. In Fig. 3 there are displayed graphs of e.c.f., *source load* and time dependent *variance functions* of NPT time series, $\mathrm{Var}_{\mathrm{NPT}}(\text{e.c.f.}, \lambda, k)$, obtained for 24 different seeds of the first pseudo-random number generator for each PSN model with $\mathcal{L}^{p}_{\square}(16, \text{e.c.f.}, \lambda)$ setup, where, respectively, e.c.f. $=$ ONE, QS, QSPO and each of them is listed under the corresponding plot.



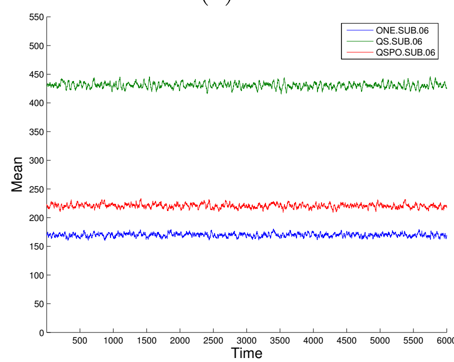
(a) λ =0.020



(b) $\lambda = 0.040$



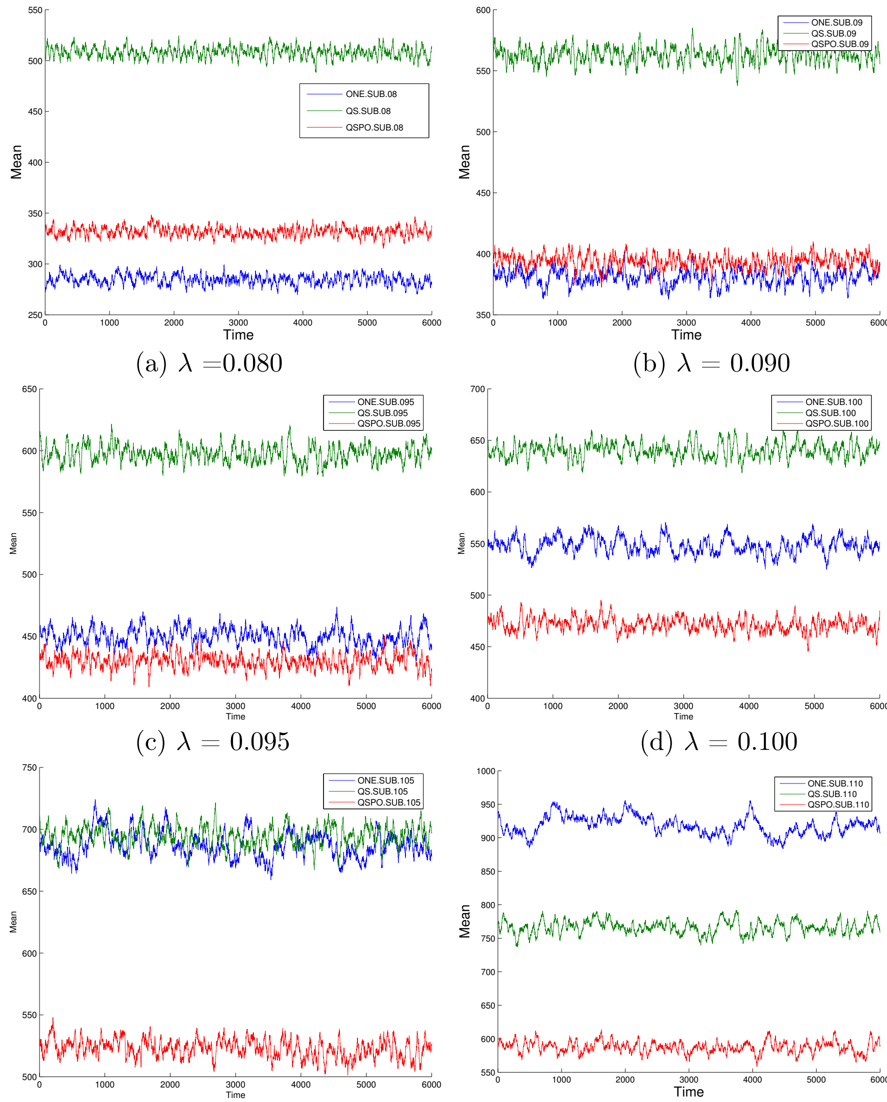
(c) $\lambda = 0.050$



(d) $\lambda = 0.060$

Fig. 1. Graphs of e.c.f., *source load* and time dependent *mean value functions* $\hat{N}(\text{e.c.f.}, \lambda, k)$ of NPT time series corresponding to 24 different seeds of the first pseudo-random number generator for each PSN model with setup $\mathcal{L}^{p}_{\square}(16, \text{e.c.f.}, \lambda)$, where e.c.f. $=$ ONE, QS, QSPO, respectively. The sub-critical values of the *source load* λ of incoming packet traffic are listed under each plot. The blue graphs correspond to the PSN model with $\mathcal{L}^{p}_{\square}(16, \text{ONE}, \lambda)$ setup, the green graphs to the one with $\mathcal{L}^{p}_{\square}(16, \text{QS}, \lambda)$ setup and the red graphs to the one with $\mathcal{L}^{p}_{\square}(16, \text{QSPO}, \lambda)$ setup.

In the plots of Fig. 1 and Fig. 2 we observe that the *mean value functions* $\hat{N}(\text{e.c.f.}, \lambda, k)$ behave as follows. They fluctuate around their respective e.c.f. and *source load* value dependent constant values that increase with the increase of the values of *source load*. For *source load* value $\lambda = 0.020$, the



(a) λ =0.080 (b) $\lambda = 0.090$

(c) $\lambda = 0.095$ (d) $\lambda = 0.100$

(e) $\lambda = 0.105$ (f) $\lambda = 0.110$

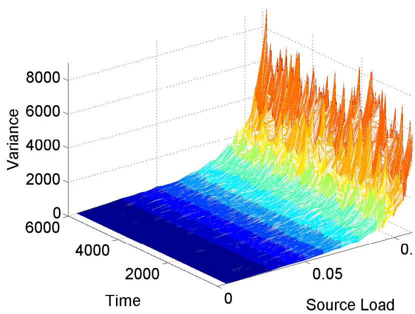
Fig. 2. Graphs of e.c.f., *source load* and time dependent *mean value functions* $\hat{N}(\text{e.c.f.}, \lambda, k)$ of NPT time series corresponding to 24 different seeds of the first pseudo-random number generator for each PSN model with $\mathcal{L}^p_\square(16, \text{e.c.f.}, \lambda)$ setup, where e.c.f. $=$ ONE, QS, QSPO, respectively. The *sub-critical* values of the *source load* λ of incoming packet traffic are listed under each plot. The blue graphs correspond to the PSN model with $\mathcal{L}^p_\square(16, \text{ONE}, \lambda)$ setup, the green graphs to the one with $\mathcal{L}^p_\square(16, \text{QS}, \lambda)$ setup and the red graphs to the one with $\mathcal{L}^p_\square(16, \text{QSPO}, \lambda)$ setup.

values $\hat{N}(\text{ONE}, 0.020, k) \approx \hat{N}(\text{QSPO}, 0.020, k)$ and they are much smaller than $\hat{N}(\text{QS}, 0.020, k)$. The reason for this is that for very small *source load* values the queue sizes at the network nodes are very small. Thus, the edge costs ONE and QSPO are very similar ones, as the dominant part of the edge cost QSPO is ONE but not QS, because many queues are almost empty. The values of $\hat{N}(\text{QS}, 0.020, k)$ are significantly higher than the values of $\hat{N}(\text{ONE}, 0.020, k) \approx \hat{N}(\text{QSPO}, 0.020, k)$ because when e.c.f. QS is used there are many routes with the same costs from packets current locations to their destinations, as many queues are almost empty. Since the packets perform almost random walks to reach their destinations, then they stay much longer in the network when e.c.f. QS is used instead of e.c.f. ONE or QSPO and this is why the values of $\hat{N}(\text{QS}, 0.020, k)$ are so high.

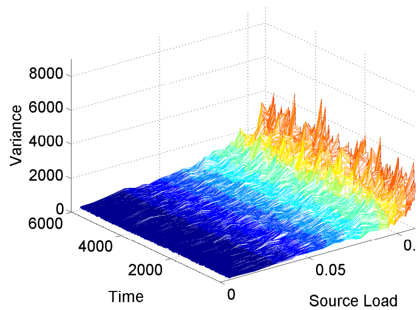
For *source load* values λ such that $0.040 \leq \lambda \leq 0.080$ we observe that values of the *mean value functions* $\hat{N}(\text{e.c.f.}, \lambda, k)$ satisfy the following inequalities $\hat{N}(\text{ONE}, \lambda, k) < \hat{N}(\text{QSPO}, \lambda, k) < \hat{N}(\text{QS}, \lambda, k)$. The reason for this is that the total amount of incoming packet traffic is still rather low. When e.c.f. ONE is used the queues that form along the shortest paths, due to the random fluctuations, are not sufficiently big to be able to slow down the delivery of packets to their destinations. Since the adaptive routings try to redistribute the packets evenly among the network routers, packets stay longer in the network on their routes to their destinations and values of the respective *mean value functions* are higher than those if the static routing is used instead. Because many queue sizes are similar ones when e.c.f. QS is used, the routing provides less clues to the packets to reach their destinations than when e.c.f. QSPO is used instead. This is because the ONE component of the e.c.f. QSPO helps to better differentiate the costs of routes and packets travel to their destinations along the routes that resemble more the shortest paths. This is why $\hat{N}(\text{ONE}, \lambda, k) < \hat{N}(\text{QSPO}, \lambda, k) < \hat{N}(\text{QS}, \lambda, k)$.

For *source load* values λ such that $0.090 \leq \lambda \leq 0.100$ we observe an interesting dynamics. The order in the magnitudes of the values of $\hat{N}(\text{ONE}, \lambda, k)$ and $\hat{N}(\text{QSPO}, \lambda, k)$ functions changes from $\hat{N}(\text{ONE}, 0.090, k) \leq \hat{N}(\text{QSPO}, 0.090, k)$ to $\hat{N}(\text{QSPO}, 0.095, k) \leq \hat{N}(\text{ONE}, 0.095, k)$ and next to $\hat{N}(\text{QSPO}, 0.100, k) < \hat{N}(\text{ONE}, 0.100, k)$, and the difference between the corresponding values becomes significantly large for each k. For the considered set of λ values the values of $\hat{N}(\text{QS}, \lambda, k)$ as still the largest ones. The described behavior of the *mean value functions* of changing the order in their values is correlated with the behavior of the *variance functions* $\text{Var}_{\text{NPT}}(\text{e.c.f.}, \lambda, k)$ explained below. In Fig. 3 we see that values of the *variance functions* $\text{Var}_{\text{NPT}}(\text{e.c.f.}, \lambda, k)$ increase with the increase of *source load* values and that the largest increase has the function $\text{Var}_{\text{NPT}}(\text{ONE}, \lambda, k)$. This means that the variability (*i.e.*, the magnitudes of fluctuations) of

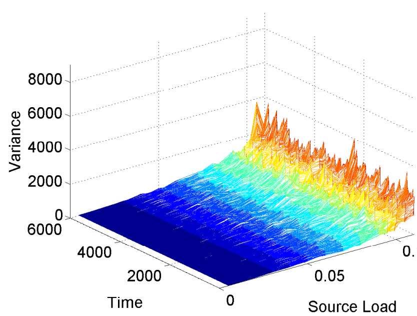
NPT time series increases with increase of the *source load* values. For the *source load* values close to the *critical source load* values λ_c the values of $\text{Var}_{\text{NPT}}(\text{ONE}, \lambda, k)$ function are very high and they are much larger than those of the other two functions $\text{Var}_{\text{NPT}}(\text{QS}, \lambda, k)$ and $\text{Var}_{\text{NPT}}(\text{QSPO}, \lambda, k)$. This means that for the considered *source load* values the NPT series of the PSN model with $\mathcal{L}^{p}_{\square}(16, \text{ONE}, \lambda)$ setup have very high variability which is much larger than those of the NPT time series of the other setups. Thus, for the values of λ that are closed to the *critical source load* values λ_c there is a very high variability among the queue sizes in the PSN model with $\mathcal{L}^{p}_{\square}(16, \text{ONE}, \lambda)$ setup which is the highest among the PSN model with $\mathcal{L}^{p}_{\square}(16, \text{e.c.f.}, \lambda)$ setups, as can be seen from [7, 8, 9].



(a) e.c.f. = ONE



(b) e.c.f. = QS



(c) e.c.f. = QSPO

Fig. 3. Graphs of e.c.f., *source load* and time dependent *variance functions* of NPT time series, $\text{Var}_{\text{NPT}}(\text{e.c.f.}, \lambda, k)$, corresponding to 24 different seeds of the first pseudo-random number generator for each PSN model with $\mathcal{L}^{p}_{\square}(16, \text{e.c.f.}, \lambda)$ setup, where respectively, e.c.f. = ONE, or QS, or QSPO and each of them is listed under the corresponding plot.

In the PSN model with $\mathcal{L}^{p}_{\square}(16, \text{ONE}, \lambda)$ setup when the values of λ are closed to the *critical source load* values λ_c the queue sizes increase due to random fluctuations and the static routing in ability to routes packets avoiding congested network nodes. Packets stay much longer in the network when the static routing is used then when an adaptive routing is used instead. The adaptive routings have the ability to route packets avoiding congested network nodes and they try to redistribute the packets evenly among the network nodes. These prevent local congestions to build up.

The above discussion explains the sequence of changes in the order of magnitudes of the *mean value functions* from $\hat{N}(\mathrm{ONE}, 0.090, k) \leq \hat{N}(\mathrm{QSPO}, 0.090, k) < \hat{N}(\mathrm{QS}, 0.090, k)$ to $\hat{N}(\mathrm{QSPO}, 0.095, k) \leq \hat{N}(\mathrm{ONE}, 0.095, k) < \hat{N}(\mathrm{QS}, 0.095, k)$, next to $\hat{N}(\mathrm{QSPO}, 0.100, k) < \hat{N}(\mathrm{ONE}, 0.100, k) < \hat{N}(\mathrm{QS}, 0.100, k)$, next to $\hat{N}(\mathrm{QSPO}, 0.105, k) < \hat{N}(\mathrm{ONE}, 0.105, k) \approx \hat{N}(\mathrm{QS}, 0.105, k)$, and finally to $\hat{N}(\mathrm{QSPO}, 0.110, k) < \hat{N}(\mathrm{QS}, 0.110, k) < \hat{N}(\mathrm{ONE}, 0.110, k)$ that can be seen in Fig. 2. Since the dynamic routing using e.c.f. QSPO takes under consideration not only the number of hops that packets must perform to reach their destinations but also the queue sizes of the routers located along the packets routes then the packets are more efficiently delivered to their destinations than when e.c.f. QS is used instead. This explains why we have always observed that $\hat{N}(\mathrm{QSPO}, \lambda, k) < \hat{N}(\mathrm{QS}, \lambda, k)$.

5. Conclusions

The discussed e.c.f., *source load* and time dependent *mean value functions* $\hat{N}(\text{e.c.f.}, \lambda, k)$ and *variance functions* $\mathrm{Var}_{\mathrm{NPT}}(\text{e.c.f.}, \lambda, k)$ are statistical aggregate measures of network performance that provide some insight into the dynamics of number of packets in transit and their variability. We have observed that these dynamics are affected by the coupling of a routing type with values of *source load*. For *sub-critical source load* values that are away from the critical points λ_c the values of mean value functions satisfy the inequalities $\hat{N}(\mathrm{ONE}, \lambda, k) \leq \hat{N}(\mathrm{QSPO}, \lambda, k) \leq \hat{N}(\mathrm{QS}, \lambda, k)$. When the *source load* values approach the critical points λ_c the order of values of the mean value functions changes first to $\hat{N}(\mathrm{QSPO}, \lambda, k) \leq \hat{N}(\mathrm{ONE}, \lambda, k) \leq \hat{N}(\mathrm{QS}, \lambda, k)$ and next to $\hat{N}(\mathrm{QSPO}, \lambda, k) \leq \hat{N}(\mathrm{QS}, \lambda, k) \leq \hat{N}(\mathrm{ONE}, \lambda, k)$. The reason for these changes is that with the increase of *source load* values an adaptive routing, *i.e.* a routing using the dynamic cost QS or QSPO have ability to route packets avoiding local congestions that arise from the increased fluctuations in number of packets in transit. The static routing using e.c.f. ONE does not have this ability. Since the dynamic routing using e.c.f. QSPO takes under consideration not only the number of hops that packets must perform to reach their destinations but also the queue sizes of the routers located along the packets routes then the packets are more efficiently delivered to their destinations than when e.c.f. QS is used instead. This explains why we have always observed $\hat{N}(\mathrm{QSPO}, \lambda, k) < \hat{N}(\mathrm{QS}, \lambda, k)$.

This work was made possible by the facilities of the Shared Hierarchical Academic Research Computing Network (SHARCNET: `www.sharcnet.ca`). The authors acknowledge the prior work of A.T. Lawniczak (A.T.L.) with A. Gerisch, B.N. Di Stefano, X. Tang and J. Xu. A.T.L. acknowledges partial financial support from SHARCNET and NSERC of Canada and S. Xie from the Univ. of Guelph and SHARCNET. The authors acknowledge use of simulation data produced by J. Xu as part of the fulfilment of a SHARCNET grant of A.T.L. The authors thank The Fields Institute for Research in Mathematical Sciences for the hospitality while writing this paper.

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