THE HAGEDORN SPECTRUM AND LARGE N_c QCD*

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This talk discusses a recent approach developed to deduce the existence of a Hagedorn spectrum — that is a hadron spectrum in which the number of hadrons grows exponentially with mass — from QCD. The approach is valid in the large N_c limit of QCD and exploits a tension between asymptotic freedom and confinement. While the approach is not rigorous in the sense of a mathematical theorem, it will hold provided certain standard assumptions made about correlation function in QCD are correct.

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1. Introduction

This workshop is entitled "Excited QCD". Perhaps the most basic question one could ask about excited QCD is the density of hadrons in the spectrum as a function of mass. That is, how many hadrons are there in a mass range from m to $m + \Delta m$? A useful way to parameterize this information is to look at the integral of the density — that is the number of hadrons with mass less than m: $N(m)$.

A Hagedorn spectrum refers to a spectrum of hadrons in which $N(m)$ grows exponentially

$$N(m) \sim m^{-2b} \exp\left(\frac{m}{T_{\rm H}}\right) , \quad (1)$$

where b is a constant, and $T_{\rm H}$, the so-called Hagedorn temperature, is a parameter with dimensions of mass. Hagedorn proposed this idea in the mid-1960s based on data from high energy collisions and a "fireball" picture of the reactions combined with statistical analysis [1,2]. Over the years this

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idea has had a wild ride and has played an important role in fields ranging from string theory to phase transitions in QCD.

Now before proceeding it is worth noting that there is some phenomenological support for a Hagedorn spectrum by looking at the hadron masses as reported by the Particle Data Group [3]. The meson masses grow rapidly with m up to above 2 GeV and appear to be consistent — at least qualitatively — with an exponential growth [4]. However, such an analysis is not fully compelling, in part because at very high masses it becomes increasingly difficult to extract hadrons from partial wave analysis of scattering experiments. Thus one cannot really check that $N(m)$ begins to very accurately approach an exponential at very large values.

2. The large N_c limit

Beyond the practical problem of being unable to extract the masses of very high-lying mesons lies a deeper problem. Recalling Voltaire's famous aphorism that the Holy Roman Empire is neither holy nor Roman nor an empire, it is worth remembering similarly that the "particle data" on mesons listed by the Particle Data Group is neither data nor is it about particles. Rather, it is model-dependent extractions from the physical scattering data of properties of hadronic resonances. The fact the mesons are resonances with widths means that in a strict sense their masses are not well defined and neither is $N(m)$. Thus at a deep theoretical level the question of whether QCD has a Hagedorn spectrum is simply not well posed. Given this problem, how can we proceed?

One way is to replace the messy and complicated world of QCD with the more orderly world of QCD in the limit of a large number of colors [5, 6]. In doing this we make the assumption that the large N_c world is similar to the real world of $N_c = 3$ with relatively small $1/N_c$ corrections. One critical thing about the large N_c limit is that the widths of mesons and glueballs are given by

$$\Gamma_{\text{meson}} \sim \frac{1}{N_c}\,, \qquad \Gamma_{\text{glueball}} \sim \frac{1}{N_c^2}\,. \tag{2}$$

Thus, meson and glueball widths go to zero at large N_c and their masses become well defined. Therefore, the question of whether large N_c QCD has a Hagedorn spectrum is at least well posed. As it happens, the large N_c limit also greatly simplifies many other aspects of the analysis needed to show the existence of a Hagedorn from QCD.

The question of whether the large N_c limit of QCD has a Hagedorn spectrum is related to a fundamental question. Namely, whether large N_c QCD acts like a string theory — at least for sufficiently high-lying states. Note that simple string theories with unbreakable strings (like, for example, flux tubes at large N_c) automatically have Hagedorn spectra [7]. Thus, if one can show that large N_c QCD does, in fact, have a Hagedorn spectrum, one has some evidence that the dynamics is stringy.

3. A theoretical argument

One can construct a theoretical argument that large N_c QCD must have a Hagedorn spectrum [8, 9]. The argument has the following input:

- Confinement. Confinement is only required in the most basic physical sense that physical states are color singlets.
- Asymptotic freedom.
- Standard assumptions about the regime of validity of perturbation theory. In particular, the assumption that perturbation theory itself is sufficient to determine the regime of validity of perturbation theory. That is, if one calculates perturbative correction to a point-to-point correlation function, one expects at sufficiently short distances the correlation function to be accurately described by the asymptotically free expression, and that if perturbative corrections to this result are computed to be small in perturbative theory, then perturbation is accurate.
- Plausible technical assumptions about the ordering of limits.

Note that this argument does not constitute a rigorous theorem since it requires the assumptions stated above. Nevertheless, the argument only really depends on assumptions which appear to be well justified in QCD.

There are a couple of key things that allow the argument to work. The first is simply that the number of local single color trace operators of fixed quantum number grows exponentially with the mass dimension of the operator. While each operator does not create a distinct hadron, this exponential growth in the number of operators is ultimately translated into an exponential growth with mass in the number of hadrons. The second is that the regime of validity of perturbation theory for the trace of the logarithm of a matrix of point-to-point correlators between operators is independent of the dimension of the operator at large N_c. Note that this latter condition is false at finite N_c.

I will not outline the details of this argument in this write-up. A quick sketch of the argument can be seen in the conference slides: `http://th.physik.uni-frankfurt.de/~excitedqcd/talkeqcd2011/monday/TomHagedorng.pdf` more thorough discussion can be found in Ref. [9].

Let me close with a couple of comments about the argument. The first is that the demonstration of a Hagedorn spectrum is made without any explicit assumption that the dynamics are in any sense "stringy". Thus the demonstration can be taken as independent evidence that the dynamics does in fact become stringy. A second comment is that confinement is only assumed in the sense that all physical states are color singlets. No explicit assumptions are made about area law of Wilson loops or an unbroken center symmetry. Finally, it should be recalled that the argument is based on taking the large N_c limit from the outset. What this analysis tells us about the real world of $N_c = 3$ remains an open question.

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