

PHENOMENOLOGICAL IMPROVEMENT OF THE LINEAR σ -MODEL IN THE LARGE- N_c LIMIT*

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The linear σ -model has been widely used to describe the chiral phase transition. Numerically, the critical temperature T_c of the chiral phase transition is in agreement with other effective theories of QCD. However, in the large- N_c limit T_c scales as $\sqrt{N_c}$ which is not in line with the NJL model and with basic expectations of QCD, according to which T_c is — just as the deconfinement phase transition — N_c -independent. This mismatch can be corrected by a phenomenologically motivated temperature dependent parameter.

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1. Introduction

The Quantum Chromodynamics (QCD) at finite temperature and finite density is a central topic in high energy physics. For small temperatures and densities the quark and gluon degrees of freedom are confined in hadrons. It is expected that there exists a region in the QCD phase diagram where quarks and gluons behave like a plasma (deconfinement) [1,2]. The QCD Lagrangian does not allow to directly calculate the confinement/deconfinement phase transition or the related chiral phase transition. The latter is mathematically well defined in the limit of zero quark masses, in which the QCD Lagrangian is invariant under chiral symmetry transformation and the chiral condensate is an exact order parameter [3, 4, 5]. Two effective models, the Nambu–Jona-Lasino model (NJL) [6, 7, 8, 9] and the linear σ -model [10, 11, 12], have been often used to study the properties of the chiral phase transition.

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Beside these phenomenological approaches to QCD there is also the large- N_c approximation [13, 14]. The number of color degrees of freedom in the QCD Lagrangian is three, but a theory with an infinite large number of color degrees of freedom shows a behavior similar to the one of a theory with three colors. In the large- N_c limit the gauge symmetry of the QCD is changed from $SU(3)$ to $SU(N_c \gg 3)$. Enlarging the number of colors also leads to a modified QCD coupling g_{QCD} in a way that for $N_c \rightarrow \infty$ the product $g_{\text{QCD}}^2 N_c$ remains constant. Quarks and gluons are still present but gluons dominate the behavior of the theory. For low temperatures there still exists an confined phase, where the degrees of freedom are mesons and baryons [15]. For high enough T it is believed that the theory is deconfined. Although also in the large- N_c limit the theory is not solvable, it is significantly simpler because only planar diagram survive.

2. Linear σ -model in the large- N_c limit

The linear σ -model [10, 11, 12], is an effective theory which is able to describe the mass splitting of the pions and the sigma via spontaneous symmetry breaking. The model is built with terms which are invariant under chiral symmetry transformation. In the vacuum the chiral symmetry is spontaneously broken and the pions emerge as Goldstone bosons. In the original form there is no explicit N_c dependency. From former studies one knows that the quark–antiquark meson masses are independent of the number of colors, but the coupling of three mesons is suppressed by a factor of $1/\sqrt{N_c}$ and the four mesons coupling by a factor of $1/N_c$ [14]. These scaling properties can be implemented by redefining the meson four point interaction $\lambda \rightarrow 3\lambda/4N_c$, while the parameter μ is not affected in the large- N_c limit: $\mu \rightarrow \mu$. The Lagrangian of the σ -model as function of N_c reads

$$\mathcal{L}_\sigma(N_c) = \frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{2}\mu^2 \Phi^2 - \frac{\lambda}{4} \frac{3}{N_c} \Phi^4, \quad (1)$$

where the scalar field σ and the pseudoscalar pion triplet $\vec{\pi}$ are described by $\Phi^t = (\sigma, \vec{\pi})$. For $\mu^2 > 0$ the chiral condensate is $\varphi_0 = \varphi(T = 0) = \mu\sqrt{N_c/3\lambda} = \sqrt{N_c/3} f_\pi$. The only scale in the Lagrangian is f_π , to be identified with the pion decay constant. Note that the chiral condensate scales with $\varphi_0 \propto \sqrt{N_c}$. The tree-level masses are not effected by the N_c -scaling and read $m_\sigma^2 = 3\lambda f_\pi^2 - \mu^2$, $m_\pi^2 = 0$.

The behavior at finite temperature can be analyzed using the Cornwall–Jackiw–Tomboulis (CJT) formalism [16]. The gap equation for the chiral condensate is

$$0 = \varphi(T) \left(\frac{3}{N_c} \lambda \varphi(T)^2 - \mu^2 + \frac{9}{N_c} \lambda \int G_\sigma + \frac{9}{N_c} \lambda \int G_\pi \right). \quad (2)$$

The full propagators G_σ and G_π have the form

$$G_i = \int_0^\infty \frac{dk}{2\pi^2} \frac{k^2}{\sqrt{k^2 + m_i^2}} \left[\exp \left(\sqrt{k^2 + m_i^2} / T \right) - 1 \right]^{-1}. \quad (3)$$

The critical temperature T_c is defined as the temperature where the condensate exactly vanishes $\varphi(T_c) = 0$. This leads to the following N_c dependent scaling of the critical temperature

$$T_c(N_c) = \sqrt{2} f_\pi \sqrt{\frac{N_c}{3}} \propto N_c^{1/2}. \quad (4)$$

For the case $N_c = 3$ one obtains the known result $T_c = \sqrt{2} f_\pi$, but it also clearly emerges that T_c increases with N_c . This means that for $N_c \gg 3$ the chiral phase transition will not take place. The result is general for all hadronic models which do not include color degrees of freedom or temperature dependent parameters. This result, first noticed in Ref. [17], contradicts the results found in the NJL-model [7] where the critical temperature T_c remains constant in the large- N_c limit, and with the fact that the related confinement/deconfinement phase transition is expected to be proportional to Λ_{QCD} , which is a large- N_c independent quantity.

3. Phenomenological modification of the linear σ -model

In order to solve this discrepancy a phenomenological approach is proposed. In Refs. [18, 19] it is argued that the T^2 scaling of order parameters is general. A phenomenological way to take this property into account is to make the parameter μ^2 temperature dependent

$$\mu^2 \rightarrow \mu(T)^2 = \mu^2 \left(1 - \frac{T^2}{T_0^2} \right). \quad (5)$$

The parameter T_0 is a new temperature scale and should be of the order of Λ_{QCD} . The Eq. (5) modifies the gap equation (2) and leads to a different critical temperature

$$T_c(N_c) = T_d \left(1 + \frac{T_d^2}{2f_\pi^2} \frac{3}{N_c} \right)^{-1/2} \propto N_c^0. \quad (6)$$

The critical temperature is constant in the limit $N_c \rightarrow \infty$.

Now we turn back to the $N_c = 3$ and study the case where the explicit symmetry breaking term $\epsilon\sigma$ is present. The complete Lagrangian including a second temperature scale reads

$$\mathcal{L}_\sigma(T_0) = \frac{1}{2}(\partial_\mu\Phi)^2 + \frac{1}{2}\mu^2\left(1 - \frac{T^2}{T_0^2}\right)\Phi^2 - \frac{\lambda}{4}\Phi^4 + \epsilon\sigma. \quad (7)$$

All parameters are fixed via the masses and the pion decay constant ($\epsilon = f_\pi M_\pi^2$, $\lambda = (M_\sigma^2 - M_\pi^2)/(2f_\pi^2)$, $\mu^2 = (M_\sigma^2 - 3M_\pi^2)/2$). The quantity T_0 is set to a value of $T_0 = 0.27$ GeV. The vacuum masses are chosen to be the following: the mass of the σ -field is $M_\sigma = 1.2$ GeV (for the discussion of the value of the σ mass in the vacuum, see Refs. [20] and refs. therein), of the π -field is $M_\pi = 0.135$ GeV and the value for the pion decay constant is $f_\pi = 92.4$ MeV.

The finite temperature behavior for the masses, see Fig. 1, is similar to the one with no temperature dependent parameters. Until the critical temperature T_c is reached the temperature dependency of the pion mass and the σ mass varies slowly. Close to T_c the mass of the σ drops and slightly above T_c the mass becomes degenerated with the pion mass. At high temperature both masses rise linearly.

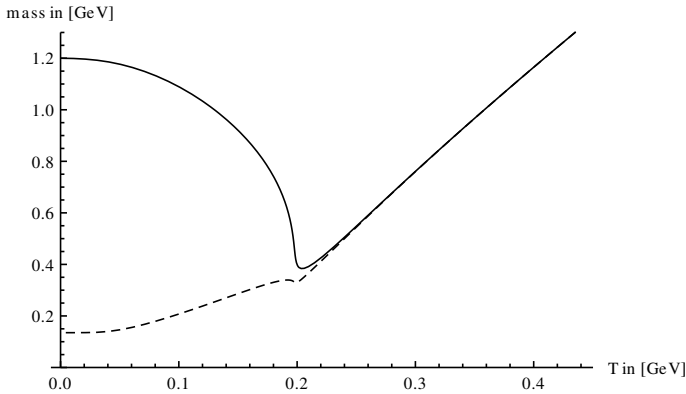


Fig.1. Finite temperature behavior of the masses. The dashed line represents the mass of the pions and the continuous line mass of the σ . Above the critical temperature of $T_c \approx 200$ MeV the masses become degenerated.

Beside these similarities there are two remarkable properties that differ. First, the order of the phase transition is changed from a first order to a crossover phase transition. Second, the critical temperature is lowered to $T_c \approx 200$ MeV. Both phenomena can be seen in Fig. 2, where the case $N_c \rightarrow \infty$ is shown: in this limit the chiral symmetry is restored through the new temperature scale T_0 and not via mesonic loops.

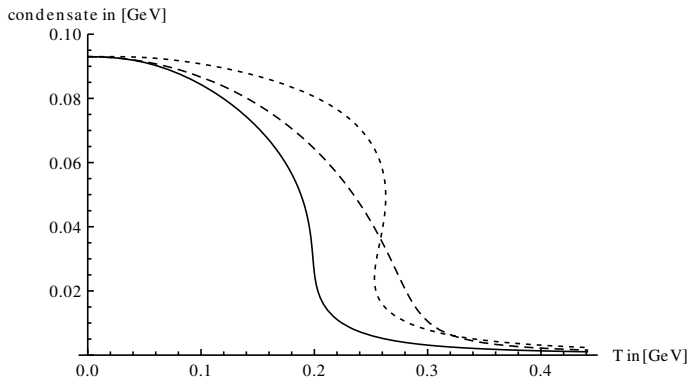


Fig. 2. The chiral condensate for different number of colors and temperature scale T_0 . Continuous line: $N_c = 3$ and $T_0 = 270$ MeV, dotted line: $N_c = 3$ and $T_0 \rightarrow \infty$, dashed line: $N_c \rightarrow \infty$ and $T_0 = 270$ MeV.

4. Conclusions

In this work the mismatch between the NJL model and purely hadronic-models in the large- N_c limit has been studied. We have found that the linear σ -model implies a scaling of T_c which is at odd with the NJL model and basic expectations [3].

In order to solve this issue, we have introduced a phenomenologically motivated temperature dependent parameter. As a result, the critical temperature remains constant in the large- N_c limit. Moreover, for $N_c = 3$ the critical temperature is lowered to $T_c \approx 200$ MeV, a value which is in line with recent model and lattice results on the chiral phase transition.

Future studies should go beyond the simple phenomenological Ansatz presented in this work and include, for instance, the coupling of hadrons to the Polyakov loop [2, 21]. Preliminary results [22] show that this approach also leads to the correct large- N_c scaling of the critical temperature $T_c \sim N_c^0$.

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