TRANSITION TEMPERATURE AND THE EQUATION OF STATE FROM LATTICE QCD, WUPPERTAL-BUDAPEST RESULTS*

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(Received July 27, 2011)

The QCD transition is studied on lattices up to $N_t = 16$. The chiral condensate is presented as a function of temperature, and the corresponding transition temperature is extracted. The equation of state is determined on lattices with $N_t = 6$, 8, 10 and at some temperature values with $N_t = 12$. The pressure and the trace anomaly are presented as functions of the temperature in the range 100–1000 MeV. Using the same configurations we determine the continuum extrapolated phase diagram of QCD on the μ -T plane for small to moderate chemical potentials. Two transition lines are defined with two quantities, the chiral condensate and the strange quark number susceptibility.

DOI:10.5506/APhysPolBSupp.4.593 PACS numbers: 12.38.Gc, 25.75.Nq

1. Introduction

The study of QCD thermodynamics is receiving increasing interest in recent years. A systematic approach to determine the properties of the deconfinement phase transition is through lattice QCD. Lattice simulations indicate that the transition at vanishing chemical potential is merely an analytic crossover [1]. Some interesting quantities that can be extracted

^{*} Presented at the Workshop "Excited QCD 2011", Les Houches, France, February 20–25, 2011.

from lattice simulations are the transition temperature T_c , the QCD equation of state and, for small chemical potentials, the phase diagram in the μ -Tplane: we review the results on these observables that have been obtained by our collaboration using the staggered stout action with physical light and strange quark masses, thus $m_s/m_{ud} \simeq 28$ [2,3]. For all details we refer the reader to Refs. [4,5,6]. For a pedagogical review see Ref. [7].

2. The QCD transition temperature

We present here the results for several quantities. We study strange susceptibility, the Polyakov-loop and the chiral condensate, and extract the value of T_c associated to these observables. The T_c values are different which reflects the nature of the crossover transition. For details we refer the reader to Ref. [4].

The strange susceptibility does not need any additional renormalization. The renormalization procedure of the Polyakov loop was given in [3]. The temperature dependences of the strange susceptibility and the Polyakov loop are shown in Fig. 1. The chiral condensate is defined as

$$\langle \bar{\psi}\psi \rangle_q = T \partial \ln Z / (\partial m_q V) \quad \text{for} \quad q = u, d, s.$$
 (1)

It is an indicator for the remnant of the chiral transition, since it rapidly changes around $T_{\rm c}$. Its renormalization is given in [1].



Fig. 1. Left: Strange quark number susceptibility as a function of temperature. Right: Renormalized Polyakov loop as a function of temperature. In both figures, the different symbols correspond to different N_t . The gray band is the continuum extrapolated result.

A similar quantity can be defined as

$$\Delta_{l,s} = \frac{\left\langle \bar{\psi}\psi \right\rangle_{l,T} - m_l/m_s \left\langle \bar{\psi}\psi \right\rangle_{s,T}}{\left\langle \bar{\psi}\psi \right\rangle_{l,0} - m_l/m_s \left\langle \bar{\psi}\psi \right\rangle_{s,0}} \tag{2}$$

for l = u, d. Since the results at different lattice spacings are essentially on top of each other, we connect them to lead the eye (see Fig. 2). The value of T_c that we obtain from the inflection point of the latter observable is $T_c = 157(3)(3)$. The lattice results can be compared with the predictions of the hadron resonance gas model. As it is shown in Fig. 3 the two results agree up to quite high temperatures.



Fig. 2. Left: Renormalized chiral condensate $\langle \bar{\psi}\psi \rangle_R$ defined in Ref. [4]. Right: Subtracted chiral condensate $\Delta_{l,s}$ defined in Eq. (2). In both figures, the different symbols correspond to different N_t . The gray band is our continuum estimate.



Fig. 3. Left: Renormalized chiral condensate as defined in Ref. [4]. Right: Subtracted chiral condensate $\Delta_{l,s}$ as defined in Eq. (2), as a function of temperature. Gray bands are the continuum results of our collaboration, obtained with the stout action. Full symbols are obtained with the asqtad and p4 actions [8,9]. In both panels, the solid line is the HRG model result with physical masses. The error band corresponds to the uncertainty in the quark mass-dependence of hadron masses. The dashed lines are the HRG+ χ PT model result with distorted masses, which take into account the discretization effects and heavier quark masses used in [8,9] for $N_t = 8$ and $N_t = 12$.

It is also instructive to compare the present results obtained on $N_t = 6$, 8, 10, 12 and 16 with the results of the HotQCD Collaboration (*cf.* [4]). Figure 4 shows this comparison. As it can be seen, the results of the HotQCD Collaboration are getting closer and closer to our predictions. The long standing discrepancy is disappearing.



Fig. 4. The subtracted chiral condensate $\Delta_{l,s}$ as a function of temperature. We show a comparison between stout, asqtad, p4 and HISQ [8,9] results. Our results are shown by colored open symbols, whereas the hotQCD results are shown by full black symbols. The gray band is our continuum result, the thin lines for the hotQCD data are intended to lead the eye. Our stout results were all obtained by the physical pion mass of 135 MeV. The full dots and squares correspond to $m_{\pi} = 220$ MeV, the full triangles and diamonds correspond to $m_{\pi} = 160$ MeV of the hotQCD Collaboration.

3. QCD equation of state

Next, we present our results regarding the equation of state. The details of this calculation can be found in [5]. It is important to emphasize that quark masses were set to their physical values and we used quite fine lattices up to $N_t = 12$. We have explicitly showed that for our systems the finite volume corrections are under control. The left plot in Fig. 5 shows two systems. One of them with a volume V the other one with a volume of 8 V. As it can be seen for the whole temperature range, there are practically no finite volume corrections, the two curves are lying on top of each other.

In the right plot in Fig. 5 we show the trace anomaly for the 2 + 1 flavor system at four different lattice spacings. Results show essentially no dependence on "a", they all lie on top of each other. Only the coarsest $N_t = 6$ lattice shows some deviation around ~ 300 MeV. On the same figure, we zoom in to the transition region. Here, we also show the results from the Hadron Resonance Gas model: a good agreement with the lattice results is found up to hight T.



Fig. 5. Left: The trace anomaly on lattices with different spatial volumes: $N_s/N_t = 3$ (red band) and $N_s/N_t = 6$ (blue points). Right: The trace anomaly $I = \epsilon - 3p$ normalized by T^4 as a function of temperature on $N_t = 6, 8, 10$ and 12 lattices.

In order to obtain the pressure, we determine its partial derivatives with respect to the bare lattice parameters. p is then rewritten as a multidimensional integral along a path in the space of bare parameters. The result is shown in Fig. 6, left. To obtain the EoS for various m_{π} , we simulate for a wide range of bare parameters on the plane of $m_{u,d}$ and β (m_s is fixed to its physical value). Having obtained this large set of data we generalize the integral method and include all possible integration paths into the analysis [5, 10]. We remove the additive divergence of p by subtracting the same observables measured on a lattice, with the same bare parameters but at a different T value. Here we use lattices with a large enough temporal extent, so it can be regarded as T = 0. The right plot of Fig. 6 shows the entropy density.



Fig. 6. The pressure (left) and the entropy density (right) normalized to temperature as a function of temperature on $N_t = 6,8$ and 10 lattices. The Stefan-Boltzmann limits $(p/T^4)_{\rm SB} \approx 5.209$ and $(s/T^3)_{\rm SB} \approx 20.836$ are indicated by an arrow. For our highest temperature T = 1000 MeV the pressure is almost 20% below this limit.

It is also instructive to compare the present results obtained on $N_t = 6$, 8, 10 and 12 with the results of the HotQCD Collaboration (*cf.* [4]). Figure 7 shows this comparison for the trace anomaly. As it can be seen, the results of the HotQCD Collaboration are still quite far away from our results. Their peak position is about at a 20 MeV higher temperature, whereas their peaks heights are about 50% larger than ours. The clarification of this discrepancy remains for the future.



Fig. 7. The normalized trace anomaly obtained in our study is compared to recent results from the "hotQCD" Collaboration [8,11].

4. The QCD phase diagram at nonzero quark density

We provided results for the transition temperatures T_c at vanishing chemical potential ($\mu = 0$). Now we move out to the $\mu \neq 0$ plane (for the details see [6]). There are several possibilities. Figure 8 shows two scenarios. In principle, it is possible that the transition temperatures (*e.g.* the one related to the chiral condensate and the one related to the strange susceptibility) are getting closer and closer to each other as the chemical potential increases. One might even end up with a critical endpoint. It is also possible that the transition is getting weaker and weaker and the difference between two transition temperatures increases as we increase the chemical potential. In this section we determine the leading order behaviour of the transition lines. Note, that this order will be not able to tell the difference between these two scenarios, thus it cannot prove or exclude the existence of the critical endpoint. To that end one has to determine higher order terms.

As we will see as one increases μ the transition temperature $T_{\rm c}(\mu^2)$ decreases. Let us parameterize the transition line in the vicinity of the vertical $\mu = 0$ axis as $T_{\rm c}(\mu^2) = T_{\rm c}(1 - \kappa \cdot \mu^2/T_{\rm c}^2)$.



Fig. 8. Two possible scenarios for the QCD phase diagram on the μ -T plane, defined using a given observable. The left panel shows a phase diagram with a transition growing stronger and possibly even turning into a real, first-order phase transition at a critical endpoint. The right panel on the other hand corresponds to a scenario with a weakening transition and no critical endpoint. The paths corresponding to systems describing the early Universe and a heavy ion collision are also shown by the arrows. Note that different observables may lead to different scenarios.

In order to determine the transition temperature as a function of μ , we use two quantities which are monotonic in the transition region and do not depend on μ for zero or infinite temperatures. The transition temperature is defined as the temperature value at which these observables take their value as given by the inflection points of the curves at $\mu = 0$.

The two observables we use are the renormalized chiral condensate and the normalized strange quark number susceptibility. In order to measure the μ dependence of these quantities, we apply reweighting. Since our lattices are quite large the full reweighting method is quite expensive. Therefore, we truncate the μ dependence of the weights at μ^2 order (this truncation of the original [12,13] method is usually called the Taylor method; for a recent application see [14]).

The strange quark number susceptibility is the second derivative of the partition function with respect to the strange chemical potential. It needs no renormalization, since it is related to a conserved current. It is useful to normalize it by T^2 , which provides a dimensionless combination. It is easy to see that at T = 0 its value is 0, whereas for infinitely large temperatures it approaches the Stefan–Boltzmann limit of 1.

For the chiral condensate we apply here a slightly different renormalization prescription than what was used for the determination of T_c . We cancel the additive divergences by subtracting the T = 0 contribution, while the multiplicative divergence due to the derivative with respect to the mass can be eliminated with a multiplication by the bare quark mass. Then, in order to have a dimensionless combination the whole expression can be divided by the fourth power of some dimensionful mass scale. In this work we use the T = 0 pion mass for the normalization. This observable has also μ independent limiting values at zero and at infinitely high temperatures (at T = 0this is true for chemical potentials smaller than the baryon mass, well within our applicability region).

Our final result for the phase diagram on the μ -T plane is shown in Fig. 9. The crossover region's extent changes little as the chemical potential increases. The two definitions give slightly different curves for $T_c(\mu)$. In this order of the $T_c(\mu)$ curve we do not see the critical endpoint. One should emphasize, that this is not a signal for non-existence of the critical point. This order neither can exclude nor prove the existence of the critical endpoint. It is useful to compare the whole picture to the freeze-out curve which summarizes experimental results on the $T-\mu$ points where hadronization of the quark-gluon plasma was observed. This curve is expected to lie in the interior of the crossover region, as is indicated by our results as well.



Fig. 9. The crossover transition between the "cold" and "hot" phases is represented by the shaded (colour on-line) area (blue and red correspond to the transition regions obtained from the chiral condensate and the strange susceptibility, respectively). The lower solid band shows the result for $T_c(\mu)$ defined through the chiral condensate and the upper one through the strange susceptibility. The width of the bands represent the statistical uncertainty of $T_c(\mu)$ for the given μ coming from the error of the curvature κ for both observables. The dashed line is the freeze-out curve from heavy ion experiments [15]. Also indicated are with different symbols the individual measurements of the chemical freeze-out from RHIC, SPS (Super Proton Synchrotron) and AGS (Alternating Gradient Synchrotron), respectively. The center of mass energies $\sqrt{s_{NN}}$ for each are shown in the legend.

Work supported in part by the EU grant (FP7/2007-2013)/ERC No. 208740. The work of C.R. is supported by funds provided by the Italian Ministry of Education, Universities and Reserach under the Firb Research Grant RBFR0814TT. The calculations have been carried out on the computer clusters at the Universities of Wuppertal and Budapest using GPU programming [16] and on the BG/P system at Forschungszentrum Julich.

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