QED3 AT FINITE TEMPERATURE*

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Novel nonperturbative properties of QED in three dimensions are examined at finite temperature. We show that infrared divergences are endemic to the theory and discuss difficulties in computing the electric screening mass.

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1. Introduction

A variety of novel features has spurred interest in low-dimensional QED for many decades [1]. For example, high temperature QCD can be represented as the dimensionally reduced QCD3. If the number of quark flavours (N_f) is large, the non-Abelian behaviour of the theory is suppressed and it may be approximated as quantum electrodynamics in three dimensions (QED3) [2]. Massless QED3 in the large N_f limit generates dynamical fermion masses that are suppressed exponentially in the fermion number. Thus this theory illustrates how large mass hierarchies can be dynamically generated [3, 4], which is of interest to BSM physics.

The extension of a reliable computational scheme for QED3 to finite temperature is of interest due to its many condensed matter applications [5]. This is a technically challenging problem, and past attempts have been forced to make many additional approximations beyond the truncation of the Schwinger–Dyson equations. The most immediate concern is the lack of covariance which makes dynamical quantities a function of two variables, p_0, $\vec{p}$, rather than simply p^2. This raises the computational requirements by one or two orders of magnitude.

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An additional concern is the presence of infrared divergences in the formalism. Infrared divergences are exacerbated at nonzero temperature because perturbative diagrams are dominated in the infrared limit by the lowest available Matsubara frequency, which is zero in bosonic sums. Thus, even though QED3 is infrared finite at zero temperature, problems may arise again at nonzero temperature. This issue has engendered some confusion in the literature. Some authors have noted that an infrared divergence exists, but have ignored it [6], or imposed an infrared cutoff [7], or assumed that higher order corrections remove the divergence [8]. Many authors simply evade the issue entirely by employing the approximation [9]

$$iD_{\mu\nu}\left(\omega,\vec{q}\,\right) \to iD_{00}\left(0,\vec{q}\,\right)\,. \tag{1}$$

2. QED3 at finite temperature

We shall argue that infrared divergences are endemic to QED3 at finite temperature. Furthermore, the problem is not alleviated by finite fermion masses. Nevertheless, observables are finite and the theory is well-defined [10].

We employ the imaginary time formalism and choose to work covariantly, which necessitates introducing a three-vector, n^μ, that represents the heat bath. Thus the full fermion propagator is

$$S = \frac{i}{\left(A_0 - A\right) p_0 \not{n} + A \not{p} - B}\,. \tag{2}$$

Here,

$$p^\mu = \left(i\omega_n, \vec{p}\,\right), \tag{3}$$

where $\omega_n = (2n+1)\pi T$ is a fermionic Matsubara frequency and A, B, and A_0 are functions of ω_n and $\vec{p}$.

2.1. Photon propagator

The vacuum polarisation tensor remains transverse at finite temperature, however, the presence of an additional three-vector permits two transverse tensors

$$P^L_{\mu\nu}(n,q) = \hat{q}^\perp_\mu \hat{q}^\perp_\nu \tag{4}$$

and

$$P^\perp_{\mu\nu}(n,q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} - P^L_{\mu\nu}(q)\,. \tag{5}$$

A transverse three-vector has been defined as

$$q^\perp_\mu = q_\mu - n_\mu \frac{q^2}{n\cdot q}\,. \tag{6}$$

With these definitions one can parameterise the photon self energy as

$$\begin{aligned}\Pi_{\mu\nu}(n,q) &= P^{\perp}_{\mu\nu}\Pi_{\perp} + P^{L}_{\mu\nu}\Pi_{L} + i\epsilon_{\mu\nu\alpha}\hat{q}^{\alpha}\tilde{\Pi} + i\hat{q}^{\perp}_{\mu}\epsilon_{\nu\alpha\beta}\hat{q}^{\alpha}\hat{q}^{\beta}_{\perp}\Pi_{4} \\ &\quad + i\hat{q}^{\perp}_{\nu}\epsilon_{\mu\alpha\beta}\hat{q}^{\alpha}\hat{q}^{\beta}_{\perp}\Pi_{5} + \epsilon_{\mu\alpha\beta}\epsilon_{\nu\alpha'\beta'}\hat{q}^{\alpha}\hat{q}^{\alpha'}\hat{q}^{\beta}_{\perp}\hat{q}^{\beta'}_{\perp}\Pi_{6}\,. \end{aligned} \tag{7}$$

Note that $\tilde{\Pi}$, Π_4, Π_5, and Π_6 are all null for four-component fermions and it is possible to combine the Π_4 and Π_5 terms into symmetric and antisymmetric tensors.

The full photon propagator is

$$\begin{aligned} iD_{\mu\nu}(n,q) &= D_{\perp}P^{\perp}_{\mu\nu} + D_{L}P^{L}_{\mu\nu} - i\xi\frac{q_{\mu}q_{\nu}}{q^{4}} + i\tilde{D}\epsilon_{\mu\nu\alpha}\hat{q}^{\alpha} + iD_{4}\hat{q}^{\perp}_{\mu}\epsilon_{\nu\alpha\beta}\hat{q}^{\alpha}\hat{q}^{\beta}_{\perp} \\ &\quad + iD_{5}\hat{q}^{\perp}_{\nu}\epsilon_{\mu\alpha\beta}\hat{q}^{\alpha}\hat{q}^{\beta}_{\perp} + D_{6}\epsilon_{\mu\alpha\beta}\epsilon_{\nu\alpha'\beta'}\hat{q}^{\alpha}\hat{q}^{\alpha'}\hat{q}^{\beta}_{\perp}\hat{q}^{\beta'}_{\perp}\,. \end{aligned} \tag{8}$$

2.2. Infrared divergences

It is known that Π_L is nonzero at zero momentum: this provides electric screening in-medium.

The one-loop expression for the Chern–Simons form factor is

$$\tilde{\Pi}^{(\mathrm{mat})}(0, q\to 0) = \alpha q\,\tanh\frac{m}{2T}\,. \tag{9}$$

This result should be compared to the zero temperature form factor

$$\tilde{\Pi}(0) = \alpha q\,\frac{m}{|m|}\,. \tag{10}$$

We remark that both of these results hold to all orders due to a theorem of Coleman and Hill [11].

An old argument due to Fradkin [12] establishes that $\Pi_{\perp}$ is zero at $(\omega,\vec{p}) = (0,0)$. This statement can be extended to

$$\Pi_{\perp}(0, q\to 0) = c_{\perp}q^{2} + O\left(q^{4}\right)\,. \tag{11}$$

This is explicitly true to $O(e^5)$ in QED4 and Blaizot *et al.* [13] argue that it is true to all orders. The basic idea is that the nonvanishing minimum fermionic Matsubara frequency makes the self energy an analytic function of q^2. An expansion about $q = 0$ then yields $\Pi_{\perp} \to 0 + O(q^2)$, with the odd terms vanishing due to rotational invariance. We note that this argument generalises directly to three dimensions. This result is important because it implies that there is no dynamical screening in the magnetic sector.

The lack of magnetic screening leads directly to an infrared divergence in the fermion self-energy, as we now demonstrate. Consider the exact expression for the fermion self energy

$$i\Sigma(p) = e^2 T \sum_n \int \frac{d^2q}{(2\pi)^2} \gamma_\nu S(q) \Gamma_\mu(p,q) D^{\mu\nu}(p-q)\,. \tag{12}$$

We employ the finite temperature version of the Ward identity to obtain the leading behaviour of the fermion self energy when $q = p - \eta$

$$\Gamma_\nu(p+\eta, p) = \frac{\partial S^{-1}(p)}{\partial p_\nu} + \eta^\alpha \frac{\Gamma_\nu}{\partial p_\alpha} + \ldots\,. \tag{13}$$

The leading infrared behaviour is obtained when ν (or q_0) is zero, we therefore set $\nu = 0$ in the following. One obtains

$$\begin{aligned} \operatorname{div} \Sigma(p) &= -ie^2 T \left[\gamma_\nu S(p) \frac{\partial S^{-1}}{\partial p_\mu}\right] \operatorname{div} \int\limits_{\Lambda_{\mathrm{IR}}} \frac{d^2\eta}{(2\pi)^2} D_{\mu\nu}(\eta) \qquad (14)\\ &= ie^2 T \left[\gamma_\nu \frac{\partial S}{\partial p_\mu} S(p)^{-1}\right] \operatorname{div} \int\limits_{\Lambda_{\mathrm{IR}}} \frac{d^2\eta}{(2\pi)^2} \\ &\quad \times \left[n_\mu n_\nu D_L + \frac{n_\mu n_\nu - g_{\mu\nu}}{2}\left(-D_\perp + D_6 - i\frac{\xi}{\eta^2}\right)\right]. \qquad (15) \end{aligned}$$

For two-component fermions only the gauge term is infrared divergent if the photon mass is nonzero. If it is zero one has

$$\operatorname{div} \Sigma(p) = ie^2 T \left[\gamma_\nu \frac{\partial S}{\partial p_\mu} S(p)^{-1}\right] \frac{n_\mu n_\nu - g_{\mu\nu}}{4\pi} \left[-i\frac{1}{1-c_\perp} - i\xi\right] \log \Lambda_{\mathrm{IR}}\,. \tag{16}$$

Thus a logarithmic infrared divergence appears in the fermion propagator. This statement is exact, only relying on the Ward identity, the existence of $1/q^2$ terms in the exact photon propagator, and general properties of the photon form factors. It is clear that a finite fermion mass does not change this conclusion. However, a finite photon mass regulates the transverse part of the propagator, leaving only the divergence in the gauge term. These expressions make it clear that the infrared divergence does not affect observables. For example, it is possible to choose a gauge to eliminate the divergences entirely.

2.3. The electric screening mass

The electric and magnetic screening masses are defined in terms of the longitudinal and transverse form factors of Eq. (7)

$$m_{\rm el}^2 = \lim_{p\to 0} \Pi_{\rm L}(\omega = 0, p)\,, \tag{17}$$

$$m_{\rm mag}^2 = \lim_{p\to 0} \Pi_{\perp}(\omega = 0, p)\,. \tag{18}$$

Of course, as stated above, the magnetic screening mass should be zero. The electric screening mass is an experimental observable and hence should be infrared and ultraviolet finite. Here we run into a problem: the naive application of Eq. (17) yields an infinite result for $m_{\rm el}$. The problem can be traced to a few linked causes:

(i) A superrenormalisable field theory can still contain divergences (although only a finite number of diagrams diverge).

(ii) The zero temperature photon self energy tensor is given in terms of a scalar function as $\Pi_{\mu\nu} = P_{\mu\nu}\Pi$, *if the regulator and truncation scheme respect gauge invariance.* If this is not the case one must write

$$\Pi_{\mu\nu} = g_{\mu\nu}\Pi_\infty + P_{\mu\nu}\Pi\,. \tag{19}$$

The new scalar function diverges. This function was neglected in previous analyses by simply projecting it away.

(iii) The projection trick no longer works at finite temperature. To obtain a finite screening mass one must renormalise properly; the final, finite, expression is

$$m_{\rm el}^2 = \lim_{|\vec{p}|\to 0} \left[\Pi_{\rm L}(0,\vec{p}) + \Pi_\infty(0,\vec{p}) - \Pi(p) - \Pi_\infty(p)\right]\,. \tag{20}$$

(In fact, the presence of the $g_{\mu\nu}$ term requires a photon mass term $\frac{1}{2}\mu A_\nu A^\nu$ in the Lagrangian and this expression is more conveniently expressed at the renormalisation point $\omega = \mu$, $\vec{p} = 0$ for $T > 0$ and $p^2 = \mu^2$ for $T = 0$.)

Unfortunately, this expression is very difficult to evaluate numerically. The divergence in the finite and zero temperature portions must cancel. A simple cutoff regulator will not do since it is implemented differently in three dimensions and $2+1$ dimensions. Numerically implementing dimensional regularisation is also not sufficient because this method can only deal with logarithmic divergences, and we have linear divergences. A third possibility is Pauli–Villars regularisation. This has the benefit of maintaining gauge invariance and separately regulates the zero and finite temperature portions of $m_{\rm el}^2$, which is desirable. Unfortunately, this doubles the computational effort as fermion functions (A_0, A, B) must be obtained numerically for the Pauli–Villars fermion. Furthermore, one must extrapolate to large Pauli–Villars mass to obtain a cutoff independent result. This is very inconvenient because obtaining accurate results for large fermion mass is difficult. A possible way out of this numerical morass is to implement a judicious subtraction.

These options are currently under investigation and we hope to be able to report first-ever predictions of the truncated Schwinger–Dyson equations without additional approximations for finite temperature gauge theories in the near future.

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