HOLOGRAPHIC STUDY OF MAGNETICALLY INDUCED QCD EFFECTS*

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We study the effect of a constant external magnetic field on the rho meson mass m_ρ on the one hand, and on the chiral symmetry restoration temperature T_χ on the other hand, using the Sakai–Sugimoto model with two quenched flavours and non-zero constituent quark masses, which is a holographic dual of a QCD-like theory in the quenched approximation and chiral limit. We find that the explicit chiral symmetry breaking caused by the presence of the magnetic field eB manifests itself in a stronger chiral magnetic catalysis effect for the up than for the down quarks, resulting in two separate chiral symmetry restoration temperatures T_χ^u and T_χ^d, with $T_\chi^u(eB) > T_\chi^d(eB) > T_{\rm c}$ (the deconfinement temperature) for each value of eB. The Landau levels described in the rho meson mass equation indicate an instability of the QCD vacuum towards condensation of rho mesons at $eB \sim 1.1\, m_\rho^2 \approx 0.67$ GeV^2, confirming a recent prediction of Chernodub.

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1. Introduction

The interest in magnetically induced QCD effects has been growing recently, as a magnetic field offers a controllable parameter that gives rise to unexpected results which might lead to new insights in QCD. The very strong magnetic fields (of the order of 10^{15} Tesla) that are expected to arise in heavy ion collisions [2] at the LHC, provide an additional incentive for investing in this research, as this creates the perfect setting for experimental searches for possible interesting QCD effects in a strong magnetic field. We investigate the possible rho meson condensation at strong magnetic field put forward by Chernodub [3] and the chiral transition's dependence on a magnetic field in a holographic setting, more specifically the Sakai–Sugimoto model [1]. We work with 3 colours, $N_c = 3$, and 2 flavours, $N_f = 2$.

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2. Effect of eB on the rho meson mass m_ρ

2.1. The Sakai–Sugimoto model at zero temperature

The Sakai–Sugimoto model [1] involves N_f pairs of D8–$\overline{\text{D8}}$ flavour branes, placed in a 10-dimensional D4-brane background, which contains a four-sphere, a cigar-shaped (u,τ)-subspace and our 4-dimensional Minkowski world 'living' at the boundary $u \to \infty$. On the stack of N_f coinciding D8–$\overline{\text{D8}}$ flavour pairs lives a $U(N_f)_\text{L} \times U(N_f)_\text{R}$ gauge theory, which is interpreted as the global chiral symmetry in the dual QCD-like theory. The cigar-shape of the (u,τ)-subspace of the D4-brane background enforces a $\cup$-shaped embedding of the flavour branes, encoded in $u(\tau)$. This embedding represents the dynamical breaking of chiral symmetry $U(N_f)_\text{L} \times U(N_f)_\text{R} \to U(N_f)$ as the merging of the D8-branes and $\overline{\text{D8}}$-branes at $u = u_0$ (see Fig. 1).

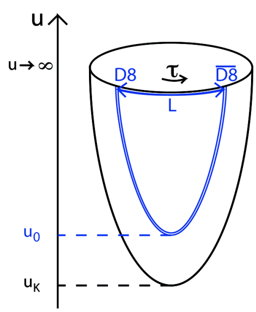

Fig. 1. The Sakai–Sugimoto model.

The backreaction of the flavour branes on the background is ignored (valid for $N_c \gg N_f$), which is equivalent to working in the quenched approximation in the dual QCD-like theory. In contradistinction to the original setup of [1] where $u_0 = u_K$, we consider the more general setting with $u_0 > u_K$ in order to allow for a non-zero constituent quark mass m_q, which is related to the distance u_0–u_K. We fixed the values of the geometric holographic parameters M_K (which is the inverse radius of the asymptotic τ-circle) and the asymptotic separation L between D8- and $\overline{\text{D8}}$-branes, indicated in Fig. 1, to $M_K = 0.7209$ GeV and $L = 1.574$ GeV^{-1}, by matching them to the known QCD values for the rho meson mass $m_\rho = 0.776$ GeV and the constituent quark mass $m_q = 0.310$ GeV [4].

2.2. The Sakai–Sugimoto model at zero temperature with magnetic field eB

The $U(N_f)$ gauge field $A_m(x^\mu, u)$ $(m = 0, 1, 2, 3, u)$ living on the D8-branes,

$$A_\mu(x^\mu, u) = \sum_{n\geq 1} V_{\mu,n}(x^\mu)\psi_n(u) + \overline{A}_\mu\,, \qquad A_u = 0\,, \tag{1}$$

describes a tower of vector mesons $V_{\mu,n}(x^\mu)$ and a constant background magnetic field $\vec{B} = B\vec{e}_3$ in the boundary field theory, through the boundary value

$$\overline{A}_\mu = A_\mu(u \to \infty) = ieQ_{\mathrm{em}}x_1 B\delta_{\mu 2} = ix_1 \begin{pmatrix} \frac{2}{3}eB & 0 \\ 0 & -\frac{1}{3}eB \end{pmatrix} \delta_{\mu 2}\,.$$

The $\{\psi_n(u)\}_{n\geq 1}$ in (1) form a complete set of eigenfunctions of u. The action for this flavour gauge field is given by the non-Abelian DBI-action. This action determines the — now eB-dependent — embedding of the flavour branes. The up- and down-brane, which are coincident at $eB = 0$, become separated if the magnetic field is switched on (see Fig. 2), representing the up-quark's stronger coupling to eB than the down-quark and the resulting explicit breaking of chiral symmetry, $U(2)_{\mathrm{L}} \times U(2)_{\mathrm{R}} \overset{eB}{\to} (U(1)_{\mathrm{L}} \times U(1)_{\mathrm{R}})^u \times (U(1)_{\mathrm{L}} \times U(1)_{\mathrm{R}})^d$. As u_0 rises with eB, the probe branes in the presence of the external magnetic field get more and more bent towards each other, which drives them further away from the chirally invariant situation of straight branes. This feature corresponds to a holographic modelling of the magnetic catalysis of chiral symmetry breaking, which is stronger for the up-quarks than for the down-quarks. The up- and down-quarks' constituent masses grow accordingly, as will the rho meson mass.

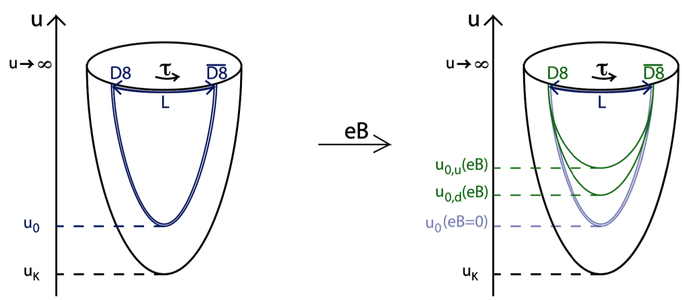

Fig. 2. Chiral magnetic catalysis.

The mass equation for the rho meson, derived from the effective 4-dimensional action after integrating out the u-dependences, describes Landau levels. The effective masses of the transverse charged rho meson combinations $\rho_1^\pm \pm i\rho_2^\pm$ in the lowest Landau level become tachyonic as a result of their magnetic moment coupling to eB. This signals a condensation in these channels at $eB_c = 1.1\, m_\rho^2 \approx 0.67$ GeV^2 [4], see Fig. 3. We are currently still investigating the possible influence on eB_c of scalar fluctuations of the embedding, which are related to the stability of the brane configuration shown in Fig. 2.

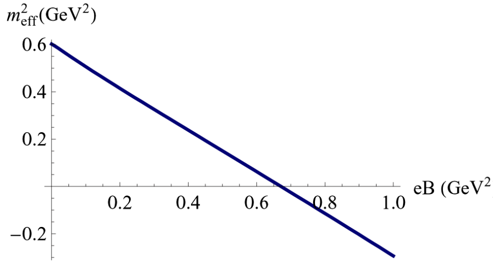

Fig. 3. The effective mass squared of the field combinations $\rho_1^\pm \pm i\rho_2^\pm$ as a function of eB goes through zero at eB_c.

3. Effect of eB on the chiral symmetry restoration temperature T_χ

3.1. The Sakai–Sugimoto model at finite temperature

At finite temperature T the time-direction in the Sakai–Sugimoto model becomes periodic with period $\beta = T^{-1}$. At the deconfinement temperature the D4-brane background transforms (see Fig. 4) whereby the cigar-shaped (u,τ)-subspace turns into a cylinder. This means the embedding of the flavour branes is no longer forced to be ∪-shaped in the deconfining phase. For $M_K = 0.7209$ GeV, we find that the deconfinement transition for the $N_f = 2$ quenched QCD-like dual of the Sakai–Sugimoto model happens at $T_{\rm c} \approx 115$ GeV [5].

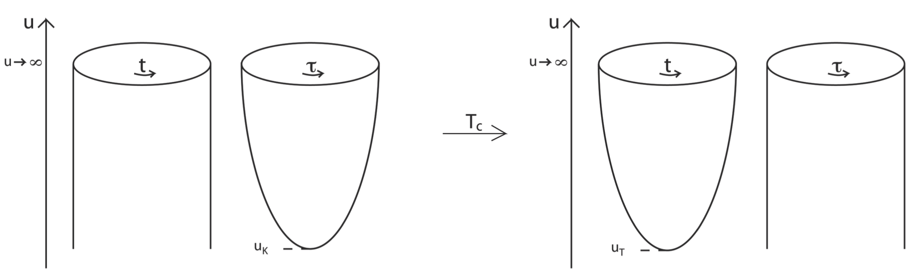

Fig. 4. The deconfinement transition at $T_{\rm c}$ in the Sakai–Sugimoto model.

At the chiral symmetry restoration temperature T_χ it becomes energetically favourable for the flavour branes to fall straight down, corresponding with restoration of chiral symmetry, see Fig. 5.

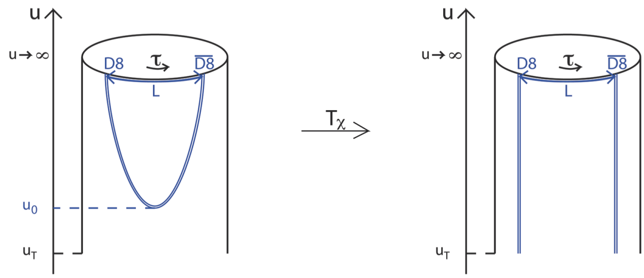

Fig. 5. The chiral symmetry restoration at T_χ in the Sakai–Sugimoto model.

3.2. The Sakai–Sugimoto model at $T > T_c$ and magnetic field eB

The deconfinement temperature T_c is independent of eB. In the phase where quarks are deconfined but the dynamically broken chiral symmetry is not yet restored ($T_c < T < T_\chi$), the effect of eB on the embedding of the flavour branes is again that u_0 rises, modelling chiral magnetic catalysis (see Fig. 6). As the magnetic catalysis of dynamical chiral symmetry breaking is

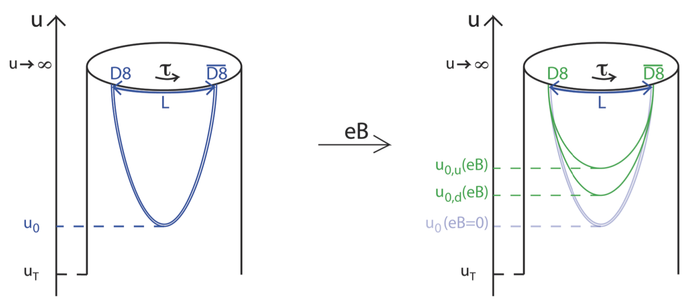

Fig. 6. Chiral magnetic catalysis.

stronger for the up-quark, it seems natural that a higher value of the temperature will be needed to restore the chiral symmetry of the up-quarks than for the down-quarks. To fix u_0 and L at $eB = 0$, we make the physical assumption, motivated by lattice input, that chiral symmetry restoration and the deconfinement transition coincide in the case of vanishing background magnetic field. The resulting chiral symmetry restoration temperatures T_χ^u and T_χ^d are plotted in Fig. 7 [5]. T_χ^u indeed is larger than T_χ^d for every value of the magnetic field $eB > 0$. In the region $T_\chi^d < T < T_\chi^u$ the chiral symmetry is partly restored to $U(1)^u \times (U(1)_L \times U(1)_R)^d$, for $T > T_\chi^u$ the chiral symmetry is restored entirely to $(U(1)_L \times U(1)_R)^u \times (U(1)_L \times U(1)_R)^d$ (see Fig. 8). Anyhow, we find a small split between chiral transition temperatures and deconfinement temperature, in agreement with *e.g.* [6].

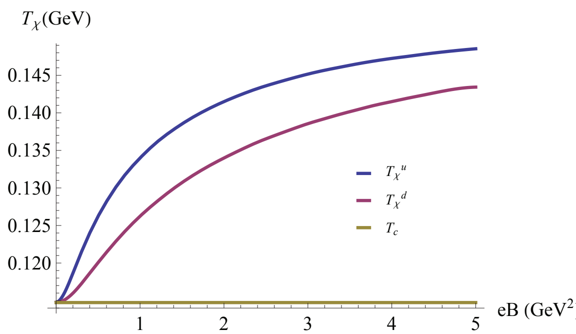

Fig. 7. T_χ^u, T_χ^d and $T_{\rm c}$ as functions of eB.

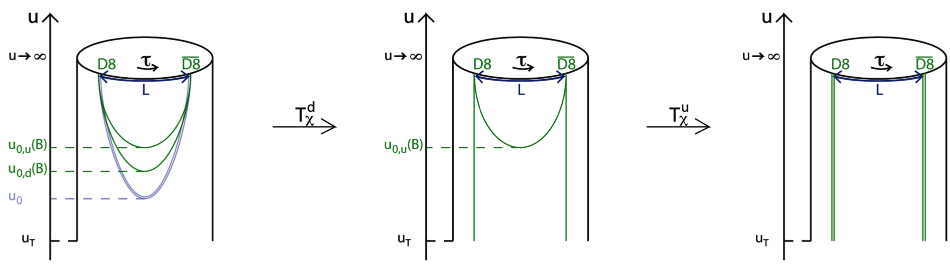

Fig. 8. The chiral symmetry restoration transitions in the Sakai–Sugimoto model at $eB \neq 0$.

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