CHIRAL PROPERTIES OF THE BARYON GROUND STATE*

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The $\mathrm{U_A}(1)$ properties of the three-quark nucleon interpolating fields have recently been used to predict the flavor-singlet (isoscalar) axial coupling of the nucleon ("the spin problem") based on chiral mixing, with F and D values as input, or *vice versa*, in reasonable agreement with experiment. Here, we derive such mixing from effective chiral Lagrangians: first we construct all $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3)$ chirally invariant non-derivative one-meson–baryon interactions and then we calculate the mixing angles in terms of baryons' masses. It turns out that these Lagrangians are subject to (strong) chiral selection rules. For example, there is only one non-derivative chirally symmetric interaction between $J = \frac{1}{2}$ fields belonging to the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$ and the $[(\mathbf{3},\overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}},\mathbf{3})]$ chiral multiplets, that is also $\mathrm{U_A}(1)$ symmetric. We use these Lagrangians to fit the mixing angles by choosing the two lowest lying observed nucleon (resonance) masses, and then predict the third $(J = \frac{1}{2}, I = \frac{3}{2})$ Δ resonance, as well as one or two flavor-singlet Λ hyperon(s), in agreement with particle data group tables.

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1. Introduction

Chiral symmetry, as one of the basic symmetries of QCD, is a key property of the strong interactions. More than 40 years ago Gerstein and Lee [1], Harari [2, 3] and Weinberg [4] considered mixing of chiral multiplets in the spontaneously broken symmetry phase a.k.a. Nambu–Goldstone phase as a way to reproduce the observed (isovector) axial coupling constant of the nucleons. Such chiral representation mixing tends to be most useful when only a few multiplets are involved: in such a case this method may have considerable predictive power. The number of chiral multiplets grows as the number of light valence quarks increases, and in the case of three light (valence)

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quarks in the baryons, this number becomes three (or six, if one counts the so-called mirror multiplets); those are the chiral multiplets $[(\mathbf{3},\overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}},\mathbf{3})]$, $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ and $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$.

The axial current coupling constants of the baryon flavor octet are well known, see Ref. [5]. The zeroth (time-like) components of these axial currents are generators of the $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3)$ chiral symmetry of QCD. The general flavor $\mathrm{SU}_F(3)$ symmetric form of the nucleon axial current contains two free parameters, called the F and D couplings, that are empirically determined as $F = 0.459 \pm 0.008$ and $D = 0.798 \pm 0.008$, [5]. The nucleon also has a flavor singlet axial coupling $g_{\mathrm{A}}^{(0)}$, that has been estimated from spin-polarized lepton–nucleon DIS data as $g_{\mathrm{A}}^{(0)} = 0.28\pm\ 0.16$ [6], or more recently as $0.33 \pm 0.03 \pm 0.05$ [7], [8], subject to certain assumptions about hyperon decays (the axial F and D values).

The basic question now is if the same chiral mixing mechanism can explain the anomalously low value of this coupling simultaneously with reasonable values of F and D? The answer manifestly depends on the $\mathrm{U_A}(1)$ chiral transformation properties of the two admixed nucleon fields. In this talk we address this question using the $\mathrm{U_A}(1)$ and $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3)$ chiral transformation properties of nucleon fields as derived from the three-quark nucleon interpolating fields in QCD in Refs. [9, 10, 11]. The answer to our question turns out in the positive, so we may speak about the chiral mixing idea being viable in QCD. Moreover, we show a model dynamics that reproduces this mixing.

2. Three-quark nucleon interpolating fields

We summarize the transformation properties of various quark trilinear forms with quantum numbers of the nucleon as shown in Refs. [9, 10]. The nucleon interpolating fields appear to be rather formal objects with little familiar intuition attached to them. So, in order to gain some physics intuition, we shall first look at the non-relativistic three-quark nucleon interpolating fields and their relation to the baryon wave functions.

2.1. Non-relativistic three-quark nucleon interpolating fields

There is nothing in the interpolating field formalism that would limit it to relativistic (Dirac) quark fields: indeed a straightforward, extension of the interpolating field formalism to three non-relativistic second-quantized Schrödinger quark fields leads to two apparently independent nucleon interpolating fields

$$\mathcal{N}_1 = \epsilon_{abc} \left(q_a^T q_b\right) q_c \,, \tag{1}$$

$$\mathcal{N}_2 = \epsilon_{abc} \left(q_a^T \tau^i \sigma^j q q_b\right) \tau^i \sigma^j q_c \,, \tag{2}$$

where ϵ_{abc} is the completely antisymmetric tensor, the indices $a, b, c = 1, 2, 3$ correspond to the color degree of freedom and σ and τ are the spin- and isospin Pauli matrices, respectively. These two field operators are equivalent, however. One can see that in two different ways:

(1) Formally we can prove this equivalence starting from the Fierz identities

$$\mathcal{F}\left[\begin{pmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \end{pmatrix}\right] = -\frac{1}{4}\begin{pmatrix} 1 & 1 \\ 9 & 1 \end{pmatrix}\begin{pmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \end{pmatrix} \tag{3}$$

which lead to the identity

$$\mathcal{N}_2 = 3\mathcal{N}_1 \, . \tag{4}$$

Therefore, only one of the two nucleon operators is independent.

(2) Intuitively, one expects that each field operator corresponds to the two spin–isospin wave functions/combinations shown on the right-hand side of Eqs. (5), (6)

$$\mathcal{N}_1 \longleftrightarrow \chi^{\rho_{12}}\phi^{\rho_{12}} + \chi^{\rho_{23}}\phi^{\rho_{23}} + \chi^{\rho_{31}}\phi^{\rho_{31}} \, , \tag{5}$$

$$\mathcal{N}_2 \longleftrightarrow \chi^{\lambda_{12}}\phi^{\lambda_{12}} + \chi^{\lambda_{23}}\phi^{\lambda_{23}} + \chi^{\lambda_{31}}\phi^{\lambda_{31}} \, . \tag{6}$$

The $\mathcal{N}_{1,2}$ are operators, however, not wave functions, that are also anti-symmetric with respect to the color degree of freedom. This means that after complete anti-symmetrization one obtains both the ρ and the λ components in the w.f.

$$\chi^{\rho_{12}}\phi^{\rho_{12}} + \chi^{\rho_{23}}\phi^{\rho_{23}} + \chi^{\rho_{31}}\phi^{\rho_{31}} = \chi^{\rho_{12}}\phi^{\rho_{12}} + \chi^{\lambda_{12}}\phi^{\lambda_{12}} \, , \tag{7}$$

$$\chi^{\lambda_{12}}\phi^{\lambda_{12}} + \chi^{\lambda_{23}}\phi^{\lambda_{23}} + \chi^{\lambda_{31}}\phi^{\lambda_{31}} = 3\left(\chi^{\rho_{12}}\phi^{\rho_{12}} + \chi^{\lambda_{12}}\phi^{\lambda_{12}}\right) \, . \tag{8}$$

Note the factor 3 in the wave functions in Eq. (8) that is directly related to the factor 3 in the Fierz identity Eq. (4). In other words, these two interpolating field operators are equivalent. This should have been expected: we knew all along that there is only one viable candidate for the non-relativistic 3-quark ground state.

This non-relativistic example is instructive because it shows how the interpolating field operators correspond to the spin-flavor wave functions and how the Pauli principle reduces their number. The same is true for relativistic interpolating fields, except in that case there is more than one independent interpolating field and more than one Dirac-flavour-spatial "wave function" ("chiral component"). This kind of mixing in the wave function is due to the fact that the $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3)$ chiral symmetry is spontaneously broken, unlike the $\mathrm{SU_{FS}}(6)$ flavor-spin symmetry of the nonrelativistic quark model.

2.2. Relativistic three-quark nucleon interpolating fields

In Table I we show the Abelian and non-Abelian chiral properties of the nucleon interpolating fields in QCD, Ref. [9,10,11]. It turns out that the relativistic nucleon, *i.e.*, spin-1/2 (Lorentz group representation $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$) and isospin 1/2, field comes in two varieties: one with "mirror" and another with "triple-naive" Abelian chiral properties. These two fields have the same non-Abelian $\mathrm{SU_L}(2)\times\mathrm{SU_R}(2)$ transformation properties [9,10], though different $\mathrm{SU_L}(3)\times\mathrm{SU_R}(3)$ transformation properties: N_1-N_2 belongs to $[(\mathbf{3},\overline{\mathbf{3}})\oplus(\overline{\mathbf{3}},\mathbf{3})]$ multiplet, N_1+N_2 to $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ and $(N_3^{'}+\frac{1}{3}N_4^{'})$ to $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$, see Ref. [11]. Moreover, bilocal baryon fields provide "mirror" images of these fields [12]. We use these results as the (theoretical) input into our chiral mixing calculations.

TABLE I

The Abelian and the non-Abelian axial charges (+ sign indicates "naive", − sign "mirror" transformation properties) and the non-Abelian chiral mutiplets of $J^P=\frac{1}{2}$, Lorentz representation $(\frac{1}{2},0)$ nucleon fields.

Case	Field	$g_A^{(0)}$	$g_A^{(1)}$	F	D	$\mathrm{SU_L}(3)\times\mathrm{SU_R}(3)$
I	N_1-N_2	-1	$+1$	0	$+1$	$[(\mathbf{3},\overline{\mathbf{3}})\oplus(\overline{\mathbf{3}},\mathbf{3})]$
II	N_1+N_2	$+3$	$+1$	$+1$	0	$[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$
III	$N_1^{'}-N_2^{'}$	$+1$	-1	0	-1	$(\overline{\mathbf{3}},\mathbf{3})]\oplus[(\mathbf{3},\overline{\mathbf{3}})$
IV	$N_1^{'}+N_2^{'}$	-3	-1	-1	0	$(\mathbf{1},\mathbf{8})]\oplus[(\mathbf{8},\mathbf{1})$
0	$\partial_\mu(N_3^\mu+\frac{1}{3}N_4^\mu)$	$+1$	$+\frac{5}{3}$	$+\frac{2}{3}$	$+1$	$[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$

3. Chiral mixing of baryons

Recent studies [13,14] point towards baryon mixing of $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ with the $[(\mathbf{3},\overline{\mathbf{3}})\oplus(\overline{\mathbf{3}},\mathbf{3})]$, $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ chiral multiplets as a mechanism underlying the baryons' axial couplings, including the "problematic" flavor-singlet axial coupling $g_A^{(0)}$, that was not considered in the mid-1960s.

Such a three-field admixture requires two free parameters: the 0–I mixing angle θ and the relative/mutual mixing angle $\varphi=\theta_{31}$, as the two nucleon fields III and I mix due to the off-diagonal chiral interaction. Thus we have two equations with two unknowns

$$\frac{5}{3}\sin^2\theta+\cos^2\theta\left(g_A^{(3)}(\mathrm{III})\cos^2\varphi+g_A^{(3)}(\mathrm{I})\sin^2\varphi\right) = 1.267\,, \tag{9}$$

$$\sin^2\theta+\cos^2\theta\left(g_A^{(0)}(\mathrm{III})\cos^2\varphi+g_A^{(0)}(\mathrm{I})\sin^2\varphi\right) = 0.33\pm0.08\,. \tag{10}$$

The values of the mixing angles θ, φ that are solutions to these equations provide input for the prediction of F and D. The values of the mixing angles (θ, φ) obtained from this fit to the baryon axial coupling constants are shown in Table II. We also show the result of the case I–IV in Table II. Note that both scenarios lead to the same values of F, D; this is due to the relation ${g_{\rm A}}^{(0)} = (3F - D)$ that holds for all three-quark fields.

TABLE II

The values of the mixing angles obtained from the simple fit to the baryon axial coupling constants and the predicted values of axial F and D couplings. The experimental values are $g^{(3)}_{\rm A\ expt.} = 1.267$, $g^{(0)}_{\rm A\ expt.} = 0.33 \pm 0.08$.

Case	θ	φ	F	D	F/D
I–III	$50.7° \pm 1.8°$	$23.9° \pm 2.9°$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04
I–IV	$63.2° \pm 4.0°$	$54° \pm 23°$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04

As the next step, we have looked for a dynamical source of the chiral mixing. One such mechanism is the simplest $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3)$ chirally symmetric *non-derivative* one-(σ, π)-meson interaction Lagrangian that was written down in Ref. [15]; it is non-derivative because only thus one can induce the baryon masses via the σ-baryon coupling. The (σ, π)-meson nonets form a chiral $[(\mathbf{3}, \overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}}, \mathbf{3})]$ multiplet.

It turns out that there are severe chiral selection rules: for example, only the mirror field $[(\overline{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \overline{\mathbf{3}})]$ may couple to the $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ baryon chiral multiplet by non-derivative terms; whereas the ordinary ("naive") multiplet $[(\mathbf{3}, \overline{\mathbf{3}}) \oplus (\overline{\mathbf{3}}, \mathbf{3})]$ requires one (or generally an odd number of) derivative(s). Moreover, this interaction also conserves the $\mathrm{U_A}(1)$ symmetry. We note that all, but one of the $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3)$ symmetric interactions, *viz.* the $[(\overline{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \overline{\mathbf{3}})] - [(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$, also conserve the $\mathrm{U_A}(1)$ symmetry.

This means that explicit $\mathrm{U_A}(1)$ symmetry breaking may occur in baryons only in so far as the $\mathrm{SU_L}(3) \times \mathrm{SU_R}(3)$ symmetry is explicitly broken, with the one exception mentioned above. This is in contrast with the $\mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$ case, where all of the interaction terms have both the $\mathrm{U_A}(1)$ symmetry-conserving and the $\mathrm{U_A}(1)$ symmetry-breaking version [13, 16]. In this sense, the three-flavor chiral symmetry is more restrictive and consequently more instructive than the two-flavor one.

Thus, we have provided a dynamical model of chiral mixing that solves both the (F, D) and the flavor-singlet axial coupling problems, assuming only three-quark baryon interpolating fields. We have found two simple solutions/fits: one that conserves the $\mathrm{U_A}(1)$ symmetry and another one that does not. This goes to show that the "QCD $\mathrm{U_A}(1)$ anomaly" may, but need not be the underlying source of the "nucleon spin problem".

Special cases of three-field chiral mixing have been considered by Harari [3], and by Gerstein and Lee [1] in the context of the ("collinear") $\mathrm{U}(3) \times \mathrm{U}(3)$ current algebra at infinite momentum, long time ago. We emphasize here that our results are based on fields that transform as the $(\frac{1}{2}, 0) + (0, \frac{1}{2})$ representation of the Lorentz group, so the currents are (fully) Lorentz covariant and therefore our results hold in any frame. Thus, the chiral multiplet mixing is a viable theoretical scenario for the explanation of the nucleon axial couplings in QCD.

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