

STUDY OF TETRAQUARKS WITH A SIMPLE MODEL*

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Tetraquarks are studied with an approximation which reduces the internal degrees of freedom of the system, using both finite differences and scattering theory. The existence of bound states and resonances is inspected and the masses and widths of the found resonances are calculated.

DOI:10.5506/APhysPolBSupp.4.689

PACS numbers: 12.38.Gc, 12.39.Mk, 12.39.Pn, 14.40.Rt

1. Introduction

For the $q_1 q_2 \bar{q}_3 \bar{q}_4$ system, there is evidence in Lattice QCD [1, 2], at least for static quarks, that the Hamiltonian, is

$$H = \sum_{i=1}^4 T_i + V_{1234} \quad (1)$$

with the potential being given by $V_{1234} = \min(V^{\text{Tetra}}, V^{M_1 M_2}, V^{M_3 M_4})$, where $V^{M_1 M_2}$ and $V^{M_3 M_4}$ are the two possible meson-meson potentials and

$$V^{\text{Tetra}} = C + \alpha_s \sum_{i < j} \frac{\lambda_i}{2} \cdot \frac{\lambda_j}{2} \frac{1}{r_{ij}} + \sigma L_{\min}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \quad (2)$$

is the tetraquark sector potential. We call V_{1234} the triple flip-flop potential.

In the tetraquark configuration, a single connected string is formed. This string is formed from five segments of string as can be seen in Fig. 1 (see [3, 4] for a detailed explanation). We have studied the colour field configurations for the tetraquark. The results are shown in Fig. 2.

* Presented at the Workshop “Excited QCD 2011”, Les Houches, France, February 20–25, 2011.

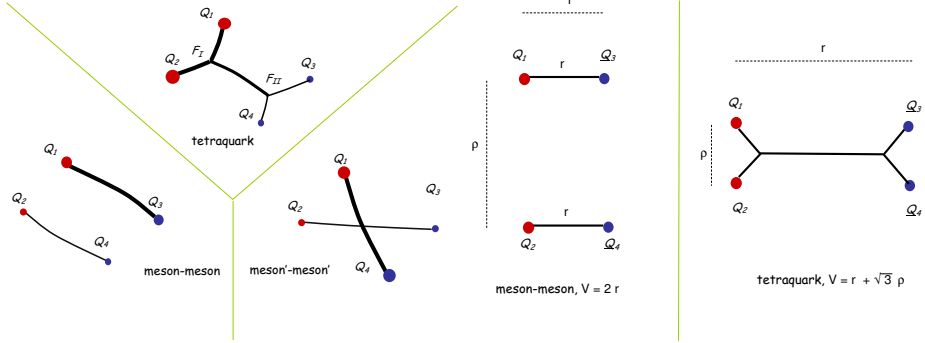


Fig. 1. Left: Triple flip-flop potential. Right: Simplified potential, which depends only on ρ and r .

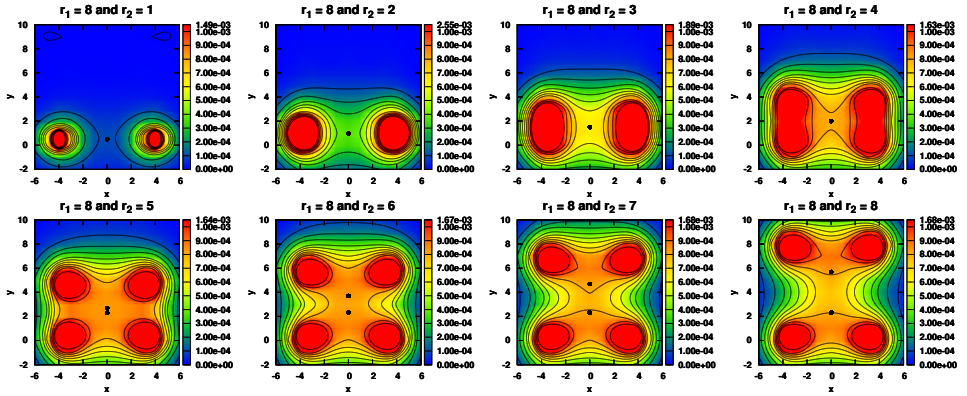


Fig. 2. Plot of Lagrangian density of a static tetraquark. r_1 is the intra-diquark distance and r_2 the inter-diquark distance.

It is hard to solve this Hamiltonian, so we will instead solve a simplified potential model which contains the essential ingredients of the triple flip-flop potential, but depends on a smaller number of variables.

We will do the following approximations. First, we will consider that all the quarks and antiquarks have an equal mass, while neglecting the exchange effects. Secondly, we will consider only the confining (linear) part of the potential and that the quarks are non-relativistic. Finally, we will also reduce the number of variables of the model.

2. A toy model for the potential

We now simplify the triple flip-flop potential, in order to reduce it to a two-dimension problem, similar to the one of a cherry in a broken glass. To simplify the triple flip-flop potential, we assume that the quark-quark

distance $\rho_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and the antiquark–antiquark distance $\rho_{34} = \mathbf{r}_3 - \mathbf{r}_4$ are equal $\rho_{12} = \rho_{34} = \rho$. Thus, we only need to consider the variable ρ and $\mathbf{r} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2} - \frac{\mathbf{r}_3 + \mathbf{r}_4}{2}$ in our model.

In this way, only one meson–meson branch is relevant, and the simplified flip–flop potential is given by $V_{\text{FF}}(r, \rho) = \min(V_{MM}, V_T)$, where $V_{MM}(r, \rho) = \sigma(2r)$ and $V_T(r, \rho) = \sigma(r + \sqrt{3}\rho)$. This potential is depicted in the right-hand side of Fig. 1.

In this way, we obtain the Schrödinger equation, for this model

$$\left[-\frac{\hbar^2}{2m} (\nabla_r^2 + \nabla_\rho^2) + V_{\text{FF}}(r, \rho) \right] \psi(r, \rho) = E\psi(r, \rho) \quad (3)$$

and what we call the outgoing spherical wave method (useful to study resonances).

3. Finite difference method

Since the Hamiltonian is invariant for independent rotations of ρ and $\mathbf{r}$, we have two conserved angular momenta $\mathbf{L}_r = \mathbf{r} \times \mathbf{p}_r$ and $\mathbf{L}_\rho = \rho \times \mathbf{p}_\rho$. So, the wavefunction can be written as $\Psi = \frac{u(r, \rho)}{r\rho} Y_{l_r m_r}(\theta_r, \varphi_r) Y_{l_\rho m_\rho}(\theta_\rho, \varphi_\rho)$ and the Schrödinger equation (3) becomes

$$-\frac{\hbar^2}{2m} \left(\frac{d^2}{dr^2} + \frac{d^2}{d\rho^2} \right) u + \frac{l_r(l_r + 1)}{r^2} u + \frac{l_\rho(l_\rho + 1)}{\rho^2} u + V_{\text{FF}}(r, \rho) u = Eu. \quad (4)$$

We now place the system in a lattice and discretise the second derivatives the usual way. Dirichlet boundary conditions are used at the origin, for the wavefunction to be regular and at the box boundary for convenience.

We then evaluate the eigenstates of the system. Only a few of them are bound or semi-localised states. They can be identified by calculating $\sqrt{\langle \rho^2 \rangle}$ and comparing it to the box size.

In the following results we consider only $l_\rho = 0$. For $l_r = 0$, only continuum states are found. However, for $l_r > 0$, we are able to found some semi-localised states, such as the one depicted on the left-hand side of Fig. 3. In the center we can see a true bound state found for $l_r = 3$.

We try to calculate the phase shifts using the finite differences results, by calculating the functions $\psi_i(\rho) = \int d^3\mathbf{r} \phi_i^*(\mathbf{r}) \Psi(\mathbf{r}, \rho)$, where the ϕ_i are the eigenfunctions of the Hamiltonian $\hat{H}_{MM} = -\frac{\hbar^2}{2m} \nabla_r^2 + 2\sigma r$ and, then by fitting the long range limit $\psi_i \rightarrow A_i \sin(k_i \rho + \delta_i)$. We obtain the results in the right-hand side of Fig. 3. As we can see there, the results become irregular when a second channel is open, due the interference of the different channels, and so we cannot use this method to calculate the phase shifts.

For that, we had to resort to a scattering theory approach.

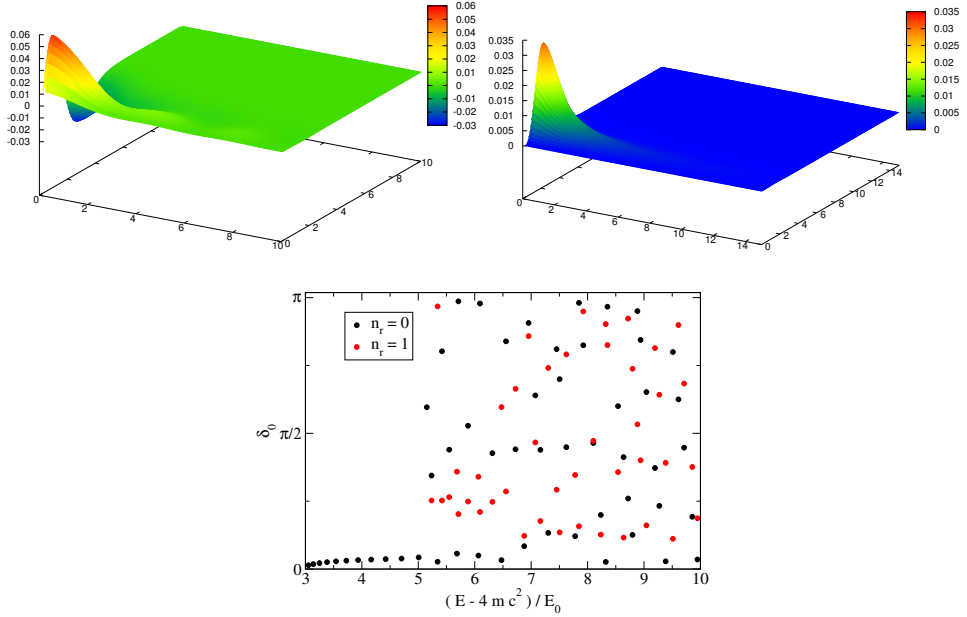


Fig. 3. Left: Semi-localized state, or resonance for $l_r = 1$. Center: Bound state for $l_r = 3$. Right: “Phase shifts” obtained from the finite differences.

4. Outgoing spherical wave method

By expanding the wavefunction as $\Psi(\boldsymbol{\rho}, \mathbf{r}) = \sum_i \phi_i(\mathbf{r}, \boldsymbol{\rho})$ we obtain the coupled channel Schrödinger equation

$$-\frac{\hbar^2}{2\mu_i} \nabla^2 \Psi_i + V_{ij} \Psi_j = (E - \epsilon_i) \Psi_i, \quad (5)$$

where $V_{ij} = \int d^3 \mathbf{r} \phi_i * (\mathbf{r}) (V_{\text{FF}} - 2\sigma r) \phi_j(\mathbf{r})$, ϵ_i are the eigenvalues of $\hat{H}_{MM}$ and $\mu_i = m$.

Considering the scattering from the channel i to the channel j , we have asymptotically $\Psi_j \rightarrow \delta_{ij} e^{ik_i z} + f_{ij}(\hat{r}) \frac{e^{ik_j r}}{r}$, if the channel j is open, otherwise vanishing. The f_{ij} can be computed by considering the outgoing solutions of the Eq. (5), $\Psi_j = e^{ik_i r} \delta_{ij} + \sum_{nm} \frac{u_n^l(r)}{r} Y_{lm}(\theta, \varphi)$. We obtain [5]

$$-\frac{\hbar^2}{2\mu_j} \frac{d^2 u_j}{dr^2} + V_{jn} u_n = (E - \epsilon_j) u_j - V_{jn} j_l(k_i r) r. \quad (6)$$

We can also obtain the phase shifts by studying the asymptotic behaviour of the solutions.

We can describe the scattering process with four quantum numbers: The scattering angular momentum l_ρ , the confined angular momentum l_r and the initial and final states radial number in the confined coordinate r , n_i and n_j .

We now compute the phase shifts, in order to search for resonances in our simplified flip–flop model.

The energies in the plots will be given in terms of $E_0 = (\hbar^2 \sigma^2 / m)^{1/3}$, which is the energy scale of the problem.

In Fig. 4 we see the phase shifts for different values of l_r , from the radial ground state $n_r = 0$. As can be seen, no resonance is formed for $l_r = 0$, but for $l_r = 1$ and $l_r = 2$ one resonance is formed since the phase shift has a jump of π . This is somewhat expected, since the centrifugal barrier has the effect of maintaining the diquarks separated, while increasing the stability of the system. We also see a bound state for $l_r = 3$, which agrees with the results obtained with the finite differences method and presented in Fig. 3. The phase shifts for scattering for $n_r = 1$ and different l_r and for $l_r = 0$ and different values of n_r are also presented.

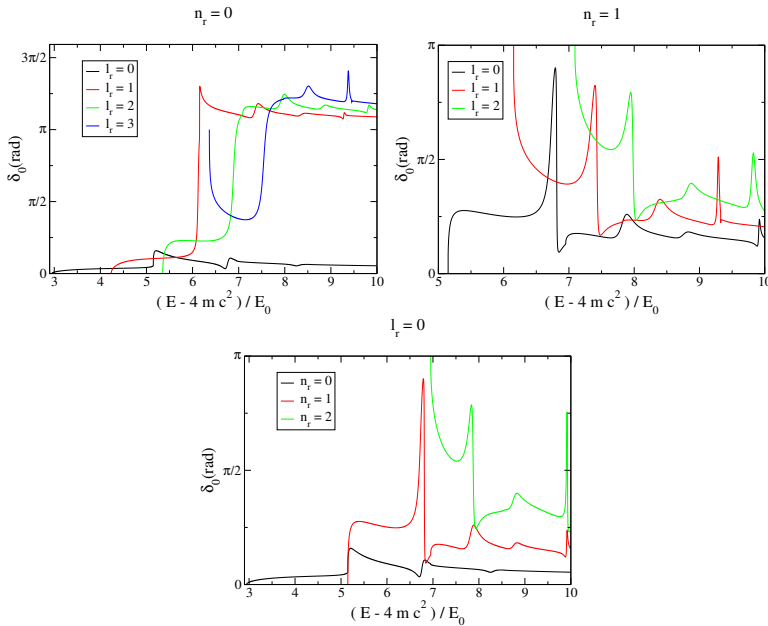


Fig. 4. Comparison of the phase shifts for $l_r = 0, 1, 2$ and 3 , with $n_r = 0$ (upper left) and $n_r = 1$ (upper right). Phase shifts for $l_r = 0$ and different n_r (bottom).

If we include the relativistic corrections for the reduced mass of the system, the μ_i becomes $\mu_i = m + \frac{\epsilon_i}{4c^2}$. Greater reduced masses could increase the binding of the system. Which is in fact what we observe in Fig. 5. For $l_r = 0$ we can see a resonance that is destroyed for masses as low as $2\sqrt{\sigma}/c^2$,

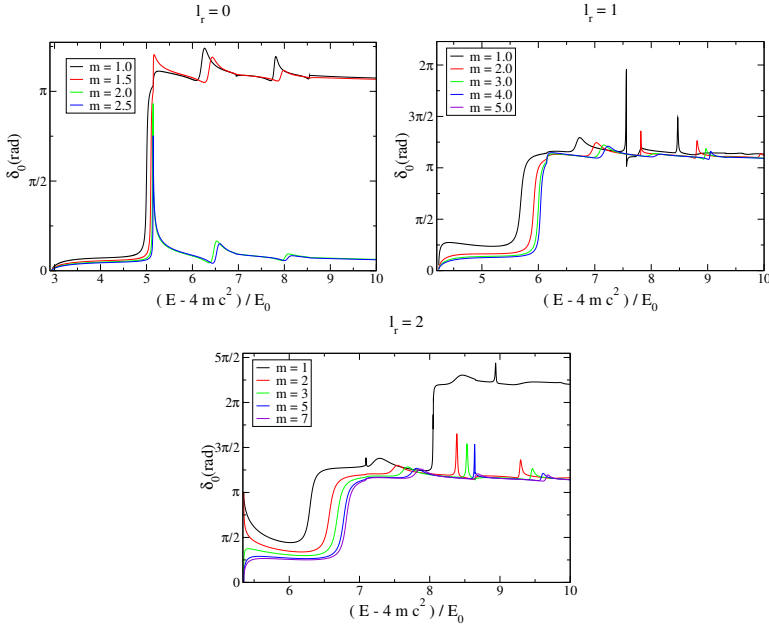


Fig. 5. Phase shifts, including the reduced mass correction for $l_r = 0, 1$ and 2 .

while for $l_r = 1$ the resonance is always present. When $l_r = 2$ for sufficiently low masses we can observe a bound state and even a resonance in addition to it.

Finally, we can compute the decay widths of the resonances by using the expression $\frac{\Gamma}{2} = \left(\frac{d\delta}{dE}\right)^{-1}$ when δ is $\pi/2$. The results are given in Table I. The width is larger for large angular momentum and small quark mass.

TABLE I

Left: Decay widths as a function of l_r . Right: Decay widths with the reduced mass correction.

l_r	$(E - 4mc^2)/E_0$	Γ / E_0	l_r	$m/\sqrt{\sigma}$	$(E - 4mc^2)/E_0$	Γ / E_0
1	6.116	0.037	0	1.0	5.001	0.039
2	6.855	0.131		1.5	5.096	0.022
3	7.462	0.352		1.0	5.659	0.137
			1	3.0	5.990	0.075
				5.0	6.053	0.053
				1.0	6.194	0.586
			2	3.0	6.634	0.209
				7.0	6.777	0.162

Using this results to estimate the physical scale of our decay widths, we obtain, in the $l_r = 1$ state, a decay width $\Gamma_l \simeq 15$ MeV for light quarks ($m \simeq 0.4$ GeV). For the charm quark we obtain $\Gamma_c \simeq 10$ MeV and for the bottom quark, the estimated width is $\Gamma_b \simeq 6$ MeV.

In the $l_r = 2$ state the estimated widths are $\Gamma_l = 52$ MeV, $\Gamma_c = 34$ MeV and $\Gamma_b = 23$ MeV, while in the $l_r = 3$ state, the values are $\Gamma_l = 140$ MeV, $\Gamma_c = 90$ MeV and $\Gamma_b = 60$ MeV.

5. Conclusion and outlook

We have constructed a simplified model for the tetraquark. With this model, we have observed the formation of a resonance for non null l_r , and even a bound state for $l_r = 3$. So we conclude that a higher angular momentum favours the formation of resonances and bound states, in agreement with Karliner and Lipkin [6]. Extrapolating this results to the real world, it seems plausible the existence of tetraquarks with high angular momentum.

The formation of resonances and bound states is also favoured by low quark masses in our model. This happens since a low quark mass produces a higher binding energy which increasing the meson masses and so favouring the localisation of resonances and bound states.

We think the methods developed here are useful and could be futrely used in the complete tetraquark problem with a triple flip–flop potential.

We thank George Rupp for useful discussions. We acknowledge the financial support of the FCT grants CFTP, CERN/FP/109327/2009 and CERN/FP/109307/2009.

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