RESONANCES $f_0(1370)$ AND $f_0(1710)$ AS SCALAR $\bar{q}q$ STATES FROM AN $N_f = 3$ SIGMA MODEL WITH VECTORS AND AXIAL-VECTORS*

Denis Parganlija

Institute for Theoretical Physics, Goethe University
Max-von-Laue-Str. 1, 60438 Frankfurt am Main, Germany
parganlija@th.physik.uni-frankfurt.de

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Features of the non-strange $\bar{n}n$ and strange $\bar{s}s$ scalar mesons are investigated in the Extended Linear Sigma Model (eLSM) with $N_f = 3$ and vector and axial-vector mesons. Our model contains a pure non-strange and a pure strange scalar state; implementing the mixing of the two states originates two new states, a predominantly non-strange and a predominantly strange one. We investigate the possibility to assign the two mixed states to experimentally well-established resonances. To this end, we calculate the masses and the two-pion decay widths of the mixed states and compare them with experimental data. The predominantly non-strange state is found to be consistent with the resonance $f_0(1370)$ and the predominantly strange state with the resonance $f_0(1710)$.

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1. Introduction

Experimental data [1] show an abundance of scalar meson resonances both in the non-strange and in the strange sectors. In particular, the kaons and other mesons containing strange quarks are expected to play an important role in vacuum phenomenology as well as in the restoration of the $\mathrm{U}(N_f)_\mathrm{L} \times \mathrm{U}(N_f)_\mathrm{R}$ chiral symmetry [2], a feature of the Quantum Chromodynamics (QCD) broken in vacuum spontaneously [3] by the quark condensate and explicitly by non-vanishing quark masses (N_f denotes the number of quark flavours). Meson phenomenology in the non-strange and strange sectors has been considered in various sigma model approaches (see Ref. [2] and references therein). In this paper, we present an Extended Linear Sigma

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Model (eLSM [4,5]) where these approaches are generalised to contain scalar, pseudoscalar, vector and axial-vector mesons both in the non-strange and strange sectors. In particular, we devote attention to the structure of scalar mesons $f_0(1370)$ and $f_0(1710)$. In Ref. [6], the resonance $f_0(1370)$ was found to be predominantly of $\bar{q}q$ nature (thus disfavouring the interpretation of the scalar state $f_0(600)$ as a $\bar{q}q$ state). However, the model of Ref. [6] contained no strange mesons and in this paper we address the question whether the conclusion of Ref. [6] regarding $f_0(1370)$ as a predominantly $\bar{q}q$ state also holds in a more general $\mathrm{U}(3)_\mathrm{L} \times \mathrm{U}(3)_\mathrm{R}$ approach that simultaneously allows for a statement regarding the structure of $f_0(1710)$ (which we find to be predominantly of $\bar{s}s$ nature).

The paper is organised as follows. In Sec. 2 we present the model Lagrangian, the results are discussed in Sec. 3, and in Sec. 4 we provide a summary and outlook of further work.

2. The model

The Lagrangian of the Extended Linear Sigma Model with $\mathrm{U}(3)_\mathrm{L} \times \mathrm{U}(3)_\mathrm{R}$ symmetry reads [2,4,5]

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D^\mu\Phi)\right] - m_0^2\,\mathrm{Tr}\left(\Phi^\dagger\Phi\right) - \lambda_1\left[\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\right]^2 - \lambda_2\,\mathrm{Tr}\left(\Phi^\dagger\Phi\right)^2 \\ &-\tfrac{1}{4}\,\mathrm{Tr}\left[(L^{\mu\nu})^2+(R^{\mu\nu})^2\right] + \mathrm{Tr}\left[\left(\tfrac{m_1^2}{2}+\Delta\right)(L^\mu)^2+(R^\mu)^2\right] \\ &+\mathrm{Tr}\left[H\left(\Phi+\Phi^\dagger\right)\right] + c_1\left[\left(\det\Phi+\det\Phi^\dagger\right)^2 - 4\det\left(\Phi\Phi^\dagger\right)\right] \\ &-2ig_2(\mathrm{Tr}\{L_{\mu\nu}[L^\mu,L^\nu]\}+\mathrm{Tr}\{R_{\mu\nu}[R^\mu,R^\nu]\}) + \tfrac{h_1}{2}\,\mathrm{Tr}\left(\Phi^\dagger\Phi\right) \\ &\times\mathrm{Tr}\left[(L^\mu)^2+(R^\mu)^2\right] + h_2\,\mathrm{Tr}\left[(\Phi R^\mu)^2+(L^\mu\Phi)^2\right] + 2h_3\,\mathrm{Tr}\left(\Phi R_\mu \Phi^\dagger L^\mu\right)\,, \end{aligned} \tag{1}$$

where

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_\mathrm{N}+a_0^0)+i(\eta_\mathrm{N}+\pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_\mathrm{S}^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_\mathrm{N}-a_0^0)+i(\eta_\mathrm{N}-\pi^0)}{\sqrt{2}} & K_\mathrm{S}^0 + iK^0 \\ K_\mathrm{S}^- + iK^- & \bar{K}_\mathrm{S}^0 + i\bar{K}^0 & \sigma_\mathrm{S} + i\eta_\mathrm{S} \end{pmatrix} \tag{2}$$

is a matrix containing the scalar and pseudoscalar degrees of freedom, $L^\mu = V^\mu + A^\mu$ and $R^\mu = V^\mu - A^\mu$ are, respectively, the left-handed and the right-handed matrices containing vector and axial-vector degrees of freedom with

$$V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_\mathrm{N}+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_\mathrm{N}-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star-} & \bar{K}^{\star 0} & \omega_\mathrm{S} \end{pmatrix}^\mu, \quad A^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{f_{1\mathrm{N}}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1\mathrm{N}}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1\mathrm{S}} \end{pmatrix}^\mu \tag{3}$$

and $\varDelta = \mathrm{diag}(\delta_u, \delta_u, \delta_s)$ describes explicit breaking of the chiral symmetry in the (axial-)vector channel. Note that the explicit symmetry breaking in the (pseudo)scalar sector is described by the term $\mathrm{Tr}[H(\varPhi + \varPhi^\dagger)]$ with $H = 1/2\,\mathrm{diag}(h_{0\mathrm{N}}, h_{0\mathrm{N}}, \sqrt{2}h_{0\mathrm{S}})$, $h_{0\mathrm{N}} = \mathrm{const.}$, $h_{0\mathrm{S}} = \mathrm{const.}$ Also, $D^\mu\varPhi = \partial^\mu\varPhi - ig_1(L^\mu\varPhi - \varPhi R^\mu) - ieA^\mu[t^3, \varPhi]$ is the covariant derivative (A^μ is the photon field); $L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu[t^3, L^\nu] - (\partial^\nu L^\mu - ieA^\nu[t^3, L^\mu])$, $R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu[t^3, R^\nu] - (\partial^\nu R^\mu - ieA^\nu[t^3, R^\mu])$ are, respectively, the left-handed and right-handed field strength tensors and the term $c_1[(\det\varPhi + \det\varPhi^\dagger)^2 - 4\det(\varPhi\varPhi^\dagger)]$ describes the $\mathrm{U}(1)_\mathrm{A}$ anomaly. Note that in this paper we are using a different way to model the chiral anomaly (see Ref. [7] and references therein) than in our previous papers [4,5,6,8,9], where the 't Hooft form of the chiral-anomaly term reading $c(\det\varPhi + \det\varPhi^\dagger)$ was used [10]. The reason is that the chiral-anomaly term now present in our Lagrangian (1) influences, as one would expect, only the phenomenology of the pseudoscalar singlets (*i.e.*, η and η'), whereas the 't Hooft form of the chiral-anomaly term influences the phenomenology of other mesons (such as, *e.g.*, the σ states) as well [11].

In the non-strange sector, we assign the fields $\vec{\pi}$ and η_N to the pion and the SU(2) counterpart of the η meson, $\eta_\mathrm{N} \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$. The fields ω^μ_N, $\vec{\rho}^\mu$, $f^\mu_{1\mathrm{N}}$ and $\vec{a}^\mu_1$ are assigned to the $\omega(782)$, $\rho(770)$, $f_1(1285)$ and $a_1(1260)$ mesons, respectively [6]. In the strange sector, we assign the K fields to the kaons; η_S is the strange contribution to the η and η' fields and the ω^μ_S, $f^\mu_{1\mathrm{S}}$, $K^{\star\mu}$ and K^μ_1 fields correspond to the $\phi(1020)$, $f_1(1420)$, $K^\star(892)$ and $K_1(1270)$ mesons, respectively.

The assignment of the scalar states in our model to physical resonances is ambiguous. In accordance with Ref. [6], we assign the $\vec{a}_0$ field to $a_0(1450)$ and, consequently, K_S to the physical $K^\star_0(1430)$ state. This, of course, presupposes that these two states above 1 GeV are $\bar{q}q$ states (as all the fields present in our model are $\bar{q}q$ states [6]) and thus one needs to determine whether such an assignment allows for a global fit with a correct description of meson phenomenology to be found.

Additionally, the Lagrangian (1) contains two isoscalar $J^{\mathrm{PC}} = 0^{++}$ states, σ_N {pure non-strange state, $\sigma_\mathrm{N} = \bar{n}n \equiv (\bar{u}u + \bar{d}d)/\sqrt{2}$ [6]} and σ_S (pure strange state, $\sigma_\mathrm{S} \equiv \bar{s}s$). As noted in Refs. [4,5], we observe mixing of σ_N and σ_S in the Lagrangian leading to the emergence of two new states, σ_1 (predominantly non-strange) and σ_2 (predominantly strange). The next section describes results regarding the masses and the two-pion decay widths of the $\sigma_{1,2}$ states that allow for an assignment of the $\sigma_{1,2}$ states to physical resonances.

In order to implement spontaneous symmetry breaking in the model, we shift σ_N and σ_S by their respective vacuum expectation values ϕ_N and ϕ_S. Mixing terms containing axial-vectors and pseudoscalars and $K^\star$ and K_S

then arise and are removed as described in Ref. [5]. Consequently, renormalisation coefficients are introduced for the pseudoscalar fields and $K_{\rm S}$ [5]. We note the following formulas for Z_π (renormalisation coefficient of the pion) and Z_K (renormalisation coefficient of the kaon): $\phi_{\rm N} = Z_\pi f_\pi$ [6] and analogously $\phi_{\rm S} = Z_K f_K/\sqrt{2}$, where $f_\pi = 92.4$ MeV and $f_K = 155.5/\sqrt{2}$ MeV are, respectively, pion and kaon decay constants [1].

The Lagrangian (1) contains 14 parameters: λ_1, λ_2, c_1, $h_{0\rm N}$, $h_{0\rm S}$, h_1, h_2, h_3, m_0^2, g_1, g_2, m_1, δ_u, δ_s. The parameter g_2 is determined from the decay width $\rho \to \pi\pi$ [6]; we set $h_1 = 0$ in accordance with large-N_c deliberations [6] and also $\delta_u = 0$ because the explicit symmetry breaking is small in the non-strange sector. All other parameters are calculated from a global fit of masses including m_π, m_K, m_η, $m_{\eta'}$, m_ρ, $m_{K^\star}$, $m_{\omega_{\rm S}\equiv\varphi(1020)}$, $m_{f_{1\rm S}\equiv f_{1\rm S}(1420)}$, m_{a_1}, $m_{K_1\equiv K_1(1270)}$, $m_{a_0\equiv a_0(1450)}$ and $m_{K_{\rm S}\equiv K_0^\star(1430)}$. We also use the full decay width of $a_0(1450)$ [6] in the fit to further constrain the parameters [$\Gamma^{\rm exp.}_{a_0(1450)} = 265$ MeV]. Note, however, that the mass terms from the Lagrangian (1) used in the fit allow only for the linear combination $m_0^2 + \lambda_1(\phi_{\rm N}^2 + \phi_{\rm S}^2)$ rather than the parameters m_0^2 and λ_1 by themselves to be determined [12].

3. The global fit and two-pion decay of the sigma mesons

Results for masses from our best fit are shown in Table I. Mixing between the pure states $\sigma_{\rm N}$ and $\sigma_{\rm S}$ is implemented analogously to the quarkonium-glueball mixing of Ref. [9]. In our case, two mixed states emerge: σ_1 (95% non-strange, 5% strange) and σ_2 (95% strange, 5% non-strange). Their masses and decay widths depend on seven parameters (m_0^2, λ_1, λ_2, g_1, $h_{1,2,3}$). The decay widths $\Gamma_{\sigma_{1,2}\to\pi\pi}$ depend on the parameters m_0^2 and λ_1 separately

TABLE I

Masses from our global fit. The value of $m_{K_{\rm S}}$ is larger than the PDG value due to the pattern of explicit symmetry breaking that in our model makes strange mesons approximately 100 MeV ($\simeq$ strange-quark mass) heavier than non-strange mesons. The relatively large value of m_{K_1} is under investigation [12]. All other values correspond very well to experimental data. Note that the fit also yields $\Gamma_{a_0(1450)} = 265 \text{ MeV} \equiv \Gamma^{\rm exp.}_{a_0(1450)}$.

Mass	m_π	m_K	m_η	$m_{\eta'}$	m_ρ	$m_{K^\star}$
PDG value (MeV) [1]	139.57	493.68	547.85	957.78	775.49	891.66
Our value (MeV)	138.65	497.96	523.30	957.79	775.49	916.52
Mass	m_φ	$m_{f_{1\rm S}}$	m_{a_1}	m_{K_1}	m_{a_0}	$m_{K_{\rm S}}$
PDG value (MeV) [1]	1019.5	1426.4	1230	1272	1474	1425
Our value (MeV)	1036.9	1457	1219	1343	1452	1550

rather than on the linear combination $m_0^2 + \lambda_1(\phi_N^2 + \phi_S^2)$. Therefore, using the value of the linear combination $m_0^2 + \lambda_1(\phi_N^2 + \phi_S^2)$ obtained from the fit allows us to substitute λ_1 by m_0^2 in the formulas for $\Gamma_{\sigma_{1,2}\to\pi\pi}$. Varying m_0^2 and consequently m_{σ_1} and m_{σ_2} leads to diagrams for the decay widths shown in Fig. 1. (Note that $m_0^2 < 0$ is required for the spontaneous symmetry breaking to be implemented correctly in the model.)

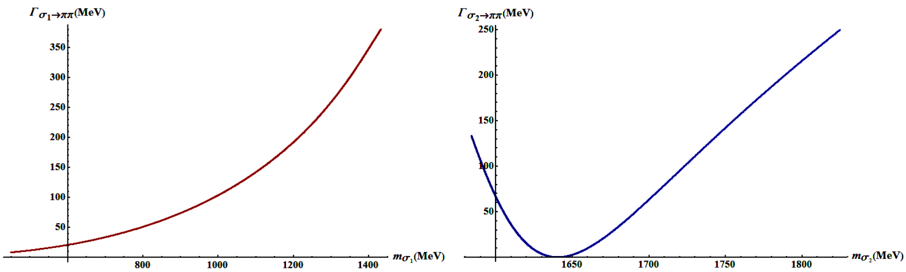

Fig. 1. $\Gamma_{\sigma_{1,2}\to\pi\pi}$ as function of $m_{\sigma_{1,2}}$.

There is a very good correspondence of our predominantly non-strange state σ_1 with $f_0(1370)$ as experimental data [1] regarding the latter state read $m_{f_0(1370)}^{\text{exp.}} = (1200\text{–}1500)$ MeV and $\Gamma_{f_0(1370)}^{\text{exp.}} = (200\text{–}500)$ MeV with a dominant two-pion decay channel. The state σ_2 corresponds well to $f_0(1710)$: experimental data regarding this resonance read $\Gamma_{f_0(1710)\to\pi\pi}^{\text{exp.}} = (29.28 \pm 6.53)$ MeV which leads to $m_{\sigma_2}^{(1)} = 1613$ MeV and $m_{\sigma_2}^{(2)} = 1677$ MeV; m_{σ_2} is very close to $m_{f_0(1710)}^{\text{exp.}} = (1720 \pm 6)$ MeV. We thus conclude that both resonances $f_0(1370)$ and $f_0(1710)$ are predominantly quarkonia: the former 95% $\bar{n}n$ and the latter 95% $\bar{s}s$. Using $m_{\sigma_2}^{(1)}$ and $m_{\sigma_2}^{(2)}$ it is possible to calculate the corresponding two values of m_0^2 and then we obtain $m_{\sigma_1}^{(1)} = 1360$ MeV, $\Gamma_{\sigma_1\to\pi\pi}^{(1)} = 309$ MeV and $m_{\sigma_1}^{(2)} = 1497$ MeV, $\Gamma_{\sigma_1\to\pi\pi}^{(2)} = 415$ MeV. Both sets of values are within PDG data.

4. Summary and outlook

We have presented a $U(3)_L \times U(3)_R$ Linear Sigma Model with (axial-)vector mesons. The model contains two isoscalar $J^{PC} = 0^{++}$ states: the pure $\bar{n}n$ state σ_N and the pure $\bar{s}s$ state σ_S. Mixing of the pure states originates a predominantly non-strange state σ_1 and a predominantly strange state σ_2. In order to assign the latter states to experimentally measured ones, we have calculated their masses and decay widths. We have determined the model parameters by using a global fit of meson masses (except the sigma masses) and the total decay width of $a_0(1450)$. Our results regarding the masses and decay widths of $\sigma_{1,2}$ lead to conclusion that σ_1 corresponds to

$f_0(1370)$ and that σ_2 corresponds to $f_0(1710)$. Conversely, the results imply that $f_0(1370)$ is a predominantly $\bar{n}n$ state and that $f_0(1710)$ is a predominantly $\bar{s}s$ state. However, one still needs to calculate other decay widths from the $N_f = 3$ sector and verify whether a fit with reasonable phenomenology can be found with assumption of scalar quarkonia in the region under 1 GeV [12].

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REFERENCES

[1] K. Nakamura *et al.* [Particle Data Group], *J. Phys. G* **37**, 075021 (2010).

[2] J.T. Lenaghan, D.H. Rischke, J. Schaffner-Bielich, *Phys. Rev.* **D62**, 085008 (2000) [`arXiv:nucl-th/0004006v1`].

[3] J. Goldstone, *Nuovo Cim.* **19**, 154 (1961); J. Goldstone, A. Salam, S. Weinberg, *Phys. Rev.* **127**, 965 (1962).

[4] D. Parganlija *et al.*, *Int. J. Mod. Phys.* **A26**, 607 (2011) [`arXiv:1009.2250v1 [hep-ph]`].

[5] D. Parganlija, F. Giacosa, P. Kovacs, G. Wolf, `arXiv:1011.6104v1[hep-ph]`.

[6] D. Parganlija, F. Giacosa, D.H. Rischke, *Phys. Rev.* **D82**, 054024 (2010) [`arXiv:1003.4934v1 [hep-ph]`].

[7] A.H. Fariborz, R. Jora, J. Schechter, *Phys. Rev.* **D77**, 094004 (2008) [`arXiv:0801.2552v2 [hep-ph]`].

[8] D. Parganlija, F. Giacosa, D.H. Rischke, *AIP Conf. Proc.* **1030**, 160 (2008) [`arXiv:0804.3949 [hep-ph]`]; D. Parganlija, F. Giacosa, D.H. Rischke, *PoS* **CONFINEMENT8**, 070 (2008) [`arXiv:0812.2183 [hep-ph]`]; D. Parganlija, F. Giacosa, D.H. Rischke, `arXiv:0911.3996v1 [nucl-th]`; D. Parganlija, F. Giacosa, D.H. Rischke, *Acta Phys. Pol. B Proc. Suppl.* **3**, 963 (2010) [`arXiv:1004.4817 [hep-ph]`]; P. Kovacs, G. Wolf, F. Giacosa, D. Parganlija, `arXiv:1102.4732v1 [hep-ph]`.

[9] S. Janowski, D. Parganlija, F. Giacosa, D.H. Rischke, `arXiv:1103.3238v1 [hep-ph]`.

[10] G. 't Hooft, *Phys. Rev.* **D14**, 3432 (1976) [*Erratum ibid.* **D18**, 2199 (1978)]; G. 't Hooft, *Phys. Rep.* **142**, 357 (1986).

[11] E. Klempt, B.C. Metsch, C.R. Munz, H.R. Petry, *Phys. Lett.* **B361**, 160 (1995) [`arXiv:hep-ph/9507449v1`]; V. Dmitrasinovic, *Phys. Rev.* **C53**, 1383 (1996); C. Ritter, B.C. Metsch, C.R. Munz, H.R. Petry, *Phys. Lett.* **B380**, 431 (1996) [`arXiv:hep-ph/9601246v1`]; V. Dmitrasinovic, *Phys. Rev.* **D56**, 247 (1997).

[12] D. Parganlija *et al.*, in preparation.