NONEXPONENTIAL DECAYS OF HADRONS*

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We analyze the survival probability of unstable particles in the context of quantum field theory. After introducing the spectral function of resonances, we show that deviations from the exponential decay law occur at short times after the creation of the unstable particle. For hadronic decays, these deviations are sizable and could lead to observable effects.

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1. Introduction

Deviations from the exponential decay law of unstable systems are a natural consequence of the postulates of Quantum Mechanics [1,2,3]: for an unstable state, whose average energy is finite, the survival probability for short times after the "creation" of the state is slower than any exponential decay law. In other terms, if we introduce an effective, time dependent decay rate $\gamma(t) = \frac{-1}{t} \log(p(t))$ one has that for $t \to 0^+$, $\gamma(0^+) = 0$ while at large times the standard exponential decay law, $\gamma(t) \simeq \Gamma$, is obtained. The initial temporal window for which deviations from the exponential law take place is usually very small: it is of the order of 10^{-15} s for electromagnetic atomic decays [4]. This explains why these deviations have never been observed in experiments before 1997 [5] when, for the first time, a cold atoms experiment has reported the evidence of such deviations for *bona fide* unstable states (tunneling of atoms out of a trap). Previously, in [6], deviations from the

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exponential law have been reported within Rabi oscillations. The short time deviations from the exponential law open up the possibility of the socalled Quantum Zeno effect [7,3,4]: by "observing" the system with pulsed measurements at short times after its preparation, the effective decay rate is reduced and eventually it vanishes for continuous measurements (Quantum Zeno paradox). Also this prediction has been recently confirmed within cold atoms experiments [8] and, moreover, the so-called Inverse Quantum Zeno effect has also been observed: in this case the measuring apparatus leads to a faster decay of the unstable state [9].

A natural question concerns the existence of deviations from the exponential law also in the context of Relativistic Quantum Field Theory (RQFT) which is the right theoretical frame for describing unstable particles. In the perturbative approaches presented in [10] no (or very much suppressed) short-time deviations from the exponential law, and thus no quantum Zeno effect, were found within RQFT. Here and in Ref. [11] we reconsider the issue of the survival probability in RQFT also by analyzing some subtleties one faces when trying to define unstable particles, such as the problem of "preparation of the system" and of the fields redefinition. We will not consider here the case of the fundamental Lagrangian of the Standard model but we limit the discussion to a toy model superrenormalizable Lagrangian. We indeed find that deviations from the exponential law occur also in a genuine RQFT context and we discuss possible implications for hadronic decays.

2. A model Lagrangian

The toy Lagrangian we use to investigate the survival probability of an unstable scalar particle S decaying into two scalars φ is given by

$$\mathcal{L} = \frac{1}{2} \left(\partial_{\mu} S \right)^2 - \frac{1}{2} M_0^2 S^2 + \frac{1}{2} \left(\partial_{\mu} \varphi \right)^2 - \frac{1}{2} m^2 \varphi^2 + g S \varphi^2 \,. \tag{1}$$

The interaction term $\mathcal{L}_{int} = gS\varphi^2$ is responsible for the decay process $S \to \varphi\varphi$, whose tree-level decay rate reads

$$\Gamma_{S\varphi\varphi}^{\rm tl} = \frac{\sqrt{\frac{M_0^2}{4} - m^2}}{8\pi M_0^2} \left(\sqrt{2}g\right)^2 \,. \tag{2}$$

The "naive", tree-level expression of the survival probability p(t) for the resonance S created at t = 0 is $p_{tl}(t) = e^{-\Gamma_{S\varphi\varphi}^{tl}t}$ and the tree-level expression of the mean life time is $\tau_{tl} = 1/\Gamma_{S\varphi\varphi}^{tl}$. Here we interpret our Lagrangian as an effective model to describe the decays of hadrons; it is therefore quite natural to introduce a cutoff Λ on the energy of the particles of the typical

mass scale of strongly interacting particles *i.e.* $\Lambda \sim 1$ GeV. To introduce the cutoff in a more consistent way one has to insert a nonlocal interaction in the Lagrangian [12]

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} S)^{2} - \frac{1}{2} M_{0}^{2} S^{2} + \frac{1}{2} (\partial_{\mu} \varphi)^{2} - \frac{1}{2} m^{2} \varphi^{2} + \mathcal{L}_{\text{int}}, \qquad (3)$$

$$\mathcal{L}_{\text{int}} = gS(x) \int d^4 y \varphi(x+y/2) \varphi(x-y/2) \Phi(y) , \qquad (4)$$

where Φ is a form factor whose Fourier transform, $f_A(q) = \int d^4 y \Phi(y) e^{-iyq}$, appears in the loop integrals and regularizes the divergences. (For nonlocal Lagrangians see also Refs. [13] and refs. therein.) In this work we will consider the case of a sharp cutoff and the case of a smooth form factor. An intermediate step to obtain the survival probability is the computation of the self energy which reads

$$\Sigma\left(x = \sqrt{p^2}, m\right) = -i \int \frac{d^4q}{(2\pi)^4} \frac{f_A\left(q^0, \overrightarrow{q}\right)^2}{\left[(q + p/2)^2 - m^2 + i\varepsilon\right]\left[(-q + p/2)^2 - m^2 + i\varepsilon\right]}$$
(5)

and modifies the propagator Δ_S of the unstable particle as usual

$$\Delta_S(p^2) = \left[p^2 - M_0^2 + \left(\sqrt{2}g\right)^2 \Sigma\left(p^2\right) + i\varepsilon\right]^{-1}.$$
 (6)

3. Spectral functions and survival probabilities

Similarly to the standard derivation within Quantum Mechanics, also in RQFT, the survival probability can be obtained by projecting the initial unstable state onto the energy eigenstates. In turn, this corresponds to the calculation of the spectral function $d_S(x)$ of the scalar field S which is proportional to the imaginary part of the propagator

$$d_S\left(x = \sqrt{p^2}\right) = \frac{2x}{\pi} \left|\lim_{\varepsilon \to 0} \operatorname{Im}\left[\Delta_S\left(p^2\right)\right]\right| \,. \tag{7}$$

The quantity $d_S(x)dx$ represents the probability that, in its rest frame, the state S has a mass between x and x + dx. It is correctly normalized for each g, $\int_0^{\infty} d_S(x)dx = 1$ and reproduces the limit $d_S(x) = \delta(x - M_0)$ for $g \to 0$ [14, 12]. Notice that there are situations in which the the spectral function can be directly pinned down by data because the the background is small and well understood: the decay $\phi \to \gamma \pi^0 \pi^0$ through the intermediate $a_0(980)$ and $f_0(980)$ mesons, the similar decay of the j/ψ charmonium, or the hadronic decay of the τ lepton into $\nu \pi \pi$, dominated by the ρ meson for an invariant $\pi \pi$ mass close to ρ mass (e.g. [15]).

The probability amplitude a(t) and the survival probability p(t) can be then expressed as

$$a(t) = \int_{-\infty}^{+\infty} dx \ d_S(x) e^{-ixt} p(t) = |a(t)|^2 \ . \tag{8}$$

The condition p(0) = 1 is fulfilled in virtue of the normalization of $d_S(x)$.

Let us now study the first derivative of p(t). We obtain that p'(t=0) = 0as a consequence of the fact that the integral $\int_0^\infty x \, d_S(x) dx$ is finite and real (it is the mean mass $\langle M \rangle$, a reasonable definition for the mass of a resonance [12]). This, in turn, implies that the function $\gamma(t) = \frac{-1}{t} \ln p(t)$ vanishes for $t \to 0^+$

$$\lim_{t \to 0^+} \gamma(t) = -\lim_{t \to 0^+} \frac{p'(t)}{p(t)} = 0.$$
(9)

We can, therefore, conclude that the Quantum Zeno effect is perfectly possible in the present RQFT context.

We show in Fig. 1 the survival probability for the case of a sharp cutoff (solid line) a smooth form factor $f_A(q) = 1/(1 + (q/\Lambda)^2)$ (thick gray line) and the standard exponential decay law (dashed line), here $\Lambda = 1.5$ GeV, $M_0 = 1$ GeV, $m = m_{\pi}$ and the tree-level mean life time $\tau_{\rm tl} = 3.27$ GeV⁻¹ (this fixes g in the two cases). Also displayed are the differences between the survival probability as calculated at one loop level and the tree-level



Fig. 1. Survival probability as a function of time. The solid line corresponds to the choice of a sharp cutoff, the thick gray line to a smooth form factor, the dashed line to the exponential decay law and the dotted black and gray lines are the differences between the survival probability as calculated from the spectral function and the exponential decay law. The deviations from the exponential law are quite sizable at short times.

exponential decay law (dotted black and gray lines). Notice that the time interval for which sizable deviations from the exponential decay law occur is of the same order of magnitude of the mean life time of the particle. This is an intriguing consequence of having strongly interacting particles and could, in principle, lead to observable effects, for instance in heavy ions collisions experiments. Moreover, the difference between the sharp and smooth cutoff is very small: this fact ensures that our results depend only slightly from the form of the cutoff function.

4. Discussion and conclusions

There is an important issue that must be considered in connection with the measurability of these deviations, which also correspond to the measurability of the spectral function.

First, we notice that for very broad resonances, for which the deviation from the exponential law are strong, one should also consider the mechanism by which these resonances are created as, for instance, the scattering $\varphi \varphi \to S \to \varphi \varphi$ [16]. One should introduce wave packets, with proper initial conditions, which substantially overlap at t = 0. In the framework of plane waves, the full state of the system can be expressed in terms of the eigenstates of the Hamiltonian H_0

$$|s(t)\rangle = \sum_{\boldsymbol{k}} c_{\boldsymbol{k}}(t) |\varphi_{\boldsymbol{k}}\varphi_{-\boldsymbol{k}}\rangle + c_{S}(t) |S\rangle .$$
(10)

The coefficient $c_S(t)$ is vanishingly small for $t \ll 0$ and only for $t \simeq 0$ it becomes significant. If it were possible to tune the starting conditions in such a way that $c_S(0) = 1$, we would have $|s(t=0)\rangle = |S\rangle$ and the survival probability of the resonance would be exactly the one presented in the previous section. However, in general the state at t = 0 is a superposition

$$|s(0)\rangle = \sum_{\boldsymbol{k}} c_{\boldsymbol{k}}(0) |\varphi_{\boldsymbol{k}}\varphi_{-\boldsymbol{k}}\rangle + c_{S}(0) |S\rangle .$$
(11)

Further evolution implies

$$e^{-iHt} |s(0)\rangle = \sum_{\mathbf{k}} c_{\mathbf{k}}(0) e^{-iHt} |\varphi_{\mathbf{k}}\varphi_{-\mathbf{k}}\rangle + c_{S}(0) e^{-iHt} |S\rangle$$

=
$$\sum_{\mathbf{k}} c_{\mathbf{k}}(0) e^{-iHt} |\varphi_{\mathbf{k}}\varphi_{-\mathbf{k}}\rangle + c_{S}(0) (a(t) |S\rangle + |\varphi\varphi\rangle) .$$

The amplitude a(t) enters in a more general expression but it is not clear a priori if the deviations from the exponential decay law are smeared out, in the final "measurement" of the decay products, or if they could provide significant effects. A careful study would be needed. Moreover, we plan also to investigate if the deviations from the exponential decay law could indeed lead to observable effects also in Particle Physics experiments.

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