DISCRETE AND CONTINUOUS MODELS OF PEDESTRIAN MOVEMENT — A COMPARISON*

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In this work, we study both continuous models of evacuation based on differential equations and discrete models based on cellular automata. Our discrete model is a floor field model with additional rules of movement (*i.e.* random movement, a form of pressure and enforced blocking of pedestrians). The continuous model is a variant of the Langevin equation. By performing simulations of evacuation from rooms with similar geometries we try to compare both approaches and find their strengths and weaknesses. In order to do that, we study evacuation times, relative evacuation times and other variables. We find that evacuation times are comparable but the results highly depend on geometry.

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1. Introduction

Whenever people gather in some place, be it an open or constrained space, there is always a possibility that due to some unpredictable circumstances (like fire, terrorist threat or even without any tangible reason) they will be forced to evacuate. Such situations are potentially very dangerous and, as we could witness in many news reports recently, can lead to injuries or even fatalities. Scientists recognised this problem and began to study it using various mathematical tools available in their fields [1]. The goal of such studies is to increase safety and our understanding of phenomena occurring during evacuation.

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Mathematical models of pedestrian movement can be roughly divided into two categories. The first category consists of continuous models in which pedestrians are treated in a similar way to particles, interacting with each other and with the surrounding environment through *social forces* [2,3,4,5,6]. The equations of motion have the form of the Langevin equations — that is, differential equations with a stochastic component. Unlike these equations, cellular automata models, the second category, are intrinsically discrete — space is divided into cells, usually located on a rectangular grid, and time is measured in evenly spaced time steps. Cellular automata models may employ various kinds of random movement [7, 8], static and dynamic floor fields [9, 10, 11] or pedestrians occupying more than one cell [12, 13].

In this work, we compare a variant of the continuous model [5] with our discrete cellular automata model [14]. Scientists have always tried to clearly define the relation between models based on cellular automata and differential equations. While we do not intend to provide an answer to this question, we believe that it is very interesting to see how (and if) the results from both kinds of models are comparable and how close they can be brought together.

2. Cellular automata based model

In this comparison, we used a variant of the cellular automata model of pedestrian movement we introduced in [14], where we studied evacuation on staircase-like geometries. The model is based on a simple floor field combined with additional rules. The floor field (formally, an assignment of real values to cells used by pedestrians as a metric which tells them how far away they are from their destination) can be calculated with the following algorithm [9]:

1. Mark all cells as inactive.
2. Assign value 1 to all destination cells (in our case — the cells the exits are comprised of) and mark them as active. Assign the highest possible floor field value to obstacle cells (*e.g.* walls, desks) and exclude them from further processing.
3. For each active cell with floor field value w, assign value $w + 1.5$ to its diagonal inactive neighbours and value $w+1$ to horizontal and vertical inactive neighbours. If this procedure leads to a conflict (two or more cells try to assign a value to a common neighbour), use the smallest floor field value as the final one.
4. Mark all active cells as processed. Mark all cells that were assigned a value during the previous step as active.
5. Repeat from step 3 as long as there are active cells.

Using this simple, iterative procedure we can create a floor field (similar to the Manhattan metric) which takes into account the diagonal movement. The properties of this floor field were discussed more broadly in [9].

A cell can be occupied by a single pedestrian. Each pedestrian in the system is characterised by actual velocity v, desired velocity v_{d} and acceleration a ($v, v_{\mathrm{d}}, a \in \mathbb{N}_0$). At the beginning of each time step a list of all pedestrians is shuffled and used to determine the order in which they are asynchronously updated. We use the following rules of movement:

1. A pedestrian adds acceleration a to his or her actual velocity v, which cannot be higher than desired velocity v_{d}. It corresponds to the assignment: $v \leftarrow \min\{v + a, v_{\mathrm{d}}\}$.

2. The pedestrian tries to move v times consecutively, choosing the next cell with probability:

   - $1 - \beta$ ($\beta \in [0, 1]$) — a neighbouring cell with the smallest floor field value (as long as this value is lower than the value of the cell the pedestrian is currently standing on);
   - β — a neighbouring cell with the smallest number of pedestrians in the neighbourhood.

   When there are two or more cells with the same smallest floor field value (or number of pedestrians in the neighbourhood), one of them is chosen at random. If the pedestrian becomes blocked by other pedestrians or obstacles and is unable to move, the actual velocity is set to 0.

The set of rules presented above, along with the floor field, fully defines a cellular automata based model of movement with discrete velocity and acceleration. In this model, pedestrians exhibit two kinds of social behaviour: they either try to move along the shortest path to the exit or try to maintain a certain amount of personal space by distancing themselves from others (which, when described in the language of physical forces, is analogous to a short range repulsion). Using parameter β we can control the way in which pedestrians switch between these behaviour.

The last thing we need to do is to define how this model and values relate to the real world. We assumed that each cell is a rectangle with length 0.5 m and each time step lasts 0.5 s. Other physical properties follow directly from these assumptions.

3. Differential model

The differential model we use in this comparison is based on a standard differential equation with a stochastic component (the Langevin equation). It should be noted that in many models (including our work) the stochastic component is usually omitted. The most general form of this equation is

$$m\frac{d\boldsymbol{v}}{dt} = \boldsymbol{F}_{\mathrm{D}} + \boldsymbol{F}_{\mathrm{S}} + \boldsymbol{F}_{\mathrm{G}} + \sqrt{\frac{2\epsilon}{\tau}}\boldsymbol{\xi}\,, \tag{1}$$

where on the right hand side all terms correspond to forces induced by some social or psychological phenomena [5]:

- $\boldsymbol{F}_{\mathrm{D}}$ — desired force

$$\boldsymbol{F}_{\mathrm{D}} = m\frac{v_{\mathrm{d}}\boldsymbol{e}_{\mathrm{d}} - \boldsymbol{v}}{\tau} \tag{2}$$

  is a force which expresses the desire of a pedestrian, whose current velocity is $\boldsymbol{v}$, to move in a certain direction (indicated by vector $\boldsymbol{e}_{\mathrm{d}}$) with desired velocity (speed) v_{d}. Parameter τ controls the characteristic acceleration time and m is the mass. In our simulations $\tau = 0.5$ s and $m = 80$ kg for all pedestrians.

  In order to make the differential model as similar as possible to the cellular automata model, we decided to divide the simulated geometries into cells with the same dimensions as in the cellular automata model. Using the floor field, we assigned each cell a desired direction $\boldsymbol{e}_{\mathrm{d}}$ — a unit vector pointing from the geometrical centre of a cell to the geometrical centre of the neighbouring cell with the lowest floor field value.

- $\boldsymbol{F}_{\mathrm{S}}$ — social force

$$\boldsymbol{F}_{\mathrm{S}} = \sum_j A\exp\left(-\frac{\varepsilon_{ij}}{B}\right)\boldsymbol{e}^n_{ij} \tag{3}$$

  is a repulsive force between pedestrians. It depends on the distance between pedestrians ($\varepsilon_{ij} = r_{ij} - R_i - R_j$, where r_{ij} is the distance between the geometrical centres of pedestrians, R_i and R_j are the radii of pedestrians). Vector $\boldsymbol{e}^n_{ij}$ is a unit vector pointing from the geometrical centre of pedestrian j to the geometrical centre of pedestrian i. Both A and B are constants. In our simulations, we assume that all pedestrians have the same radius $R = 0.25$ m, $A = 1000$ N and $B = 0.08$ m.

- $\boldsymbol{F}_\mathrm{G}$ — granular force

$$\boldsymbol{F}_\mathrm{G} = \sum_j \left[(-\varepsilon_{ij}k_n - \gamma v_{ij}^n)\boldsymbol{e}_{ij}^n + (v_{ij}^t \varepsilon_{ij} k_t)\boldsymbol{e}_{ij}^t\right] g(\varepsilon_{ij}) \tag{4}$$

  is a force inspired by granular interactions and it acts only when $\varepsilon_{ij} < 0$. For the sake of similarity between the differential and discrete model we decided to omit this term. Still, given the short-range nature of granular interactions, the results from this modified version should be in agreement with the results from the unmodified model for small velocities and small concentrations of pedestrians. However, it is impossible to observe phenomena dependent on granular interactions (*e.g.* clogging). A detailed description of all parameters and values of constants used in the omitted term can be found in [5].

4. Results

We tested both models on two geometries — an empty room (*i.e.* without obstacles) and a typical classroom. The empty room is a square consisting of 10×10 cells (Fig. 1). At the beginning of the simulation the room is filled randomly with 25 pedestrians. The classroom consists of three rows (Fig. 2). There are five desks in each row and at the beginning of the simulation two students are placed behind each desk. The teacher's desk is located near the blackboard in front of the class. In both the empty room and classroom the width of the exit is 2 cells. Pedestrians are removed from the exit at the beginning of each time step (integration step for the differential model).

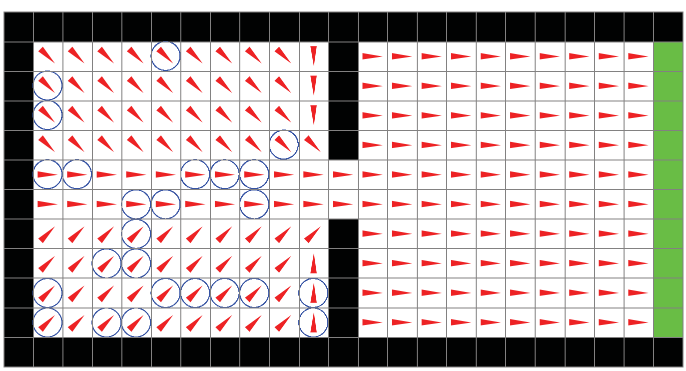

Fig. 1. Empty room filled with 25 randomly distributed pedestrians. Arrows point in the desired direction.

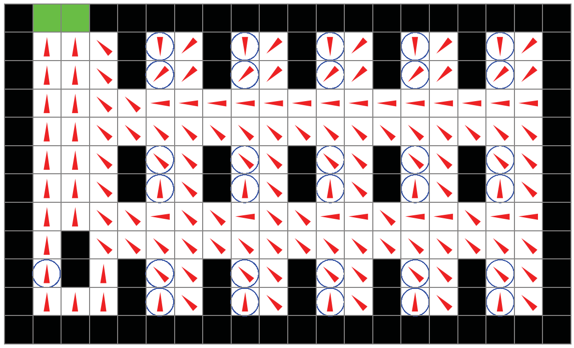

Fig. 2. Classroom with 30 students and one teacher. Arrows point in the desired direction.

In terms of average evacuation times the results we get from both models and empty room geometry are consistent (Figs. 3 and 4). When $\beta = 0$ (which means that pedestrians follow the shortest path to the exit and do not try to separate from others) pedestrians in the CA model evacuate in slightly shorter time. However, evacuation time increases with β and for $\beta = 0.5$ average evacuation times in both models are almost identical. When $\beta > 0.5$ pedestrians take longer time to evacuate. Minimal evacuation times

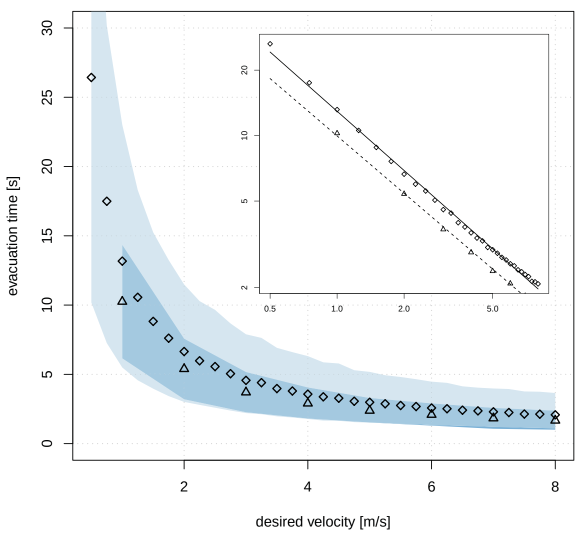

Fig. 3. Empty room: average evacuation time as a function of desired velocity for the differential model (diamonds) and CA model with $\beta = 0$ (triangles). The filled parts of the plot mark the maximal and minimal evacuation times for the differential model (light grey) and CA (dark grey). The inner plot shows the same relation between average evacuation time and desired velocity on a logarithmic scale.

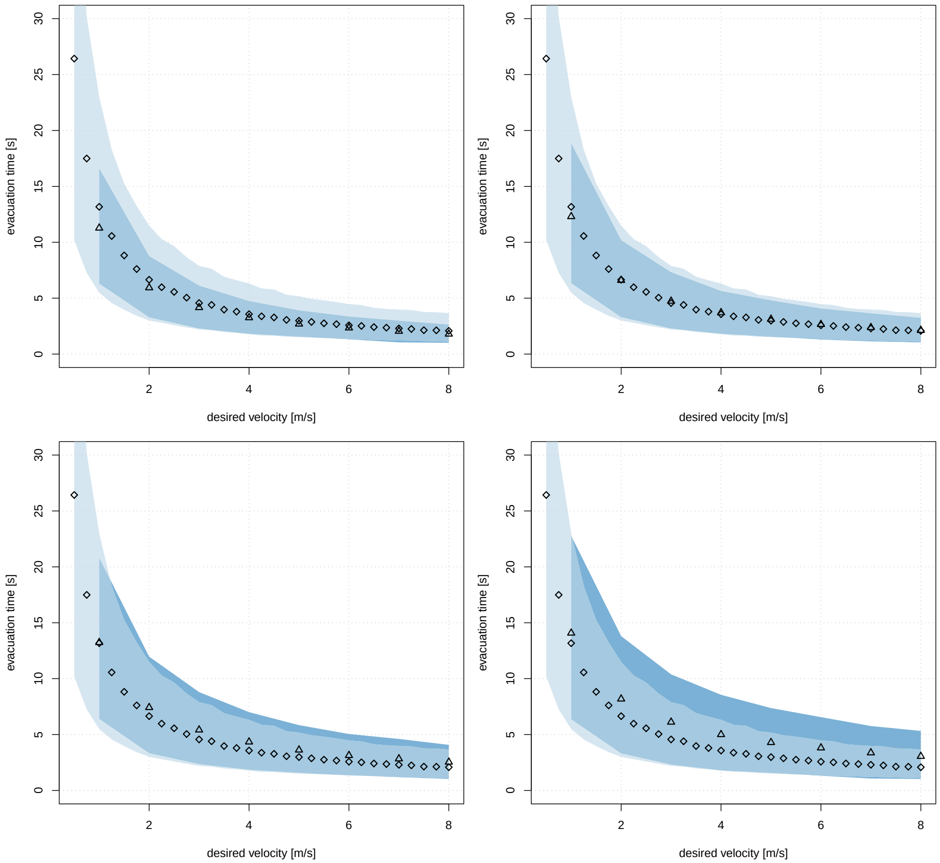

Fig. 4. Empty room: average evacuation time as a function of desired velocity for the differential model (diamonds) and CA model (triangles) with $\beta = 0.25$ (top left), $\beta = 0.50$ (top right), $\beta = 0.75$ (bottom left), $\beta = 1.00$ (bottom right). The filled parts of the plot mark the maximal and minimal evacuation times for the differential model (light grey) and CA (dark grey).

are independent of β and also, especially for high desired velocities, nearly equal for both models. It can be easily explained by the fact that minimal evacuation time is determined by the pedestrian closest to the exit and he or she often manages to leave the room without interacting with other pedestrians. Maximal evacuation time increases with β and is more susceptible to the change of this parameter than average evacuation time. When $\beta = 0.5$, much like with average evacuation time, maximal evacuation time is almost equal for both models. It is also interesting that in both models average evacuation time from an empty room as a function of desired velocity follows the relation: $\bar{t} \propto v_{\mathrm{d}}^{\alpha}$.

Given the similarity of evacuation times, one could expect that other properties will also be, at least, comparable. Unfortunately, if we look at average relative evacuation time (Fig. 5), we can clearly see this is not the case. Average relative evacuation time is a quantity which measures how other pedestrians influence our evacuation. It is calculated by dividing the actual evacuation time of a pedestrian by the evacuation time of the same pedestrian measured in a system consisting of only that one pedestrian. For the kind of cellular automata model we use the relation between average relative evacuation time and β is typical — the former increases with the latter. Average relative evacuation time also increases with desired velocity. However, when the differential model is used, the reverse is true. It should not be surprising, because both models handle interactions in a very different way. One possible explanation is that in the differential model pedestrians located further away from the exit can push forward pedestrians closer to the exit.

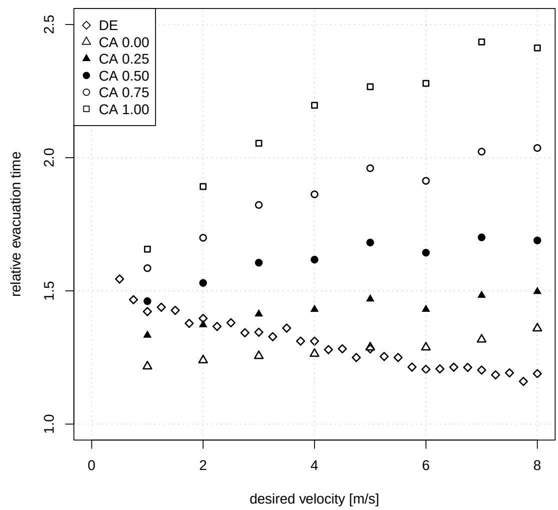

Fig. 5. Empty room: average relative evacuation time as a function of desired velocity for the differential model and CA model for different values of β.

While in both models average velocity increases with desired velocity, pedestrians in the differential model are never able to move with their desired velocity for the entire duration of the simulation (Fig. 6). It is possible in the cellular automata model because pedestrians, especially when β is high, can move relatively freely without blocking one another. Such behaviour also explains why higher velocity of cellular automata pedestrians does not necessarily translate into shorter evacuation times.

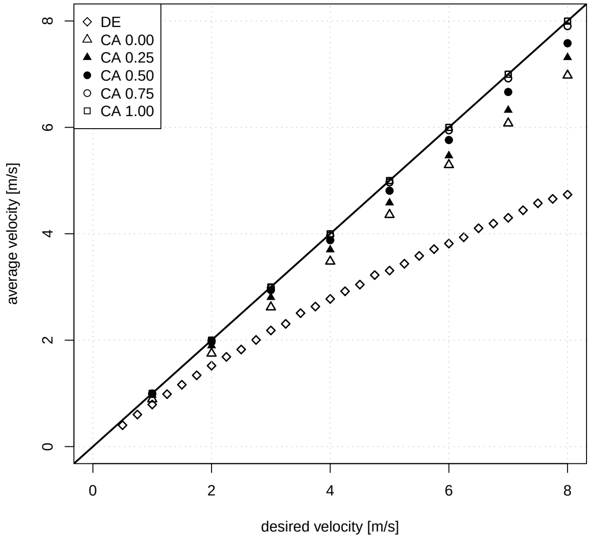

Fig. 6. Empty room: average velocity as a function of desired velocity for the differential model and CA model for different values of β.

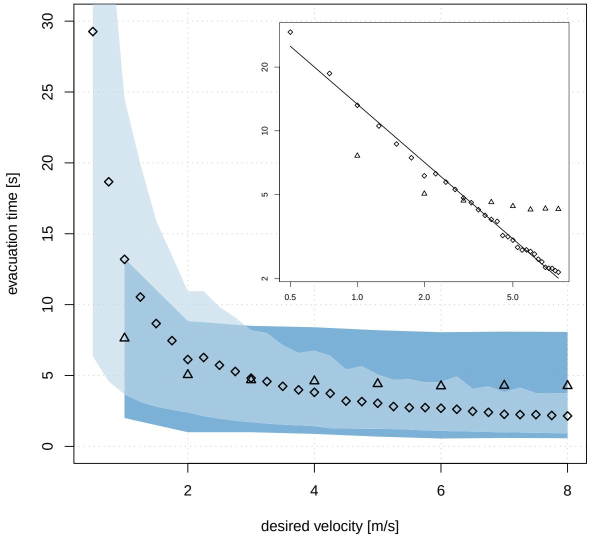

Fig. 7. Classroom: average evacuation time as a function of desired velocity for the differential model (diamonds) and CA model with $\beta = 0$ (triangles). The filled parts of the plot mark the maximal and minimal evacuation times for the differential model (light grey) and CA (dark grey). The inner plot shows the same relation between average evacuation time and desired velocity on a logarithmic scale.

It is also worth mentioning that in both models pedestrians form clusters around the exit (if their density is high enough). However, due to the lack of necessary interactions it is impossible to achieve clogging. Pedestrians are always able to leave the room in a coherent stream.

Figures 7–10 contain results equivalent to the ones presented above but for a classroom geometry. The average evacuation times, while still in the same range for $\beta = 0.50$ and $\beta = 0.75$, lack the consistency of the results obtained for the empty room geometry. The same can be said about the minimal and maximal evacuation times. While the minimal times are close together, the maximal times in the cellular automata model are much higher.

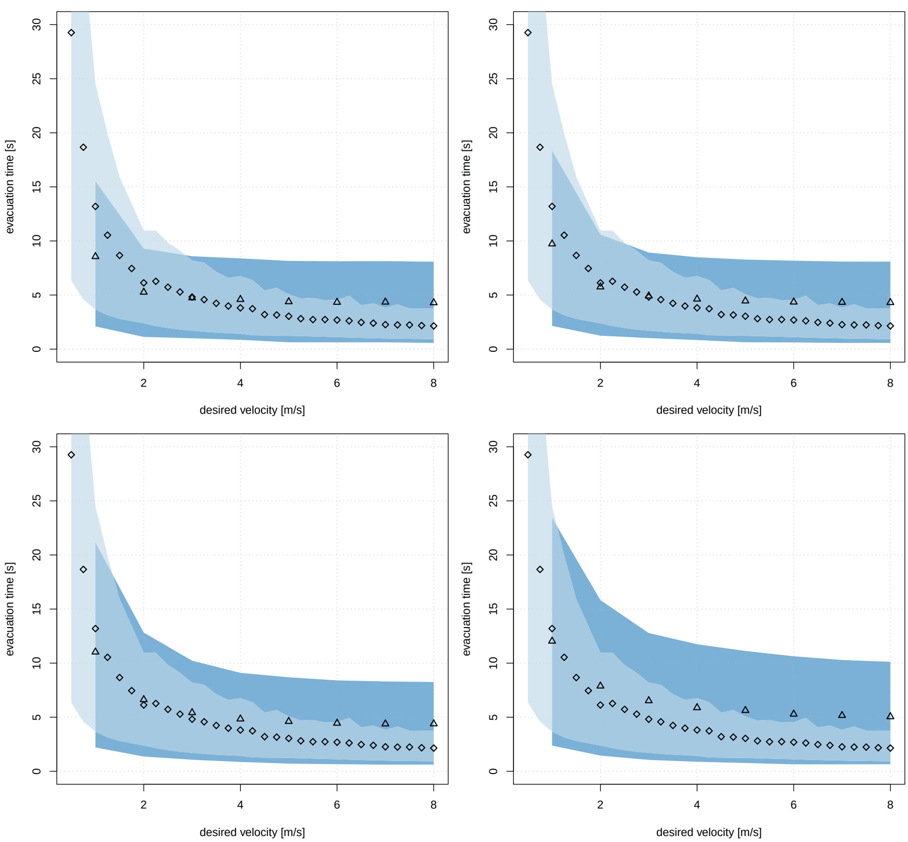

Fig. 8. Classroom: average evacuation time as a function of desired velocity for the differential model (diamonds) and CA model (triangles) with $\beta = 0.25$ (top left), $\beta = 0.50$ (top right), $\beta = 0.75$ (bottom left), $\beta = 1.00$ (bottom right). The filled parts of the plot mark the maximal and minimal evacuation times for the differential model (light grey) and CA (dark grey).

Also, the average evacuation time no longer scales like $\bar{t} \propto v_{\mathrm{d}}^{\alpha}$ in the cellular automata model. However, we think that the results in terms of evacuation times are still comparable.

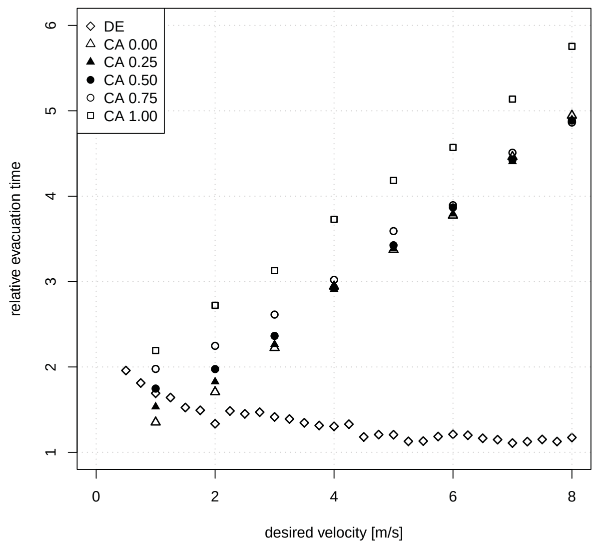

Fig. 9. Classroom: average relative evacuation time as a function of desired velocity for the differential model and CA model for different values of β.

Much like for the empty room geometry, average relative evacuation time increases (in this case linearly) with β in the cellular automata model and decreases in the differential model. What is surprising is that when $\beta = 0$ (there are no explicit interactions between pedestrians) average velocity as a function of desired velocity is almost the same for both differential and cellular automata models (Fig. 10). Still, it does not translate into similar evacuation time.

In both models pedestrians form clusters around the exit. The difference lays in the way they get to the door. In the cellular automata model pedestrians are able to move more or less freely near the desks and each other. In the differential model, the repulsive force between pedestrians and desks prevents them from crossing the border marked by each row and forces them to form queues between rows. It is yet another example of how different handling of local interactions can change the overall dynamics and one of the factors responsible for the discrepancies in the evacuation times.

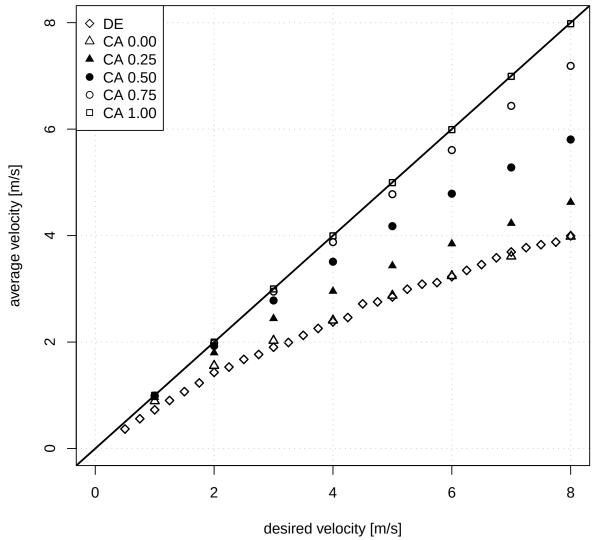

Fig. 10. Classroom: average velocity as a function of desired velocity for the differential model and CA model for different values of β.

5. Conclusions

Our research shows that it is possible to achieve a satisfying level of similarity between the differential and discrete models of evacuation. The two models we studied, at least in terms of average evacuation times, produce comparable results. Because of the differences in handling of local interactions, other properties and phenomena depend strongly on the underlying geometry. However, we believe that through some clever manipulation of the transition rules, the differential and discrete models can be brought even closer together without resorting to an overzealous discretisation of the former.

The results of our simulations indicate that discrete velocity can be easily added to cellular automata models in a plausible way. While the differential models of evacuation will always have the advantage, as they incorporate velocity and other similar physical quantities more naturally, cellular automata models, in our opinion, can be a valid alternative in cases where exact and continuous values of velocity are not required.

As for future work, we are currently trying to modify the way local interactions are defined in our cellular automata model. After some adjustments, it should be possible to mimic the phenomena (*e.g.* clogging, "faster is slower" behaviour) that can be observed in the differential model with the full granular interactions term.

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