LABOUR AND GOODS MARKET DYNAMICS USING AN ABSTRACT MICROECONOMICAL MODEL*

Ranaivo M. Razakanirina†, Bastien Chopard

Computer Science Department, University of Geneva
7 route de Drize, 1227 Carouge, Switzerland

(Received December 20, 2011)

This paper presents a multilayer cellular automata on a graph to model the exchanges of working hours against salary coupled with the exchanges of cash against goods, thus creating an artificial labour and goods markets. During the time evolution, the cooperation and the competition between the individuals create rich behaviours: the strongly connected components (SCC) of the whole market emerge, a steady state or chaotic state appears, poor and rich cells emerge. When reaching the steady state, we show also that the distribution of cash is, in average, proportional to the in-degree of the cells.

DOI:10.5506/APhysPolBSupp.5.131
PACS numbers: 05.45.–a, 89.65.Gh

1. Introduction

The approach we consider to model the dynamic of an artificial market is based on the microscopic behaviours of each agent of the market and the dynamic of the interactions between them. We model such system using the Multilayer Cellular Automata on a Graph formalism (MCAG formalism) [1], a formalism which merges recent advances on the topological analysis of complex network [2, 3, 4, 5, 6] and on the cellular automata on irregular topology [7, 8].

The main interest of MCAG is its ability to model systems consisting of the superposition of cellular automata on graph layers, each layer having distinct irregular neighbourhood topology built upon a graph. We propose in this paper a model of agents who exchange working hours against salary and cash against goods. The model, denoted Labour–Goods Market (LGM) consists of an artificial labour market coupled with an artificial goods market. The neighbourhood topologies of both markets are distinct.

* Presented at the Summer Solstice 2011 International Conference on Discrete Models of Complex Systems, Turku, Finland, June 6–10, 2011.

† Corresponding e-mail address: ranaivo.razakanirina@unige.ch

This paper is organized as follows: we first describe the LGM model then, we present the parameters used during the simulations and finally, we discuss the obtained results.

2. Labour–Goods Market model formalism (LGM model)

Let us denote by V the set of agents that may represent individuals. The structures of the labour and goods markets are built upon a directed graphs denoted respectively by $G_\ell(V, E_\ell)$ and $G_g(V, E_g)$. E_ℓ and E_g are, respectively, the sets of directed edges of G_ℓ (black edges in Fig. 1) and G_g (light grey edges in Fig. 1).

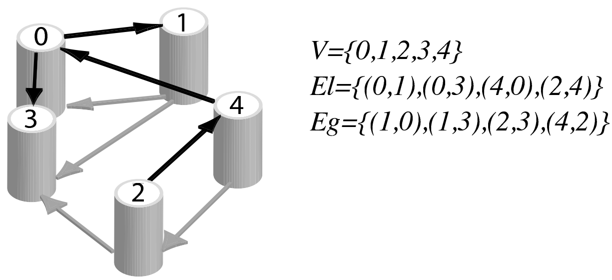


Fig. 1. An example of LGM model with five agents labelled 0 to 4. V={0, 1, 2, 3, 4}. The set of black edges E_ℓ models the structure of labour market and the set of light grey edges E_g models the structure of goods market.

Graphs G_ℓ and G_g define the roles of each individual and its neighbourhood. The directed edge $(i, j) \in E_\ell$ means that i is the employer of j. The directed edge $(k, \ell) \in E_g$ means that k buys goods from ℓ. Let us consider cell 2 of Fig. 1 as an example. Cell 2 is the employer of 4 for the labour market. Cell 2 buys goods from 3 and sells goods to 4 for the goods market.

We assume that individuals are not a self-employed workers and do not buy or sell goods to themselves. Therefore, G_ℓ and G_g are a simple directed graphs [9]. We denote respectively by a_{ij} and b_{ij} the elements of the adjacency matrices [4] of G_ℓ and G_g.

At each time iteration t, the state of each individual i consists of the individual's wealths which are (1) its available working hours $h_i(t)$, (2) its cash $c_i(t)$ and (3) the quantity of goods $g_i(t)$ it owns. These quantities are infinitely divisible. The working hours cannot be saved and are reset to value δ_i at each time iteration. For instance, if the time step is one day, $\delta_i = 8$ hours of work. For the labour market, the cash is used to pay the salaries of workers. For the goods market, the cash is used to buy goods from sellers.

During the evolution, the agents do not create or delete cash. Thus the total cash $c_{\text{tot}} = \sum_{\forall \ell} c_\ell(t), \forall t$ is conserved. However, agents produce and consume goods.

The transition function of LGM model is split naturally into two main phases which are the labour market dynamics and the goods market dynamics. This transition function is applied locally and synchronously on all the agents. These phases are stated as follows:

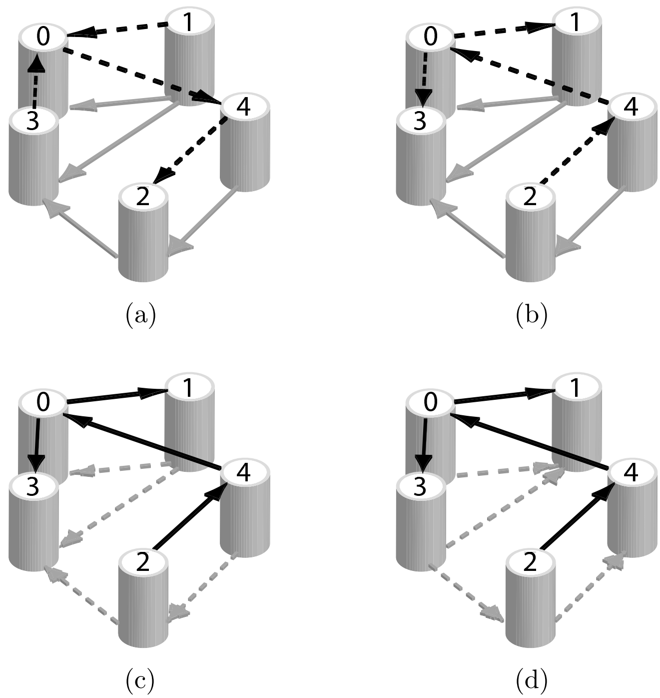


Fig. 2. Flows of hours (dark grey dashed edges in (a)), cash (dark grey dashed edges in (b) and light grey dashed edges in (c)) and goods (light grey dashed edges in (d)) during the transition rule of the LGM-model depicted in Fig. 1.

2.1. Labour market dynamics

During this phase, each individual spends time to work for its employers. In exchange cash is returned by the employers. The working hours spent by the individual are transformed to goods for the benefit of the employers. Let us consider a link (i, j) in the labour market. h_{ji} is the flow of hours (dark grey dashed edges of Fig. 2 (a)), c_{ij} is the flow of cash (dark grey dashed edges of Fig. 2 (b)) and w_{ij} is the hourly wage offered by i to j. This dynamics is split as follows:

2.1.1. Exchanges of hours

Each individual $j \in V$ spends $h_j(t)$ hours working for its employers. An individual may start with multiple employers. In this configuration, the repartition of working hours is made in proportion to the hourly wages offered by the employers. In other words, an individual spends more working time for high-paid job than for low-paid job. We have $h_{ji} = K_1(t)w_{ij}$, where $K_1(t) = \frac{h_j(t)}{\sum_{\forall \ell} a_{\ell j} w_{\ell j}}$. The goal of the individual is to maximize the amount of cash received from its employers. As the offered hourly wages are decided by the employers, the individual try to achieve this goal by the proportionality rule.

2.1.2. Salary payment

Each individual $i \in V$ pays its workers. Only a fraction $\gamma c_i(t)$ $(0 \leq \gamma \leq 1)$ of its cash is used for salary. The repartition of cash is made in proportion to the working hours spent by the workers. We have $c_{ij} = K_2(t)h_{ji}$, where $K_2(t) = \frac{\gamma c_i(t)}{\sum_{\forall \ell} a_{i\ell} h_{\ell i}}$.

2.1.3. Computation of hourly wage

After salary payment, each employee j can compute the hourly wage w_{ij} offered by employer i as the ratio of the received salary to the amount of working hours. We have $w_{ij} = \frac{c_{ij}}{h_{ji}} = K_2(t)$. As a result of our rule, one can show that w_{ij} is identical for all j. From now on, we simplify the notation by $w_{ij} = w_i$.

Each employee j cuts its links with employers offering the worst hourly wages. We will see later that this strategy confers sensitivities to the dynamics and the system tends to be chaotic due to the competition between the employers.

2.1.4. Production of goods

Each employer i produces an amount P_i of goods as a result of the working hours that its employees spend for him. $P_i = \pi_i \sum_{\forall \ell} a_{i\ell} h_{\ell i}$, where π_i is the production capacity.

2.2. Goods market dynamics

During this phase, each individual buys goods from its sellers. Thus cash is given to its sellers and in exchange the sellers give goods. The received goods are partially consumed by the individual who bought them within the iteration.

The cash c_i^* available to individual i for the goods market is $c_i(t) + \sum_{\forall \ell} a_{\ell i} c_{\ell i} - \sum_{\forall \ell} a_{i\ell} c_{i\ell}$, where the second term accounts for the salary received by i and the third term is the salary paid by i. Similarly, the quantity of goods g_i^* at the disposal of each individual i is its current quantity of goods augmented by its production. We have $g_i^* = g_i(t) + P_i$.

We use the superscript * to denote the state of each cell during the goods market dynamics.

Let us consider a link (i, j) in the goods market. c_{ij}^* is the flow of cash (light grey dashed edges of Fig. 2 (c)), g_{ji}^* is the flow of goods (light grey dashed edges of Fig. 2 (d)) and p_{ji}^* is the unit price of goods proposed by j to i. This dynamics is split as follows:

2.2.1. Buying goods

Each individual $i \in V$ buys goods from its sellers. It offers a fraction λc_i^* $(0 \leq \lambda \leq 1)$ of its available cash to the sellers. The repartition is made in inverse proportion to the unit price of goods proposed by sellers. Thus the individual gives more cash to the low-price seller than to the high-price seller. Here, the goal is to maximize the quantity of goods with less cash. We have $c_{ij}^* = K_3(t) p_{ji}^{*-1}$, where $K_3(t) = \frac{\lambda c_i^*}{\sum_{\forall \ell} b_{i\ell} p_{\ell i}^{*-1}}$.

2.2.2. Goods delivery

In exchange of the previously given cash, each individual $j \in V$ gives goods to its buyers. Only a fraction μg_j^* $(0 \leq \mu \leq 1)$ of its available quantity of goods is given during this exchange. The repartition of goods is made in proportion to the cash given by the buyers. We have $g_{ji}^* = K_4(t) c_{ij}^*$, where $K_4(t) = \frac{\mu g_j^*}{\sum_{\forall \ell} b_{\ell j} c_{\ell j}^*}$.

2.2.3. Computation of unit price of goods

After the exchange of goods against cash, buyer i can determine the actual price p_{ji} of the transaction with seller j, as the ratio of the amount of cash he paid to the amount of goods he received. $p_{ji}^* = \frac{c_{ij}^*}{g_{ji}^*} = \frac{1}{K_4(t)}$. This quantity reduces to $K_4(t)$, showing that the price offered by j is the same for all i.

Upon computing the prices of each seller, the buyer decides to stop interacting with the sellers which propose too high unit price of goods in comparison with its others sellers.

2.2.4. Consumption of goods

Each individual $i \in V$ consumes with a rate ϕ_i its balance of goods. This balance is the difference between the sum of goods received from its sellers $\sum_{\forall \ell} b_{i\ell} g^*_{\ell i}$ and the sum of quantity of goods given to its buyers $\sum_{\forall \ell} b_{\ell i} g^*_{i\ell}$ added to the current quantity of goods at its disposal. Thus, the quantity of goods C_i consumed by individual i within the iteration is $C_i = \phi_i(g^*_i + \sum_{\forall \ell} b_{i\ell} g^*_{\ell i} - \sum_{\forall \ell} b_{\ell i} g^*_{i\ell})$.

2.3. State of each agent at the next time iteration

At the next time iteration, each individual i resets its available quantity of hours. Therefore

$$h_i(t+1) = \delta_i \, . \tag{1}$$

The balance of cash of the individual i at the next time iteration $t+1$ is

$$c_i(t+1) = c_i(t) - \sum_{\forall \ell} a_{i\ell} c_{i\ell} + \sum_{\forall \ell} a_{\ell i} c_{\ell i} - \sum_{\forall \ell} b_{i\ell} c^*_{i\ell} + \sum_{\forall \ell} b_{\ell i} c^*_{\ell i} \, . \tag{2}$$

From the Eq. (2), we can prove analytically that the total cash c_{tot} in the whole LGM model is conserved.

From Sec. 2.2.4 the balance of goods of the individual i at the next time iteration is

$$g_i(t+1) = (1-\phi_i)\left(g^*_i + \sum_{\forall \ell} b_{i\ell} g^*_{\ell i} - \sum_{\forall \ell} b_{\ell i} g^*_{i\ell}\right) . \tag{3}$$

3. Simulations setup

The parameters γ, λ, μ adjust the level of wealth that each individual puts at the disposal of the LGM. Our goal is to observe the market dynamics based on agents that have the same resource repartition strategy. We do not consider the imbalance of resource repartition due to local choice of agents. Therefore, we set these parameters to homogenous values. More particularly, setting all of these values to zero stops the exchanges between the agents and freezes the markets. With all of these values set to one, each individual spreads at each time iteration all of its wealth to all of its neighbours. With this local behaviour, the system becomes oscillatory. The interesting behaviours presented in this work are observed when these parameters are set between zero and one. We consider that agents invest a large part of their cash and their quantities of goods for labour and goods market ($\gamma = 0.9$, $\lambda = 0.9$, $\mu = 0.9$).

Here, we consider a specific instance of our model by simulating the exchanges between firms and employees. The firms play only the role of employer and seller of goods. The employees play only the role of worker and buyer of goods. Thus, the structure of the labour market (G_ℓ) is a bipartite graph where the direction of edges goes from the firms to the employees and the structure of the goods market (G_g) is an another distinct bipartite graph where the direction of edges goes from the employees to the firms (Fig. 3).

From now on we denote generally each firm by the label i and each employee by the label j.

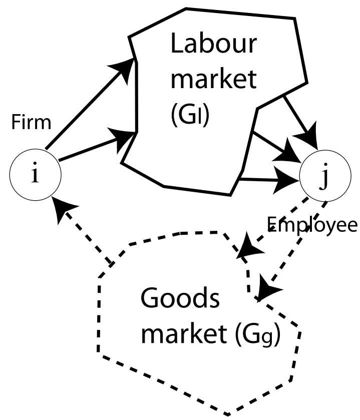


Fig. 3. Firm i is an employer in the labour market and a seller in the goods market. Employee j is a worker in the labour market and a buyer in the goods market.

The parameters π, ϕ and δ allow us to make the difference between the behaviour of firms and the behaviour of employees. Firms use only the resources proposed by their employees ($\delta_i = 0$). At each time iteration, they produce hourly and homogeneously three units of goods ($\pi_i = 3$). Employees work at each time iteration during eight hours ($\delta_j = 8$). They do not produce goods ($\pi_j = 0$). Employees consume more goods than firms ($\phi_i = 0.1$, $\phi_j = 0.9$).

Each agent starts with homogeneous values of cash, with homogeneous values of hourly wage and with homogeneous values of unit price of goods. Initially, there is no goods in the market. Goods are created during the production phase of the transition rule.

The topologies of G_ℓ and G_g are random bipartite graphs [10, 11] with, respectively, probability p_ℓ and p_g. These probabilities are chosen to have sparse graphs. Additional rules are applied on these topologies which insure that a minimal economic interaction will take place during the simulation. We assume that each firm has at least one employee and sells goods to at least one employee. We assume also that each employee works for at least one firm and buys goods from at least one firm.

4. Results and discussions

During the transient regime, the labour and goods market links between the individuals can be cut as described in Secs. 2.1.3 and 2.2.3. The evolution of the market topologies consists of two regimes.

During the first regime, the strongly connected components (SCC) of the whole market $G = G_\ell \cup G_g$ emerge (Fig. 4 and 5).

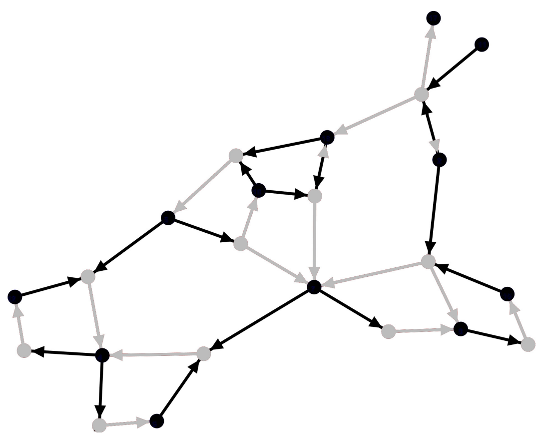


Fig. 4. Initial topology of LGM-model with 11 firms (light grey nodes) and 12 employees (black nodes). The light grey edges are the flows of cash for the labour market G_ℓ and the black edges are the flows of cash for the goods market G_g.

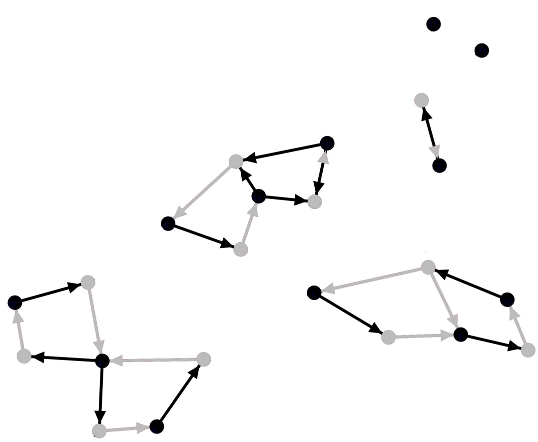


Fig. 5. The topology at $t = 100$ of the LGM-model described in Fig. 4. The SCC of the initial topology emerge. Each SCC are then split to more little parts called sub-SCC when the competition between the firms/sellers tends to have uniform wage and uniform price of goods in each SCC.

During the second regime, the competition and the cooperation between the cells tend to split the SCC created during the first regime to more little components that we denote by sub-SCC.

Let us consider an emerging SCC H which consists of firms and employees. Employees cooperate with firms according to the labour links. Employees favour topologically the employer offering the highest hourly wage and the seller proposing the lowest unit price of goods and otherwise the relations. This cooperation creates competition between the firms. The divergence of hourly wages and unit price of goods between the firms of H measures the level of competition. If this level is higher than the cutting edge levels H is split in more little parts. Otherwise, the links between the agents of H are conserved.

From now on, let us observe and consider each sub-SCC, result of the evolution of market topology. Sub-SCC may reach two behaviours. The first one is the stationary state which is characterised by the stable values of the state of each individual. An example of stable state is depicted in Fig. 6 (a). The second one is the chaotic behaviour where the state of each individual does not tend to a stable value (Fig. 6 (b)).

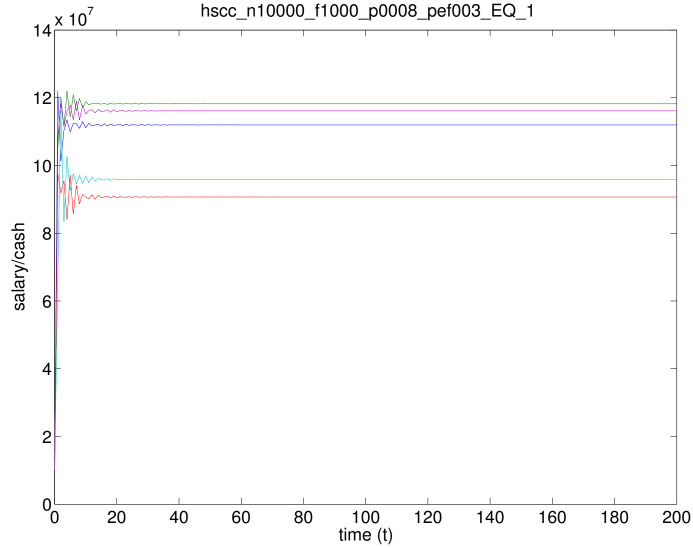
Stationary state emerges when the hourly wages offered by the firms and the unit price of goods proposed by the firms become more or less homogenous. Chaotic behaviour happens when the divergence between the hourly wages and the unit price of goods is high and that the dynamics of edges cannot split the sub-SCC in more little parts to adjust the level of hourly wages and unit price of goods in more homogeneous value.

At the stationary state, we observe that the individuals are categorised into two main classes which are the few “richer” individuals and the large number of “poorer” individuals (Fig. 7 (b)). The firms are the “richer” individuals and the employees are the “poorer” individuals.

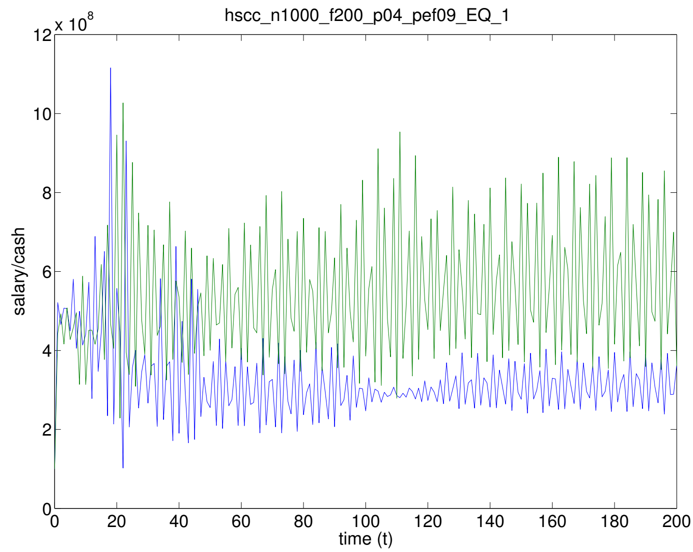
We observe also that the cash owned by each individual i is proportional to the number of buyers connected to him (Fig. 8). We estimate the distribution as $c_i = ak_i^* + b$, where k_i^* is the number of buyers of agent i (G_g) and c_i the cash at the stationary state. $\sum_i c_i = c_{\text{tot}} = a\sum_i k_i^* + \sum_i b$. Therefore, $a = \frac{c_{\text{tot}} - nb}{m^*}$, where n is the total number of agents, m^* is the size of the goods market (G_g). As a large number of employees j are poor ($k_j^* = 0$), the value of b corresponds to the mean value of cash of employees at the stationary state. We have $b = \bar{c_e} = \frac{c_e}{n_e}$, where n_e is the number of employees and $c_e = \sum_j c_j$ is the total cash of employees j. Thus

$$c_i = \left(\frac{c_{\text{tot}} - n\bar{c_e}}{m^*}\right) k_i^* + \bar{c_e}\,. \tag{4}$$

This equation fits well the mean distribution of cash *versus* the number of buyers (Fig. 8).

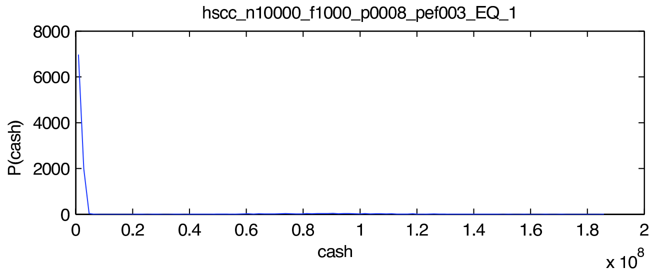


(a)

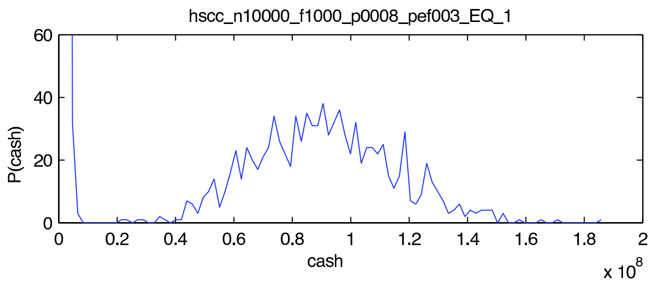


(b)

Fig. 6. Sample of cash value *versus* time iteration t (a) when converging to the stationary state (LGM model initially with 1000 firms, 9000 employees, $p_\ell = 0.0008$ and $p_g = 0.003$) (b) when become chaotic (LGM model initially with 200 firms, 800 employees, $p_\ell = 0.04$ and $p_g = 0.09$).



(a)



(b)

Fig. 7. The distribution of cash at the stationary state of LGM model with 1000 firms, 9000 employees, $p_\ell = 0.0008$ and $p_g = 0.003$. (a) Large number of individuals are poor and few individuals are rich. (b) Zoom in to see the distribution of cash among the "richer" agents.

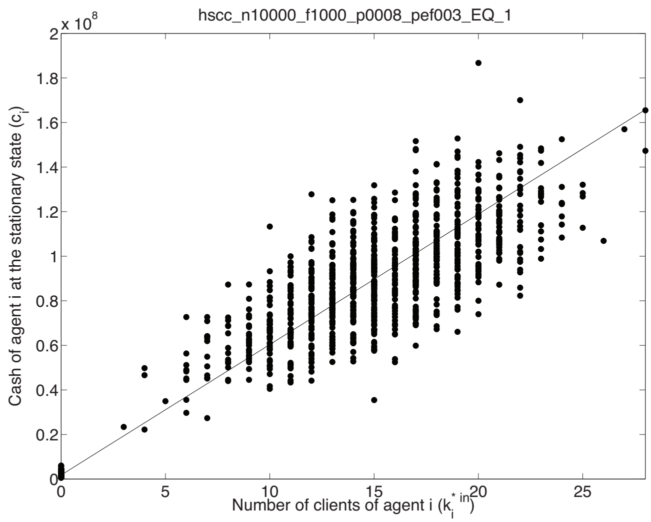


Fig. 8. Dots are the cash *versus* the number of connected buyers. The line is the mean distribution of cash *versus* the number of clients computed with Eq. (4).

One can calculate numerically the distribution of cash when the hourly wages become uniform and the unit prices of goods become uniform. The values of cash are obtained by solving the following system of linear equations expressing the rule of the model (the details of the calculations are given in [12])

$$\lambda c_j = (1-\lambda)\gamma \sum_{\forall m} \psi_{mj} c_m \,, \qquad \text{where} \qquad \psi_{ij} = \frac{\frac{a_{ij}}{k_j^{in}}}{\sum_{\forall \ell} \frac{a_{i\ell}}{k_\ell^{\text{in}}}} \,, \tag{5}$$

$$c_i = \sum_{\forall k} \sum_{\forall m} \theta_{ki} \psi_{mk} c_m \,, \qquad \text{where} \qquad \theta_{ij} = \frac{b_{ij}}{k_i^{*\text{out}}} \,, \tag{6}$$

$$c_{\text{tot}} = \sum_{i(\text{firms})} c_i + \sum_{j(\text{employees})} c_j \,, \tag{7}$$

where i and j denote respectively the labels of firms and the labels of employees. With LGM model, cash value is a centrality measure qualifying the weight of each individual in the whole network. As described above, the number of buyers connected with the individual impacts on the distribution of cash. We show that the mean distribution of cash (Eq. (4)) is closely related to the so-called page rank of each individual.

5. Conclusion and future work

We have studied the behaviours of a particular multilayer cellular automata on a graph (MCAG) denoted LGM model which models the labour and goods market dynamics. This model merges the hour against salary/cash dynamics (labour market) and the cash against goods dynamics (goods market). The topologies of labour and goods markets are distinct making the MCAG formalism useful.

We found that the SCC of the labour and goods market taken together emerge. Each SCC tends to be split in more little subcomponents depending on the level of competition between the firms in the SCC. This level is determined by the divergence of hourly wage and unit prices of goods between the firms. This competition impacts also the final state of the subcomponents which may be stationary or may be chaotic. Chaotic behaviour appeared when competition takes place but cannot split the SCC in more little subcomponents. The steady state emerge when the wages and the unit prices of goods proposed by the firms become homogeneous, allowing a perfect competition between the firms in each SCC.

Considering each subcomponent at the stationary state, two main classes of cells emerge which are the richer cells and the poorer cells. The richer ones are the employers and the poorer ones are the workers. We found also that the distribution of the amount of cash is proportional to the number of the buyers of the cell.

As an extension, firms could buy goods from other firms or work for them, thus modelling more levels of commercial relationship between them.

We acknowledge financial support from the Swiss National Science Foundation.

REFERENCES

[1] R.M. Razakanirina, B. Chopard, *Multilayer Cellular Automata on a Graph Applied to the Exchanges of Cash and Goods*, Journal of Cellular Automata, OCP Science, 2011 (in press).

[2] J.B. Glattfelder, "Ownership Networks and Corporate Control: Mapping Economic Power in a Globalized World", PhD thesis, ETH Zurich, Department of Management, Technology, and Economics, 2010.

[3] C.S. Wagner, L. Leydesdorff, *Research Policy* **34**, 1608 (2005).

[4] G. Caldarelli, *Scale-Free Networks: Complex Webs in Nature and Technology (Oxford Finance)*, Oxford University Press, USA, 2007.

[5] M. Serrano, M. Boguñá, A. Vespignani, *Journal of Economic Interaction and Coordination* **2**, 111 (2007).

[6] R. Albert, A.-L. Barabási, *Phys. Rev. Lett.* **85**, 5234 (2000).

[7] A. Hoekstra, J. Kroc, P. Sloot, *Modelling Complex Systems with Cellular Automata*, Springer-Verlag, 2010.

[8] R.M. Razakanirina, B. Chopard, *Using Cellular Automata on a Graph to Model the Exchanges of Cash and Goods*, ACRI proceedings, pp. 163–172, 2010.

[9] S.S. Epp, *Discrete Mathematics with Applications*, Richard Stratton, 2011.

[10] R.D. Richard Durrett, M. Stein, *Random Graph Dynamics*, Cambridge University Press, 1 edition, 2010.

[11] S.N. Dorogovtsev, J.F.F. Mendes, A.N. Samukhin, *Phys. Rev.* **E64**, 025101 (2001).

[12] R.M. Razakanirina, B. Chopard, "Hour and Salary Complex Model at the Steady State: Detailed Analytical Calculations", 2011, `http://www.ambohimalaza.org/phd/hsc/hscsteadystate.pdf`