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GENERALISED MICROCANONICAL STATISTICS AND FRAGMENTATION IN ELECTRON–POSITRON COLLISIONS*

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A statistical fragmentation model based on the microcanonical ensemble and on a Koba–Nielsen–Olesen (KNO) type scaling of the multiplicity distribution of charged hadrons in electron–positron collisions is presented.

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1. Introduction

The energy scale of hadronisation processes in high energy collisions is of the order of $\Lambda_{\rm QCD} \approx 200\,\mathrm{MeV}$ and thus the processes cannot be described by perturbative quantum chromodynamics (pQCD). However, an understanding of the mechanism of hadronisation is essential for the interpretation of some hadronic observables such as single particle spectra measured in proton–proton or in heavy-ion reactions. In pQCD-improved parton model calculations [1], hadron production is described by fragmentation functions [2, 3]. Though the evolution of these fragmentation functions with the scale Q^2 can be understood within the framework of pQCD [4,5,6], their actual form at a given scale $Q^2 = s_0$ cannot be deduced. In this paper, we point out that the distribution of hadrons emitted by the leading parton of a jet can be deduced from simple statistical considerations too.

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2. Microcanonical jet fragmentation

If the process of the creation of hadrons $h_1, \ldots, h_N$ by the leading parton p_i of a jet with multiplicity N is such, that the corresponding cross-section

$$d\sigma^{h_1,\ldots,h_N} = |M|^2 \, \delta^{(4)}\left(\sum_j p^\mu_{h_j} - P^\mu_{p_i} \right) d\Omega \propto \delta\left(\sum_j \epsilon_{h_j} - E_{\rm jet} \right) d\Omega \quad (1)$$

is simply proportional to the phase space available for the hadrons, restricted only by the energy conservation, then the hadrons created throughout the fragmentation process form a microcanonical enlemble. In Eq. (1), Ω is the phase space of the created hadrons, $p^\mu_{h_j}$ and ϵ_{h_j} are the four-momentum and energy of the hadron h_j, $P^\mu_{p_i}$ is the four-momentum of the leading parton p_i, $E_{\rm jet} = P^0_{p_i}$ is the energy of the jet, finally M is the matrix amplitude describing the process. Microcanonical treatment of hadron production has also been proposed in [7, 8, 9, 10] for proton–proton reactions, and in [11, 12, 13] for e^+e^- reactions.

The energy distribution of a hadron inside the jet with multiplicity N reads [13]

$$f_N(\epsilon) = A_{\rm mc}\,(1-x)^{D(N-1)-1}\,, \quad (2)$$

where $x = \epsilon/E_{\rm jet}$, ϵ is the energy of the hadron, D is the effective dimensionality of the jet and $A_{\rm mc} = \binom{D\,N-1}{D\,(N-1)-1} D/(\,k_D\,E^D_{\rm jet}\,)$ follows from the normalisation condition

$$1 = \int d\Omega_p \int dp\, p^{D-1} f_N(\epsilon)\,, \quad (3)$$

with $k_D = \int d\Omega_p$ being the angular part of the momentum space integral. Eq. (2) follows from the microcanonical momentum space volume at fixed energy and multiplicity,

$$\Omega_N(E) = \frac{1}{N!}\int \prod d^D p_i\, \delta\left(E - \sum \epsilon_i\right) = \frac{k_D^N\,\Gamma(D)}{N!} E^{N\,D-1}\,, \quad (4)$$

whence the one-particle distribution is obtained as

$$f_N(\epsilon) \propto \frac{\Omega_{N-1}(E-\epsilon)}{\Omega_N(E)}\,. \quad (5)$$

If the particles in the ensemble interact so that the one-particle energies ϵ_i and the total energy are connected via the formula

$$L(E) = \sum L(\epsilon_i)\,, \quad (6)$$

with $L(\epsilon) = (1/a)\ln(1 + a\epsilon)$, as has been proposed in [14, 15, 16], then the N particle constrained phase space volume $\Omega_N(E)$ becomes

$$\begin{aligned}
D &= 1\,, \quad \Omega_N(L) \propto L^{N-1}\, e^{a\,L}\,,\\
D &= 2\,, \quad \Omega_N(L) \propto \sum_{j=0}^{N-1} \frac{(2N-j-2)!}{j!\,(N-j-1)!}\,(-)^N\,[a\,L]^j\left\{e^{a\,L} + (-)^{j+1}e^{2a\,L}\right\}\,,\\
D &= 3\,, \quad \Omega_N(L) \propto \sum_{j=0}^{N-1}\sum_{k=0}^{N-j-1} \frac{(N+k-1)!}{j!\,k!}\,\frac{(2N-j-k-2)!}{(N-j-k-1)!}\,(-)^{N-j-1}\\
&\qquad \times [a\,L]^j\left\{\frac{(-)^{N-j-1}}{2^{2N-j-k-1}}e^{a\,L} + (-)^{j+k+1}e^{2a\,L} + \frac{e^{3a\,L}}{2^{N+k}}\right\}\,,
\end{aligned} \tag{7}$$

in $D = 1$, 2 and 3 dimensions. The energy distribution of a single particle follows from Eqs. (5) and (7). Results of numerical simulations and analytical calculations for $D = 3$ are shown in Fig. 1.

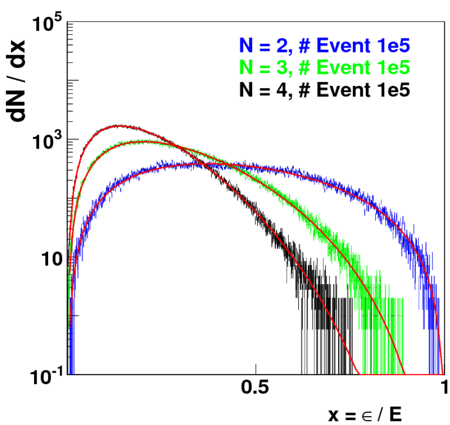

Fig. 1. Single particle distributions in ensembles consisting of 2, 3 and 4 particles with interaction described in Eq. (6), in $D = 3$ dimensions. Solid curves are analytic results obtained from Eqs. (5) and (7); histograms are results of 10^5 simulated events each.

3. Multiplicity averaged hadron spectra

It has been shown in [17, 18, 19, 20, 21, 22] that special event-by-event fluctuation patterns of the temperature or of the particle multiplicity can result in power-law tailed average particle spectra even if in each event particles are distributed according to the Boltzmann–Gibbs distribution. In papers [23, 24, 25, 26, 27] it was argued that an approximate Koba–Nielsen–Olesen (KNO) scaling of the multiplicity distribution of charged hadrons holds for electron–positron collisions (though the scaling is weakly violated by the scale evolution of the strong coupling, $\alpha_s(Q^2)$). In [13] it is shown that the average hadron spectrum takes the form

$$\frac{1}{\sigma}\frac{d\sigma}{dx} = \sum_N p(N) N f_N(\epsilon) \approx \frac{A\,x^{D-1}(1-x)^{D(N_0-1)-1}}{\left(1 - \frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}\,, \tag{8}$$

if, in each event, hadrons are distributed according to the microcanonical ensemble and the hadron multiplicity fluctuates as

$$p(N) \propto (N-N_0)^{\alpha-1} e^{-\beta\,(N-N_0)}\,. \tag{9}$$

Remarkably, if particles in each event interact as described in Eq. (6), the average hadron spectrum would take a form very similar to that of Eq. (8) in one ($D=1$) dimension

$$\frac{1}{\sigma}\frac{d\sigma}{dx} \approx \frac{A}{1+aEx}\frac{(1-y)^{N_0-2}}{\left[1 - \frac{q-1}{T/(\sqrt{s}/2)}\ln(1-y)\right]^{1/(q-1)}}\,, \tag{10}$$

but in a new variable $y = \ln(1+aE\,x)/\ln(1+a\,E)$.

Fits of Eq. (8) and Eq. (10) to the measured fragmentation functions published in [28] are shown in Figs. 2 and 3. Parameters used in Eq. (8) are: $D = 3$, $N_0 = 1 + 2/D$, $q_\Lambda = 1.297 \pm 0.002$, $q_p = 1.273 \pm 0.0006$, $q_{K^0} = 1.301 \pm 0.001$, $T_\Lambda = (0.20 \pm 0.01)$ GeV, $T_p = (0.228 \pm 0.001)$ GeV, $T_{K^0} = (0.16 \pm 0.01)$ GeV.

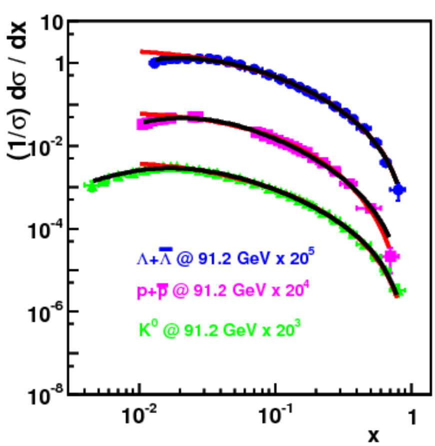

Fig. 2. Fragmentation functions of Λs, protons and K^0s measured at $\sqrt{s} = 91.2$ GeV collision energy (the data are from [28]) with the fitted distributions (black curves: Eq. (8), grey (red) curves: Eq. (10)).

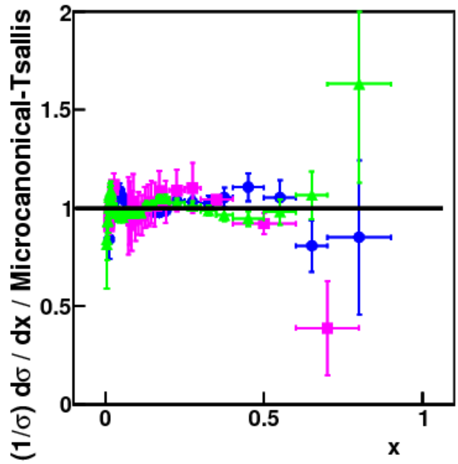

Fig. 3. Ratio of fragmentation functions of Λs, protons and K^0s measured at $\sqrt{s} = 91.2$ GeV collision energy (data of graphs are published in [28]) and fitted distribution Eq. (8) with $D = 3$ and $N_0 = 1 + 2/D$.

Parameters used in Eq. (10) are: $a = 1/E$, $q_\Lambda = 1.8 \pm 0.1$, $q_p = 1.7 \pm 0.2$, $q_{K^0} = 1.8 \pm 0.1$, $T_\Lambda = (2.9 \pm 0.4)$ GeV, $T_p = (4 \pm 1)$ GeV, $T_{K^0} = (2.8 \pm 0.4)$ GeV, $N_{0\Lambda} = 3.6 \pm 0.2$, $N_{0p} = 4.6 \pm 0.5$, $N_{0K^0} = 3.7 \pm 0.3$.

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