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FINITE PRESSURE CORRECTIONS TO THE PARTON STRUCTURE OF BARYON INSIDE A NUCLEAR MEDIUM*

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Our model calculations performed in the frame of the Relativistic Mean Field (RMF) approach show how important are the modifications of a baryon Structure Function (SF) and a nucleon mass in Nuclear Matter (NM) above the saturation point. They originated from the conservation of a parton longitudinal momenta — essential in the explanation of the EMC effect at the saturation point of NM. For higher density the finite pressure corrections emerge from the Hugenholtz–van Hove theorem valid for NM. Here, we show that the course of Equation of State (EoS) in our modified Walecka model is very close to the phenomenlogical non-relativistic expansion of single particle energies in powers of Fermi momentum. The increasing pressure between nucleons starts to increase nucleon Fermi energies e_{F} in comparison to average nucleon energies e_{A}, and consequently the Momentum Sum Rule (MSR) is broken by the factor $e_{\mathrm{F}}/e_{\mathrm{A}}$ in the RMF models. To compensate this factor which increases the longitudinal momentum for nuclear partons, the baryon SF in the nuclear medium and their masses have to be adjusted. We assume that, independently from nuclear density, quarks and gluons carry the same amount of a total longitudinal momenta — the similar assumption is used in the most nuclear models with parton degrees of freedom.

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1. The nuclear deeply inelastic limit — nuclear equilibrium

In the deep inelastic scattering on nuclei a time-distance resolution can be connected to the famous Bjorken variable x given by variable z [1,2]

$$z = 1/(xM_N) \tag{1}$$

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which measures the propagation time of the hit quark caring the x fraction of the longitudinal momentum of the nucleon of mass M_B. For $x > 0.05$ the partonic mean free paths z are shorter then the average distances between nucleons. This means that for such a short snapshot nucleons can be well treated as separated objects remaining on the energy shell. In the light cone formulation [1,3], x_A corresponds to the nuclear fraction of quark longitudinal momentum $p_q^+ = p_q^0 + p_q^3$ and is equal (in the nuclear rest frame) to the ratio $x_A = p_q^+/M_A$ with the nuclear mass M_A. But the composite nucleus is made of hadrons which are distributed with longitudinal momenta p_h^+, where $h = N, \pi, \ldots$ stands for nucleons, virtual pions *etc.* In the convolution model [1,3], a fraction of parton longitudinal momenta x_A in the nucleus is given as the product $x_A = x_h y_h$ of the fraction of parton momentum in a hadron $x_h \equiv Q^2/(2M_h\nu) = p_q^+/p_h^+$ and the fraction of longitudinal momentum of the hadron in the nucleus $y_h = p_h^+/M_A$. The nuclear dynamics of given hadrons in the nucleus is described by the distribution function $\rho^h(y \equiv y_h)$ and SF $F_2^h(x \equiv x_h)$ describing its parton structure. In the convolution model restricted to nucleons and pions (lightest virtual mesons) the nuclear SF F_2^A is described by the formula

$$F_2^A(x_A) = \int y dy \int dx \delta(x_A - xy) \left(\rho^N(y) F_2^{\mathrm{B}}(x) + \rho^\pi(y) F_2^\pi(x)\right) , \tag{2}$$

where F_2^π and F_2^{B} are the parton distributions in the virtual pion and in the bound nucleon. The nucleon distribution ρ^A in the basic convolution formula can be simplified in the RMF to the form [4]

$$\begin{aligned} \rho^A(y) &= \frac{4}{\rho} \int\limits_{|p|>p_{\mathrm{F}}} \frac{S_N(p) d^3p}{(2\pi)^3} \left(1 + \frac{p_3}{E_p^*}\right) \delta\left(y - \frac{p^+}{\varepsilon_N}\right) \\ &= \frac{3}{4} \left(\frac{\varepsilon_N}{k_{\mathrm{F}}}\right)^3 \left[\left(\frac{p_{\mathrm{F}}}{\varepsilon_N}\right)^2 - \left(y - \frac{e_{\mathrm{F}}}{\varepsilon_N}\right)^2\right] . \end{aligned} \tag{3}$$

Here, the nucleon spectral function was taken in the impulse approximation: $S_N = n(p)\delta(p^0 - (E^*(p) + U_V))$ and $E^*(p) = \sqrt{M_N^2 + p^2}$. e_{F} is the nucleon Fermi energy and y takes the values given by the inequality $(e_{\mathrm{F}} - p_{\mathrm{F}})/\varepsilon_N < y < (e_{\mathrm{F}} + p_{\mathrm{F}})/\varepsilon_N$. The flux factor $(1+\frac{p_3}{E_p^*})$ was recognized [3] as an important relativistic correction.

The MSR for the nucleonic part is sensitive to the Fermi energy as can be seen from the integral

$$\int dy\, y \rho^A(y) = \frac{e_{\mathrm{F}}}{\varepsilon_N} . \tag{4}$$

Thus the nucleonic part of MSR gives a factor $e_{\rm F}/\varepsilon_N$ which is equal to 1 at the saturation point [5]. Good description [6] of these deeply inelastic processes (with 1% admixture of nuclear pions [7]) and without gluon degrees of freedom allows us to assume that fraction of momentum carried by quarks does not change from nucleon to nucleus ($\sim$ one half, the rest is carried by gluons) also above the saturation point of NM. Here, the Fermi energy is no longer equal to the average binding energy and it will modify ρ^A in Eq. (4).

2. Non-equilibrium correction to nuclear distribution

For a finite pressure, the well known Hugenholtz–van Hove relation connecting $e_{\rm F}$, ε_N and pressure p [5] is very important. The Fermi energy is defined as density derivative of the total nuclear energy $E = A\varepsilon_N$

$$
\begin{aligned}
e_{\rm F} &= \frac{d}{d\varrho}\left(\frac{E}{\Omega}\right), \\
e_{\rm F} &= \varepsilon_N + \varrho\frac{d\varepsilon_N}{d\varrho} = \varepsilon_N + p/\varrho\,,
\end{aligned}
\tag{5}
$$

where $A/\varrho = \Omega$ gives the volume. At the saturation point $e_{\rm F} = \varepsilon_A$. But for positive pressure p

$$
\int dy\, y\rho^A(y) = \frac{e_{\rm F}}{\varepsilon_N} < 1\,. \tag{6}
$$

For positive pressure the average distances between nucleons are smaller and the pion effective cross section is strongly reduced at high nuclear densities above the threshold in $N+N = N+N+\pi$ reaction calculated in Dirac–Brueckner approach [8] (also with RPA insertions to self energy of N and Δ [9] included). Therefore, for positive pressure, not very far from the saturation density, the nuclear pions carry less then 1% [7] of the nuclear longitudinal momentum and dealing with a non-equilibrium correction to the nuclear distribution (2) we will restrict considerations to the nucleon part. In our approach, we consider the change of the nucleon mass with the change of the parton distribution (nucleon SF) above the saturation point. The increasing pressure between nucleons starts to increase the $e_{\rm F}$ (5) and consequently the sum rule (4) is broken by the factor $e_{\rm F}/\varepsilon_N > 1$. To compensate this factor, which increases the longitudinal momentum of nuclear partons, the nucleon SF in the nuclear medium has to be changed. For good estimate, in order to proceed without new parameters, assume that the changes of SF will be included through the changes of Bjorken x in the medium. Multiplying the argument of the SF by a factor $e_{\rm F}/\varepsilon_N$ the SF will be squeezed towards smaller x and the total fraction of longitudinal momentum will be smaller by a factor $\varepsilon_N/e_\mathrm{F}$

$$\int\limits_0^1 F_2^N\left(\frac{e_\mathrm{F}}{\varepsilon_N}x_N\right)dx_N = \frac{\varepsilon_N}{e_\mathrm{F}}\int\limits_0^{\frac{e_\mathrm{F}}{\varepsilon_N}} F_2^N(x)\,dx \cong \frac{\varepsilon_N}{e_\mathrm{F}}\int\limits_0^1 F_2^N(x)\,dx\,. \tag{7}$$

Here in the integral we neglect the small contributions from $x > 1$ region originated from NN correlations. Now, with the help of Eq. (4) and Eq. (7) the nuclear MSR

$$\int\limits_0^A F_2^A(x)\,dx = \int\limits_0^1 F_2^N(x)\,dx \tag{8}$$

is satisfied if quarks in the nucleus carry the same fraction of longitudinal momentum as in bare nucleons. On the other hand, the integral (7) corresponds to the total sum of the quark longitudinal momenta $p_q^+ = p_q^0 + p_q^3$ inside a nucleon, which is proportional to a total nucleon rest energy or the nucleon mass. Consequently, the nucleon mass M_N will be changed for $\varrho \geq \varrho_0$ to the mass M_med by the gradually decreasing factor $\varepsilon_N/e_\mathrm{F}$

$$M_\mathrm{med} = \frac{\varepsilon_N}{e_\mathrm{F}}M_N = \frac{M_N}{1+\frac{\varrho}{\varepsilon_N}\frac{d\varepsilon_N}{d\varrho}} \simeq M_N\left(1-\frac{p}{\varrho\varepsilon_N}\right), \tag{9}$$

which decreases as the pressure increases. This explicit mass dependence on the energy ε_N and energy derivative (9) is plugged into the following standard Walecka RMF equations [10] for nucleon energy ε_N and effective mass M^*

$$\begin{aligned}\varepsilon_N &= C_1^2\rho + \frac{C_2^2}{\rho}(M_\mathrm{med} - M^*)^2 + \frac{\gamma}{(2\pi)^3\rho}\int d^3p\sqrt{(p^2+M^{*2})}\,,\\ M^* &= M_\mathrm{med} - \frac{\gamma}{2C_2^2(2\pi)^3}\int d^3p\frac{M^*}{\sqrt{(p^2+M^{*2})}}\,,\end{aligned} \tag{10}$$

where γ denotes the level degeneracy ($\gamma = 2$ for neutron matter) and the two coupling constants: vector C_v^2 and scalar C_s^2, were fitted [10] at the saturation point of nuclear matter (in the formula $2C_1^2 = C_\mathrm{v}^2/M_N^2$ and $2C_2^2 = M_N^2/C_\mathrm{s}^2$).

In the Walecka model $M_\mathrm{med} = M_N$. In our model the finite pressure corrections to M_med (9) convert the recursive equation (10) to a differential equation above the saturation density ρ_0 in the general form

$$f\left(\varepsilon_N, \frac{d}{d\varrho}(\varepsilon_N)\right) = 0 \quad \text{for} \quad \rho \geq \rho_0\,. \tag{11}$$

The Equation of State (EOS) for NM has to match the saturation point with compressibility $K^{-1} = 9\varrho^2 \frac{d^2}{d\varrho^2}\frac{E}{A}$ but then the behavior for higher densities is different for different RMF models. We compare here the stiff Walecka model [10] with our corrected version [12] and with the model assuming the non-relativistic expansion in powers of Fermi momentum [13]. These two models and parametrization in Fermi momentum have two free parameters which are fixed by the empirical binding energy 15.7 MeV at saturation density $k_{\mathrm{F}} = 1.4\,\mathrm{fm}$. The EoS plots are displayed in Fig. 1.

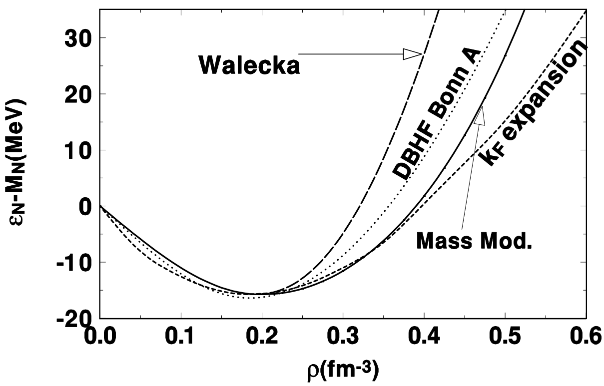

Fig. 1. The nucleon energy $\varepsilon_N - M_N$ as a function of NM density for RMF models; scalar–vector Walecka (long dashed line above the saturation point), our Modified Mass approach (solid) with ($\rho_0 = 0.19\,\mathrm{fm}^{-3}$) [10]. Results for full DBHF [11] (dotted marked line) calculation using the Bonn A NN interaction are displayed for comparison. The expansion in Fermi momentum of a single nucleon energy in NM is also shown (short dashed line).

3. Results and conclusion

We know that NM compressibility in the standard Walecka model is too large ($K^{-1} \simeq 560\,\mathrm{MeV}$). Our model with the nucleon mass modification makes the EoS significantly softer, close to the non-relativistic parametrization which fitted to the binding energy $-15.7\,\mathrm{MeV}$ of NM for $k_{\mathrm{F}} = 1.4\,\mathrm{fm}$ gives the proper value $K^{-1} = 240\,\mathrm{MeV}$. A similar dependence on density appears for the energy calculated within the DBHF method with the realistic Bonn A potential [11]. The right value of compressibility is then obtained due to additional parameters of NN interaction.

Our significant improvement of the simple scalar–vector RMF model of Walecka without additional parameters concerns the fulfillment of the parton MSR with the corrected value of nuclear stiffness. It, therefore, suggests similar modifications of a nucleon mass above the saturation density in any RMF model with a constant nucleon mass and unmodified parton SF. Such a correction is absent in the non-relativistic mean field models where the flux factor [3] in the nuclear distribution is usually not taken into account [15] and consequently the factor $e_{\mathrm{F}} = \varepsilon_N$ is not present in Eq. (7). It is related to a different off-shell behavior of effective interactions in relativistic when compared to non-relativistic mean field approaches. We note that the stiffness of EoS is very important in the context of studies compact stars as recently discussed in [16].

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