MOTT–HAGEDORN RESONANCE GAS AND LATTICE QCD RESULTS*

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A combined effective model reproducing the equation of state of hadronic matter as obtained in recent lattice QCD simulations is presented.

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1. Introduction

We construct a combined effective model reproducing the equation of state of hadronic matter as obtained in recent lattice QCD simulations [1,2]. The model should reproduce basic physical characteristics of processes encountered in the dense hadronic matter, from the hot QCD phase through the critical temperature region till the lower temperature hadron resonance gas phase. It has been shown in [3] that the equation of state derived from that time QCD lattice calculation [4] can be reproduced by a simple hadron gas resonance model below critical temperature $T_{\rm c}$. For higher temperatures the model is modified by introducing finite widths of heavy hadrons [5] with a heuristic ansatz for the spectral function which reflects medium modifications of hadrons. This Mott–Hagedorn type model is constructed to fit nicely the lattice data, also above $T_{\rm c}$ where it does so because it leaves light hadrons below a mass threshold of $m_0 = 1\,{\rm GeV}$ unaffected. The description of the lattice data at high temperatures is accidental because the effective number of those degrees of freedom approximately coincides with that of

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quarks and gluons. In order to remove this unphysical aspect of the otherwise appealing model, one has to extend the spectral broadening also to the light hadrons and thus describe their disappearance due to the Mott effect while simultaneously the quark and gluon degrees of freedom appear at high temperatures due to chiral symmetry restoration and deconfinement. In the present contribution, we will report first results obtained by introducing a unified treatment of all hadronic resonances with a state-dependent width $\Gamma_i(T)$ in accordance with the inverse hadronic collision time scale from a recent model for chemical freeze-out in a resonance gas [6]. The appearance of quark and gluon degrees of freedom is introduced by the Polyakov-loop improved Nambu–Jona-Lasinio (PNJL) model [7, 8]. The model is further refined by adding perturbative corrections to $\mathcal{O}(\alpha_{\rm s})$ for the high-momentum region above the three-momentum cutoff inherent in the PNJL model. One obtains eventually a good agreement with lattice QCD data, comparable with all important physical characteristics taken into account.

2. Extended Mott–Hagedorn resonance gas

We introduce the width Γ of a resonance in the statistical model through the spectral function

$$A(M,m) = N_m \frac{\Gamma\, m}{(M^2-m^2)^2+\Gamma^2\, m^2}\,, \tag{1}$$

where N_m is the standard normalization factor. The model ansatz for the resonance width Γ is given by [5]

$$\Gamma(T) = C_\Gamma \left(\frac{m}{T_H}\right)^{N_m} \left(\frac{T}{T_H}\right)^{N_T} \exp\left(\frac{m}{T_H}\right)\,, \tag{2}$$

where $C_\Gamma = 10^{-4}$, $N_m = 2.5$, $N_T = 6.5$ and $T_H = 165\,\mathrm{MeV}$. For simplicity, we assume $n_S = 0$ for the strangeness number density and $n_B = 0$ for the baryon number density. Then the respective chemical potentials $\mu_B = 0$ and $\mu_S = 0$ always, so that the temperature is the only significant statistical parameter here.

The energy density of this model can then be cast in the form

$$\varepsilon(T) = \sum_{m_i<m_0} g_i\, \varepsilon_i(T;m_i) + \sum_{m_i\ge m_0} g_i \int_{m_0^2}^{\infty} d\left(M^2\right) A(M,m_i)\, \varepsilon_i(T;M)\,, \tag{3}$$

where $m_0 = 1\,\mathrm{GeV}$ and the energy density per degree of freedom with a mass M is

$$\varepsilon_i(T;M) = \int \frac{d^3k}{(2\pi)^3} \frac{\sqrt{k^2+M^2}}{\exp\left(\frac{\sqrt{k^2+M^2}}{T}\right)+\delta_i} \tag{4}$$

with the degeneracy g_i. For mesons, $\delta_i = -1$ and for baryons $\delta_i = 1$. For our hadronic gas kept at fixed volume we have the relation

$$\varepsilon = T\,s - P = T\frac{\partial P}{\partial T} - P\,, \tag{5}$$

where $P = P(T)$ and $s = s(T)$ are the pressure and entropy density.

In the left panel of Fig. 1, the results for the pressure and energy density of the model at this stage are shown. Although providing us with an excellent fit of the lattice data, the high-temperature phase of this model is unphysical. Imposing that all mesons lighter than $m_0 = 1\,\mathrm{GeV}$ are stable provides us with a SB limit at high temperatures which mimics that due to quarks and gluons in the case for three flavors [9]. In reality, due to the chiral phase transition at T_c, the quarks loose their mass and, therefore, the threshold of the continuum of quark–antiquark scattering states is lowered. The light meson masses, however, remain almost unaffected by the increase in the temperature of the system. Consequently, they merge the continuum and become unbound — their spectral function changes from a delta-function (on-shell bound states) to a Breit–Wigner-type (off-shell, resonant scattering states).

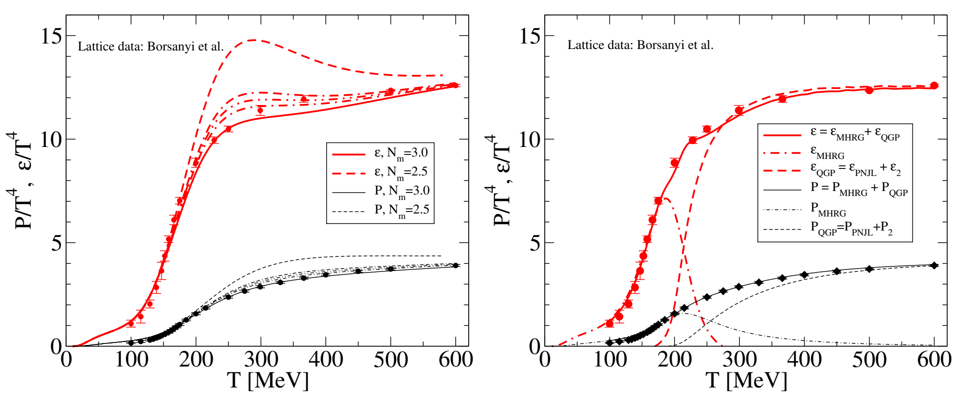

Fig. 1. Left: Thermodynamic quantities for the old Mott–Hagedorn Resonance Gas model [5]. Different line styles correspond to different values for the parameter N_m in the range from $N_m = 2.5$ (dashed line) to $N_m = 3.0$ (solid line). Lattice QCD data are from Ref. [1]. Right: Thermodynamic quantities for the new Mott–Hagedorn Resonance Gas, where quark-gluon plasma contributions are described within the PNJL model including α_s corrections (dashed lines). Hadronic resonances are described within the resonance gas with finite width, as an implementation of the Mott effect (dash-dotted line). The sum of both contributions (solid lines) is shown for the energy density (thick lines) and pressure (thin lines) in comparison with the lattice data from [1].

This phenomenon is the hadronic analogue [10] of the Mott–Anderson transition for electrons in solid state physics (insulator-conductor transition). It has been first introduced for the hadronic-to-quark-matter transition in [11]. Later, within the NJL model, a microscopic approach to the thermodynamics of the Mott dissociation of mesons in quark matter has been given in the form of a generalized Beth–Uhlenbeck equation of state [12], see also [13].

As a microscopic treatment of the Mott effect for all resonances is presently out of reach, we introduce an ansatz for a state-dependent hadron resonance width $\Gamma_i(T)$ given by the inverse collision time scale recently suggested within an approach to the chemical freeze-out and chiral condensate in a resonance gas [6]

$$\Gamma_i(T) = \tau^{-1}_{\mathrm{coll},i}(T) = \sum_j \lambda \left\langle r_i^2 \right\rangle_T \left\langle r_j^2 \right\rangle_T \; n_j(T)\,, \tag{6}$$

which is based on a binary collision approximation and relaxation time ansatz using for the in-medium hadron–hadron cross sections the geometrical Povh–Hüfner law [14]. In Eq. (6) the coefficient λ is a free parameter, $n_j(T)$ is the partial density of the hadron j and the mean squared radii of hadrons $\langle r_i^2 \rangle_T$ obtain in the medium a temperature dependence which is governed by the (partial) restoration of chiral symmetry. For the pion this was quantitatively studied within the NJL model [15] and it was shown that close to the Mott transition the pion radius is well approximated by

$$r_\pi^2(T) = \frac{3}{4\pi^2} f_\pi^{-2}(T) = \frac{3M_\pi^2}{4\pi^2 m_q} |\langle \bar{q}q \rangle_T|^{-1}\,. \tag{7}$$

Here the Gell-Mann–Oakes–Renner relation has been used and the pion mass shall be assumed chirally protected and thus temperature independent.

For the nucleon, we shall assume the radius to consist of two components, a medium independent hard core radius r_0 and a pion cloud contribution $r_N^2(T) = r_0^2 + r_\pi^2(T)$, where from the vacuum values $r_\pi = 0.59\,\mathrm{fm}$ and $r_N = 0.74\,\mathrm{fm}$ one gets $r_0 = 0.45\,\mathrm{fm}$. A key point of our approach is that the temperature dependent hadronic radii shall diverge when hadron dissociation (Mott effect) sets in, driven basically by the restoration of chiral symmetry. As a consequence, in the vicinity of the chiral restoration temperature all meson radii shall behave like that of the pion and all baryon radii like that of the nucleon. The resulting energy density and pressure behavior is shown in the right panel of Fig. 1. This part of the model we call Mott–Hagedorn-Resonance-Gas (MHRG). When all hadrons are gone at $T \sim 250\,\mathrm{MeV}$, we are clearly missing degrees of freedom!

3. Quarks, gluons and hadron resonances

We improve the PNJL model over its standard versions [7, 8] by adding perturbative corrections in $\mathcal{O}(\alpha_{\rm s})$ for the high-momentum region above the three-momentum cutoff Λ. In the second step, the MHRG part is replaced by its final form, using the state-dependent spectral function for the description of the Mott dissociation of all hadron resonances above the chiral transition. The total pressure obtains the form

$$P(T) = P_{\rm MHRG}(T) + P_{\rm PNJL,MF}(T) + P_2(T)\,, \tag{8}$$

where $P_{\rm MHRG}(T)$ stands for the pressure of the MHRG model, accounting for the dissociation of hadrons in hot dense, matter.

The $\mathcal{O}(\alpha_{\rm s})$ corrections can be split in quark and gluon contributions

$$P_2(T) = P_2^{\rm quark}(T) + P_2^{\rm gluon}(T)\,, \tag{9}$$

where $P_2^{\rm quark}$ stands for the quark contribution and $P_2^{\rm gluon}$ contains the ghost and gluon contributions. The total perturbative QCD correction to $\mathcal{O}(\alpha_{\rm s})$ is

$$P_2 = -\frac{8}{\pi}\alpha_{\rm s}T^4\left(I_\Lambda^+ + \frac{3}{\pi^2}\left(\left(I_\Lambda^+\right)^2 + \left(I_\Lambda^-\right)^2\right)\right)\,, \tag{10}$$

where $I_\Lambda^\pm = \int_{\Lambda/T}^\infty \frac{dx\;x}{e^x \pm 1}$. The corresponding contribution to the energy density is given in standard way by Eq. (5).

We will now include an effective description of the dissociation of hadrons due to the Mott effect into the hadron resonance gas model by including the state dependent hadron resonance width (6) into the definition of the HRG pressure

$$P_{\rm MHRG}(T) = \sum_i \delta_i d_i \int \frac{d^3p}{(2\pi)^3} dM\; A_i(M) T \ln\left(1 + \delta_i e^{-\sqrt{p^2+M^2}/T}\right)\,. \tag{11}$$

From the pressure as a thermodynamic potential all relevant thermodynamical functions can be obtained. Combining the $\alpha_{\rm s}$ corrected mean field PNJL model for the quark–gluon subsystem with the MHRG description of the hadronic resonances we obtain the results shown in the right panel of Fig. 1, where the resulting partial contributions in comparison with lattice QCD data from Ref. [1] are shown.

We see that the lattice QCD thermodynamics is in full accordance with a hadron resonance gas up to a temperature of ~ 170 MeV which corresponds to the pseudocritical temperature of the chiral phase transition. The lattice data saturate below the Stefan–Boltzmann limit of an ideal quark-gluon gas at high temperatures. The PNJL model, however, attains this limit by construction. The deviation is to good accuracy described by perturbative corrections to $\mathcal{O}(\alpha_s)$ which vanish at low temperatures due to an infrared cutoff procedure. The transition region $170 \leq T[\mathrm{MeV}] \leq 250$ is described by the MHRG model, resulting in a decreasing HRG pressure which vanishes at $T \sim 250\,\mathrm{MeV}$.

We present two stages of an effective model description of QCD thermodynamics at finite temperatures which properly accounts for the fact that in the QCD transition region it is dominated by a tower of hadronic resonances. To this end we further develop a generalization of the Hagedorn resonance gas thermodynamics which includes the finite lifetime of hadronic resonances in a hot and dense medium by a model ansatz for a temperature — and mass dependent spectral function.

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