THE QCD CRITICAL END POINT IN THE CONTEXT OF THE POLYAKOV–NAMBU–JONA-LASINIO MODEL*

P. Costa, C.A. de Sousa, M.C. Ruivo

Centro de Física Computacional, Departamento de Física
Universidade de Coimbra, 3004-516 Coimbra, Portugal

H. Hansen

IPNL, Université de Lyon/Université Lyon 1, CNRS/IN2P3
4 rue E. Fermi, 69622 Villeurbanne Cedex, France

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We investigate the phase diagram of the so-called Polyakov–Nambu–Jona-Lasinio model at finite temperature and nonzero chemical potential with three quark flavors. Chiral and deconfinement phase transitions are discussed, and the relevant order parameters are analyzed. A special attention is payed to the critical end point (CEP): the influence of the strangeness on the location of the CEP is studied; also the strength of the flavor-mixing interaction alters the CEP location, since when it becomes weaker the CEP moves to low temperatures and can even disappear.

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Understanding the QCD phase structure is one of the most important topics in the physics of strong interactions. The developed effort on both the theoretical point of view (by using effective models and lattice calculations) and the experimental point of view (it is one of the main goals of the heavy ion collisions program) has proved very fruitful, shedding light on properties of matter at finite temperature and density.

Confinement and chiral symmetry breaking are two of the most important features of QCD which is the fundamental theory of strong interactions. Its basic constituents are quarks and gluons that are confined in hadronic

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matter. At high temperatures and densities hadronic matter should undergo a phase transition into the quark-gluon plasma (QGP). A challenge of theoretical studies based on QCD is to predict the equation of state, the critical end point and the nature of the phase transition.

The fundamental theory of QCD is known to be highly difficult for the analytical analysis because of the strong coupling regime. Meanwhile, as this regime triggers the most interesting phenomena in QCD, it has become a common practice to replace the low-energy QCD by some effective models. Examples of such descriptions include NJL type models which have been developed providing guidance and information relevant to observable experimental signs of deconfinement and QGP features.

NJL type models, that take into account only quark degrees of freedom, give the correct chiral properties; static gluonic degrees of freedom are then introduced in the NJL Lagrangian through an effective gluon potential in terms of Polyakov loop Φ with the aim of including features of both chiral symmetry breaking and deconfinement.

Our calculations are performed in the framework of an extended SU(3) PNJL Lagrangian, which includes the 't Hooft instanton induced interaction term that breaks the $\mathrm{U}_A(1)$ symmetry, and the quarks are coupled to the (spatially constant) temporal background gauge field A_μ [1,2]

$$\begin{aligned}\mathcal{L} \;=\; & \bar{q}\,(i\gamma^\mu D_\mu - \hat{m})\,q + \tfrac{1}{2}\,g_S \sum_{a=0}^{8}\left[(\bar{q}\,\lambda^a\,q)^2 + (\bar{q}\,i\,\gamma_5\,\lambda^a\,q)^2\right] \\ & + g_D\,\{\det\,[\bar{q}\,(1+\gamma_5)\,q] + \det\,[\bar{q}\,(1-\gamma_5)\,q]\} - \mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right)\,. \end{aligned} \tag{1}$$

The covariant derivative is defined as $D^\mu = \partial^\mu - iA^\mu$, with $A^\mu = \delta^\mu_0 A_0$ (Polyakov gauge); in Euclidean notation $A_0 = -iA_4$. The strong coupling constant g is absorbed in the definition of $A^\mu(x) = g\mathcal{A}^\mu_a(x)\frac{\lambda_a}{2}$, where $\mathcal{A}^\mu_a$ is the $(\mathrm{SU}(3)_c)$ gauge field and λ_a are the (color) Gell-Mann matrices.

The effective potential for the (complex) field Φ adopted in our parametrization of the PNJL model reads

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{a\,(T)}{2}\bar{\Phi}\Phi + b(T)\ln\left[1-6\bar{\Phi}\Phi + 4\left(\bar{\Phi}^3+\Phi^3\right) - 3(\bar{\Phi}\Phi)^2\right]\,, \tag{2}$$

where

$$a\,(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 \;\;\text{and}\;\; b(T) = b_3\left(\frac{T_0}{T}\right)^3\,. \tag{3}$$

The parameters of the effective potential $\mathcal{U}$ are given by $a_0 = 3.51$, $a_1 = -2.47$, $a_2 = 15.2$ and $b_3 = -1.75$. These parameters have been fixed in order to reproduce the lattice data for the expectation value of the Polyakov loop and QCD thermodynamics in the pure gauge sector. When quarks are added, the parameter T_0, the critical temperature for the deconfinement phase transition (that manifests itself as a breaking of the center symmetry) within a pure gauge approach, was fixed to 270 MeV, according to lattice findings. This choice ensures an almost exact coincidence between chiral crossover and deconfinement at zero chemical potential, as observed in lattice calculations.

The parameters of the NJL sector are: $m_u = m_d = 5.5\,\text{MeV}$, $m_s = 140.7\,\text{MeV}$, $g_S \Lambda^2 = 3.67$, $g_D \Lambda^5 = -12.36$ and $\Lambda = 602.3\,\text{MeV}$, which are fixed to reproduce the values of the coupling constant of the pion, $f_\pi = 92.4\,\text{MeV}$, and the masses of the pion, the kaon, the η and η', respectively, $M_\pi = 135\,\text{MeV}$, $M_K = 497.7\,\text{MeV}$, $M_\eta = 514.8\,\text{MeV}$ and $M_{\eta'} = 960.8\,\text{MeV}$.

The inclusion of the Polyakov loop effective potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$, that can be seen as an effective pressure term mimicking the gluonic degrees of freedom of QCD, is required to get the correct limit. Indeed, in the NJL model the ideal gas limit is far to be reached due to the lack of gluonic degrees of freedom.

The location and even the existence of the CEP in the phase diagram is a matter of debate. While different lattice calculations predict the existence of a CEP [3], the absence of the CEP in the phase diagram is still a possibility as was seen in some lattice QCD results [4], where the first order phase transition region near $\mu = 0$ shrinks in the quark mass in μ space when μ is increased [4].

In figure 1 the phase diagram in the PNJL model is presented. As the temperature increases the chiral transition is a first order one and persists up to the CEP. At the CEP the chiral transition becomes a second order one. The location of the CEP is found at $T^{\text{CEP}} = 155.80\,\text{MeV}$ and $\mu^{\text{CEP}} = 290.67\,\text{MeV}$ ($\rho_B^{\text{CEP}} = 1.87\rho_0$). For temperatures above the CEP, there is a crossover whose location is calculated making use of $\partial^2 \langle \bar{q}q \rangle / \partial T^2 = 0$, *i.e.* the inflection point of the quark condensate $\langle \bar{q}q \rangle$.

The transition to the deconfinement is given by $\partial^2 \Phi / \partial T^2 = 0$, and is represented by the gray (magenta) line. The surrounding shaded area that limits the region, where the crossover takes place is determined as the inflection point of the susceptibility $\partial \Phi / \partial T$.

Due to the importance of the location of the CEP from the experimental point of view, let us investigate the influence of several parameters which can lead to a significant change in the CEP's localization. We study the influence of strangeness on the location of the CEP (or tricritical point (TCP) in the chiral limit).

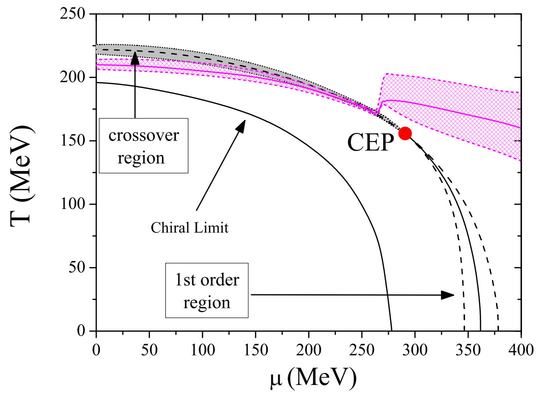

Fig. 1. The phase diagram in the SU(3) PNJL model: the location of the CEP is found at $T^{\mathrm{CEP}} = 155.80$ MeV and $\mu^{\mathrm{CEP}} = 290.67$ MeV (see details in the text).

A first point to be noticed is that in the PNJL model, when the full chiral limit is considered ($m_u = m_d = m_s = 0$), the phase diagram does not exhibit a TCP: chiral symmetry is restored via a first order transition for all baryonic chemical potentials and temperatures (see left panel of figure 2). On the contrary, in the chiral limit only for the SU(2) (light) sector ($m_u = m_d = 0$, $m_s \neq 0$) a TCP can be found [5]. Both situations are in agreement with what is expected: the chiral phase transition at the chiral limit is a second order for $N_f = 2$ and first order one for $N_f \geq 3$ [6].

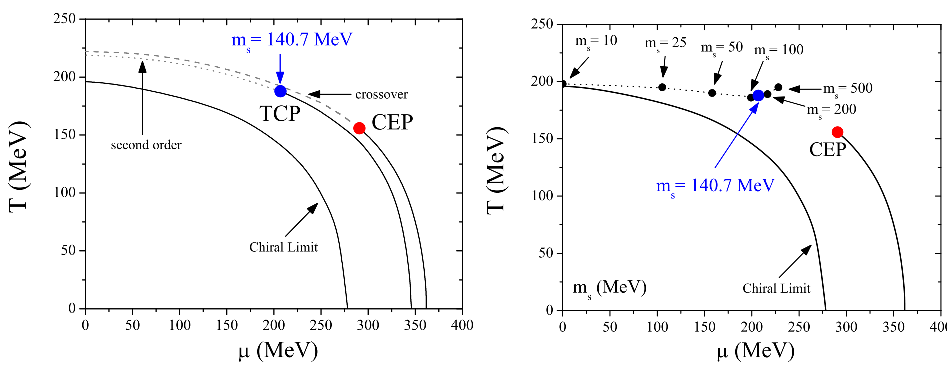

Fig. 2. Left panel: the phase diagram in the SU(3) PNJL model. The solid lines represent the first order phase transition, the dotted line the second order phase transition, and the dashed line the crossover transition. Right panel: the phase diagram and the "line" of TCPs for different values of m_s (the dotted lines are just drawn to guide the eye); the TCPs in both figures are obtained in the limit $m_u = m_d = 0$ and $m_s \neq 0$.

To study the influence of strangeness on the location of the critical points, we vary the current quark mass m_s, keeping the $\mathrm{SU}(2)_\mathrm{f}$ sector in the chiral limit and the other model parameters fixed. In figure 2 (right panel), we plot the phase diagram of the model in the (T, μ) plane, for various values of the the current quark mass m_s.

The pattern of chiral symmetry restoration via first order phase transition remains for $m_u = m_d = 0$ and $m_s < m_s^{\rm crit}$. The value for $m_s^{\rm crit}$ is a subject of debate; we found $m_s^{\rm crit} \approx 9\,\mathrm{MeV}$ in our model, lower than lattice values [7] and half of the value obtained in NJL model ($m_s^{\rm crit} = 18.3\,\mathrm{MeV}$ [5]). When $m_s \geq m_s^{\rm crit}$, at $\mu = 0$, the transition is a second order one and, as μ increases, the line of the second order phase transition will end in a first order line at the TCP. Several TCPs are plotted for different values of m_s in the right panel of figure 2. As m_s increases, the value of T for this "line" of TCPs decreases as μ increases getting closer to the CEP and, when $m_s = 140.7\,\mathrm{MeV}$, it starts to move away from the CEP. The TCP for $m_s = 140.7\,\mathrm{MeV}$ is the closest to the CEP and is located at $\mu^{\rm TCP} = 206.95\,\mathrm{MeV}$ and $T^{\rm TCP} = 187.83\,\mathrm{MeV}$. If we choose $m_u = m_d \neq 0$, instead of a second order transition we have a smooth crossover for all the values of m_s and the "line" of TCPs becomes a "line" of CEPs.

Also the change of the $\mathrm{U}_A(1)$ anomaly strength has a strong influence on the localization of the CEP in the (T, μ) plane. In figure 3, we show the location of the CEP for several values of g_D compared to the results for g_{D_0}, the value used for the vacuum. As already pointed out by Fukushima in [1], we also observe that the location of the CEP depends on the value of g_D. In fact, our results show that the existence or not of the CEP is determined by the strength of the anomaly coupling, the CEP getting closer to the μ axis as g_D decreases.

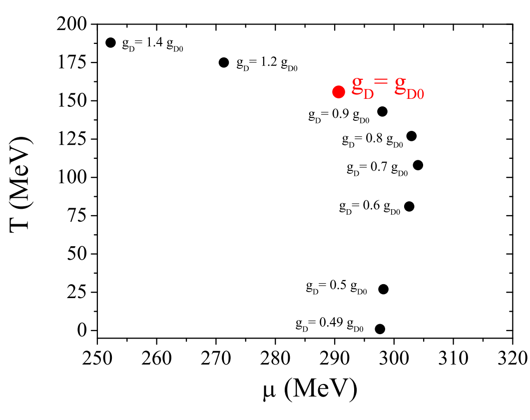

Fig. 3. Dependence of the location of the CEP on the strength of the 't Hooft coupling constant g_D.

As a conclusion, we investigated the phase diagram of the so-called PNJL model at finite temperature and nonzero chemical potential with three quark flavors. Chiral and deconfinement phase transitions are discussed, and the relevant order parameters are analyzed.

The chiral phase transition is a first order one in the chiral limit. Working out of the chiral limit, at which both chiral and center symmetries are explicitly broken, a CEP which separates first and crossover lines is found, and the corresponding order parameters are analyzed.

A special attention is payed to the critical end point. We studied the influence of strangeness on the location of the critical point varying the current quark mass m_s, keeping the $\mathrm{SU}(2)_\mathrm{f}$ sector in the chiral limit and the other model parameters fixed. When $m_s \geq m_s^{\mathrm{crit}}$, at $\mu = 0$, the transition is of the second order one and, as μ increases, the line of the second order phase transition will end in a first order line at the TCP. If we choose $m_u = m_d \neq 0$, instead of a second order transition we have a smooth crossover for all the values of m_s and the TCP becomes a CEP which depends strongly on the the value of m_s.

We also analyzed the effect of the anomalous coupling strength on the location of the CEP. We showed that the location of the CEP depends on the value of g_D and, as this strength of the flavor-mixing interaction becomes weaker, the CEP moves to low temperatures and can even disappear.

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