FLUCTUATIONS AND CORRELATIONS IN POLYAKOV LOOP EXTENDED CHIRAL FLUID DYNAMICS*

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We study nonequilibrium effects at the QCD phase transition within the framework of Polyakov loop extended chiral fluid dynamics. The quark degrees of freedom act as a locally equilibrated heat bath for the sigma field and a dynamical Polyakov loop. Their evolution is described by a Langevin equation with dissipation and noise. At a critical point we observe the formation of long-range correlations after equilibration. During a hydrodynamical expansion nonequilibrium fluctuations are enhanced at the first order phase transition compared to the critical point.

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1. Introduction

Presently the knowledge of the QCD phase diagram is still limited. While lattice QCD calculations tell us that at vanishing baryochemical potential there is a crossover [1], only model studies allow to explore the high baryon density regions. There one expects a first order transition at large μ ending at a critical point (CP) [2]. The CP is expected to be detected in heavy-ion collisions through event-by-event fluctuations of quantities like transverse

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momentum or particle multiplicity [3,4]. Nevertheless, phenomena like critical slowing down as well as the finite system lifetime and size prevent the correlation length from diverging [5] which thus weakens these signals. It is therefore important to include nonequilibrium effects and the dynamics of the system to estimate the experimental signatures of the critical end point. A promising ansatz to study the QCD phase transition in such a setting is provided by the framework of chiral fluid dynamics [6,7,8]. Here, the basic idea is to propagate the order parameter of chiral symmetry explicitly by a Langevin equation, while the heat bath is given by a fluid dynamically expanding medium made out of quarks. Presently, we extend this model with the Polyakov loop to consider both the chiral and the deconfinement transition. Here, the Polyakov loop is treated as an effective field which is propagated by a phenomenological Langevin equation. We present results of temperature quenches in a box and study the evolution of fluctuations during the expansion of a hot plasma droplet.

2. Chiral fluid dynamics with a Polyakov loop

For investigation we use the Polyakov loop extended quark meson model [9] with the Lagrangian

$$\mathcal{L} = \bar{q}\left[i\left(\gamma^\mu\partial_\mu - ig_{\mathrm{QCD}}\gamma^0 A_0\right) - g\sigma\right]q + \tfrac{1}{2}\left(\partial_\mu\sigma\right)^2 - U\left(\sigma\right) - \mathcal{U}\left(\ell,\bar{\ell}\right)\,, \quad (1)$$

where $q=(u,d)$ is the constituent quark field, A_0 the temporal component of the color gauge field and σ the mesonic field. The pion degrees of freedom are neglected in the present study. The potential for the sigma field reads

$$U\left(\sigma\right) = \frac{\lambda^2}{4}\left(\sigma^2-\nu^2\right)^2 - h_q\sigma - U_0\,. \quad (2)$$

The temperature dependent Polyakov loop potential is chosen in a polynomial form following [9,10]

$$\frac{\mathcal{U}}{T^4}\left(\ell,\bar{\ell}\right) = -\frac{b_2(T)}{4}\left(|\ell|^2+|\bar{\ell}|^2\right) - \frac{b_3}{6}\left(\ell^3+\bar{\ell}^3\right) + \frac{b_4}{16}\left(|\ell|^2+|\bar{\ell}|^2\right)^2\,. \quad (3)$$

Integrating out the quark degrees of freedom, which will constitute the heat bath, we obtain the grand canonical potential. At $\mu_B=0$, $\ell=\bar{\ell}$ and in the mean-field approximation it is [9]

$$\Omega_{\bar{q}q} = -4N_f T\int\frac{d^3p}{(2\pi)^3}\ln\left[1+3\ell e^{-\beta E}+3\ell e^{-2\beta E}+e^{-3\beta E}\right]\,. \quad (4)$$

For different coupling strengths g the effective potential $V_{\mathrm{eff}} = U + \mathcal{U} + \Omega_{\bar{q}q}$ shows different characteristic shapes at the transition temperature: for

$g = 4.7$ one obtains two degenerate minima, see Fig. 1 (left), while for $g = 3.52$ the potential is very broad and flat at the minimum, Fig. 1 (right). This resembles the situation at a critical point. These choices of the quark–meson coupling allow a first qualitative study of effects at several types of transition[1].

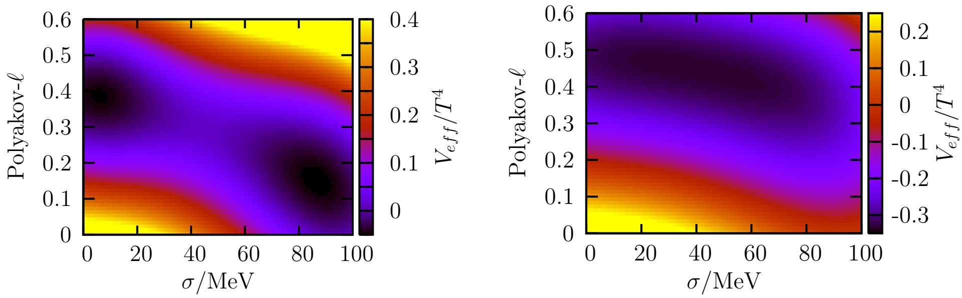

Fig. 1. Left: Effective potential at $g = 4.7$, corresponding to a first order phase transition at $T_{\mathrm{c}} = 172.9$ MeV. Right: Effective potential at $g = 3.52$, corresponding to a critical point and $T_{\mathrm{c}} = 180.5$ MeV.

In [8] the coupled dynamics for the sigma field and the quark fluid were derived self-consistently. The sigma field is propagated by a Langevin equation

$$\partial_\mu \partial^\mu \sigma + \eta_\sigma \partial_t \sigma + \frac{\partial V_{\mathrm{eff}}}{\partial \sigma} = \xi_\sigma \,. \tag{5}$$

The explicit form of the temperature dependent damping coefficient η_σ together with the correlator for the stochastic noise field have been derived in [8].

We allow for a dynamical evolution of the Polyakov loop by adding a kinetic term in the equation of motion [11]. For the full nonequilibrium description we also need to add a damping term $\eta_\ell \sim 1/\mathrm{fm}$ [12] and impose the dissipation-fluctuation relation

$$\frac{2N_{\mathrm{c}}}{g^2_{\mathrm{QCD}}} \partial_\mu \partial^\mu \ell T^2 + \eta_\ell \partial_t \ell + \frac{\partial V_{\mathrm{eff}}}{\partial \ell} = \xi_\ell \,, \tag{6}$$

$$\left\langle \xi_\ell(t) \xi_\ell\left(t'\right) \right\rangle = \frac{1}{V} \delta\left(t - t'\right) 2\eta_\ell T \,. \tag{7}$$

Note at this point that the Polyakov loop is originally defined only in equilibrium and it is not *a priori* clear what the correct dynamics are [10]. This approach is, therefore, purely phenomenological.

[1] Note that in principle one has to choose g such that $g\sigma$ reproduces the constituent quark mass in vacuum. This would give a value of $g \sim 3.2$.

The quark fluid is propagated via the equations of ideal relativistic fluid dynamic using the energy-momentum tensors of the liquid, the σ-field and the Polyakov-loop

$$\partial_\mu \left(T_q^{\mu\nu} + T_\sigma^{\mu\nu} + T_\ell^{\mu\nu}\right) = 0\,. \tag{8}$$

3. Numerical results

3.1. Temperature quench in a box

We put fields and fluid in a box with periodic boundary conditions. The fields are initialized at a temperature above T_c. Then the temperature is quenched to a value below the transition point when the quark bath is added. The fields lose energy through damping, transferring this amount of energy to the fluid via equation (8) which leads to a subsequent increase of the temperature, see also [13]. We find quench temperatures such that the system relaxes near the transition temperature. In the Polyakov loop, we can observe an interesting phenomenon. Figure 2 shows the value of ℓ in x-direction $y = z = 0$ evolving in time for both transition scenarios during the relaxation process. At the critical point, one observes the formation of long-range fluctuations over both space and time, an effect that does not occur at the first order transition.

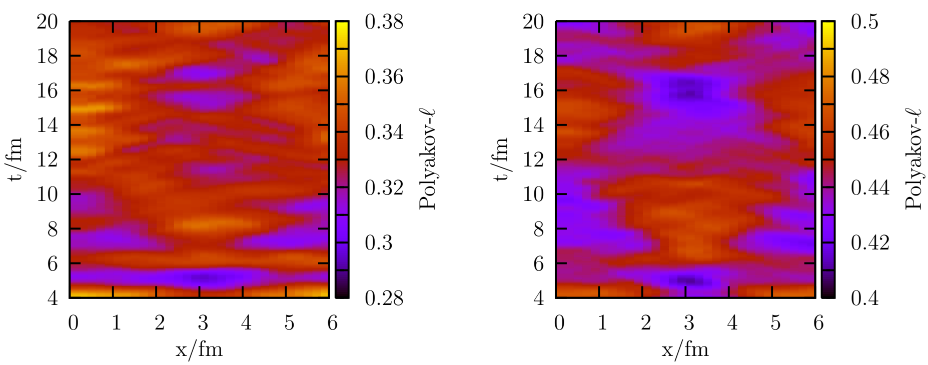

Fig. 2. Left: Fluctuations at the first order transition during relaxation process, quench from $T = 180$ MeV to $T = 140$ MeV. Right: Long-range fluctuations at the critical point or during relaxation process, quench from $T = 186$ MeV to $T = 166$ MeV.

3.2. Fluid dynamic expansion

For this simulation we initialize an ellipsoidal region with a temperature of $T = 200$ MeV, smoothed by a Woods–Saxon function at the edges. Initially, the fields and fluid are set to their equilibrium values throughout the lattice. We let the system expand by full $3+1$ dimensional fluid dynamics and measure the average temperature in a fixed central volume as a function of time. The result is shown in Fig. 3. While for the critical point T decreases monotonically with time, we can observe a reheating at the first order transition as a consequence of the forming of a supercooled phase below the transition temperature that finally decays to the global minimum and transfers its energy into the fluid. In Fig. 4, we show the evolution of nonequilibrium fluctuations for σ and ℓ. These are defined as $\langle\Delta\sigma\rangle = \sqrt{\langle(\sigma - \sigma_{\mathrm{eq}})^2\rangle}$ and $\langle\Delta\ell\rangle = \sqrt{\langle(\ell - \ell_{\mathrm{eq}})^2\rangle}$, respectively. For both order parameters we find a large increase of the nonequilibrium fluctuations in a scenario with a first order transition compared to the scenario with a critical point. This is again caused by the large deviations from equilibrium that occur during supercooling. We see in both figures a second smaller increase, this arises when parts of the system cross the transition temperature a second time after the reheating.

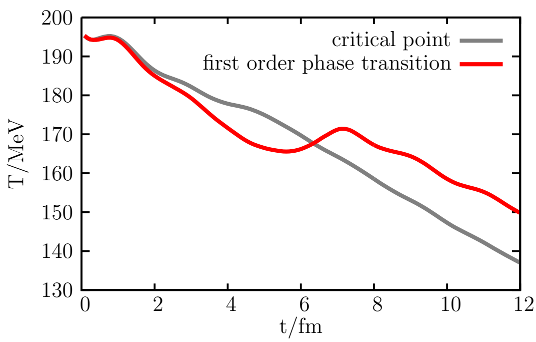

Fig. 3. Temperature of the quark fluid, for the first order scenario reheating after formation of a supercooled phase is observed.

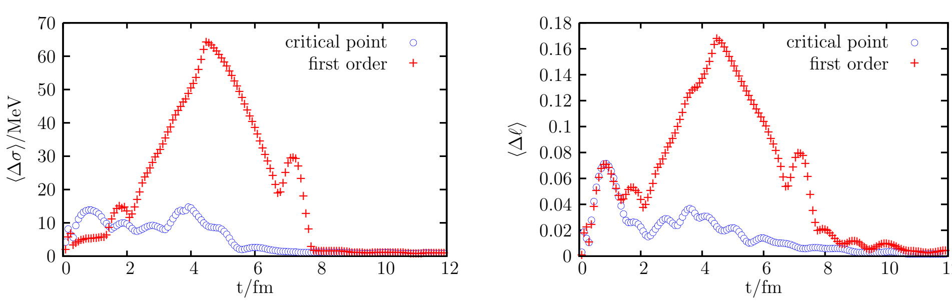

Fig. 4. Left: Sigma fluctuations at the first order transition compared to the critical point. Right: Polyakov loop fluctuations at the first order transition compared to the critical point.

4. Conclusions

We presented a dynamical model to study the chiral and deconfinement phase transition of QCD. The nonequilibrium evolution of both order parameters and their interaction with the quark fluid was described by Langevin equations. At the critical point, we observed the formation of long-range fluctuations. During the hydrodynamical expansion of a finite size system our model shows nonequilibrium effects like supercooling and reheating of the quark heat bath at the first order phase transition. There we also found an enhancement of nonequilibrium fluctuations in a first order phase transition scenario compared to an evolution through the critical point.

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