VISCOSITY COEFFICIENTS IN QUASIPARTICLE MODELS*

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Bulk and shear viscosity coefficients for systems composed of quasiparticles with medium-modified dispersion relations are determined within an effective kinetic theory approach of Boltzmann–Vlasov type. Local conservation of energy and momentum, which is self-consistently embedded in the kinetic theory, implies in thermal equilibrium thermodynamic consistency in quasiparticle approaches.

1. Introduction

Transport coefficients such as bulk and shear viscosities describe the hydrodynamic response of a system to energy and momentum fluctuations. In quantum field theories, they can be calculated within the framework of

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linear response theory, *i.e.* within the Kubo formalism [1,2]. Equally, kinetic theory, *e.g.* in form of a linearized Boltzmann equation, is applicable as a rigorous tool for systems with weak interaction strength [3,4].

The essential quantity in these considerations is the symmetric energy-momentum tensor, $T^{\mu\nu}$, describing the system. Local conservation of energy and momentum are then comprised in the relativistic equations of motion $\partial_\mu T^{\mu\nu} = 0$. In the absence of additional conservation laws, *i.e.* of additional conserved currents related to internal symmetries, these are the only equations determining the hydrodynamical evolution of the system.

The energy-momentum tensor can be decomposed [3,4,5] into a thermal equilibrium part $T^{\mu\nu}_{(0)}$ and dissipative corrections $T^{\mu\nu}_{(1)}$, *i.e.* $T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)}$, where $T^{\mu\nu}_{(0)} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu}$. When including only the first-order gradients of the fluid four-velocity field $u^\mu(x)$, then $T^{\mu\nu}_{(1)} = \zeta\Delta^{\mu\nu}\partial_\alpha u^\alpha + \eta S^{\mu\nu}_{\alpha\beta}\partial^\alpha u^\beta$. Here, ϵ and P denote energy density and thermodynamic pressure of the system in thermal equilibrium, respectively, which are related through the equation of state, $P = P(\epsilon)$. Moreover, ζ and η represent the bulk and shear viscosity coefficients, respectively. The projector $\Delta^{\mu\nu}$ is defined as $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, where u^μ is normalized by $u_\mu u^\mu = 1$ and $g^{\mu\nu}$ denotes the metric tensor with signature $(+,-,-,-)$. This projector is orthogonal to u^μ, *i.e.* $u_\mu \Delta^{\mu\nu} = 0$, and obeys in addition $\Delta_{\mu\nu}\Delta^\nu_\rho = \Delta_{\mu\rho}$. The projector $S^{\mu\nu}_{\alpha\beta} = \Delta^\mu_\alpha\Delta^\nu_\beta + \Delta^\mu_\beta\Delta^\nu_\alpha - 2\Delta^{\mu\nu}\Delta_{\alpha\beta}/3$ is also u^μ-orthogonal and satisfies $S^{\rho\omega}_{\alpha\beta}S^{\mu\nu}_{\rho\omega} = 2S^{\mu\nu}_{\alpha\beta}$, $\Delta^\alpha_\mu\Delta^\beta_\nu S^{\mu\nu}_{\alpha\beta} = 10$ and $\Delta_{\mu\nu}S^{\mu\nu}_{\alpha\beta} = 0$. With these definitions, the trace of $T^{\mu\nu}$ reads $T^\mu_\mu = \epsilon - 3P + 3\zeta\partial_\alpha u^\alpha$.

The temperature T and the fluid four-velocity u^μ can be defined in such a way that the local fluid rest frame is determined from a vanishing local momentum density T^{0i}. With the above decomposition of $T^{\mu\nu}$, the energy density can be defined by the projection $\epsilon = T^{\mu\nu}u_\mu u_\nu = T^{\mu\nu}_{(0)}u_\mu u_\nu$, *i.e.* $T^{\mu\nu}_{(1)}u_\mu u_\nu = 0$. This is the Landau–Lifshitz condition [3,5], which provides the only uniform definition of local flow for systems without additional conserved currents [6].

In this work, the viscosity coefficients for systems of quasiparticle excitations with medium-dependent dispersion relations are studied. The quasiparticle concept was shown to be a powerful tool to describe properties of strongly correlated many-particle systems [7]. The consideration of elementary, collective motions in terms of quasi-stationary single-particle states with limited life-times leads to a significant simplification in the description of such systems.

We restrict our investigations to thermal systems with ζ and η as the only independent transport coefficients in $T^{\mu\nu}_{(1)}$. Here, the explicit derivation of ζ and η follows the theoretical framework outlined in Ref. [8]. Accordingly,

by considering quantum statistics, the individual components of $T^{\mu\nu}$, *i.e.* in particular of $T^{\mu\nu}_{(1)}$, are determined self-consistently. The usual procedure is then to obtain ζ and η from individual components of $T^{\mu\nu}_{(1)}$ by using a particular velocity profile [9]. Here, instead, we determine the viscosity coefficients from the full covariant form of the energy-momentum tensor distortion from equilibrium by employing suitable projections.

2. Energy-momentum tensor

For systems of quasiparticles with dispersion relation $E = \sqrt{\vec{p}^2 + \Pi}$, where Π denotes a momentum independent effective quasiparticle mass, which in thermal equilibrium depends on T, the energy-momentum tensor emerges from an effective kinetic theory [10, 11]. The latter takes into account that the temperature is space-time dependent such that the quasiparticle energy $E(x)$ depends also on space and time via $\Pi(x)$. Then, the space-time behavior of the single-particle distribution function $f(x,p)$, which accounts for the phase-space probability density of quasiparticles, is governed by the Boltzmann–Vlasov type equation [3]

$$\left(p^\alpha(x)\partial_\alpha + \sqrt{\Pi(x)}F^\alpha(x)\frac{\partial}{\partial p^\alpha(x)} \right) f(x,p) = \mathcal{C}[f(x,p)]. \qquad (1)$$

Here, $p^\alpha(x) = (E(x), \vec{p})$ and the gradient of $\Pi(x)$ acts as an external force, $F^\alpha(x) = \partial^\alpha \Pi(x)/(2\sqrt{\Pi(x)})$ with $p_\alpha F^\alpha = 0$, which is changing the four-momenta of quasiparticles between collisions. The collision term $\mathcal{C}[f(x,p)]$, which is not influenced by $F^\alpha(x)$, is a functional of $f(x,p)$ and comprises all relevant microscopic scattering processes, which conserve locally energy and momentum. Note, that p^0 and $\vec{p}$ in $F^\alpha(\partial/\partial p^\alpha)$ in (1) must be considered as independent variables [3].

Regarding the space-time dependence of $p^0(x)$, the kinetic equation (1) may be written as

$$\partial_\mu \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} p^\mu(x)p^\nu(x)f(x,p) - \frac{1}{2}g^{\mu\nu}\left(\partial_\mu \Pi(x)\right) \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} f(x,p) = 0. \qquad (2)$$

Here, integrals containing partial derivatives of $f(x,p)$ were reformulated into vanishing surface integrals over a remote surface in momentum space [4]. From (2), the energy-momentum tensor of an isotropic fluid composed of quasiparticle excitations with medium-dependent dispersion relation emerges as

$$T^{\mu\nu}(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} p^\mu(x)p^\nu(x)f(x,p) + g^{\mu\nu}B[\Pi(x)]. \qquad (3)$$

This expression for $T^{\mu\nu}$ fulfills the equations of motion $\partial_\mu T^{\mu\nu} = 0$ if the mean field $B[\Pi(x)]$ satisfies the consistency condition

$$\partial^\nu B[\Pi(x)] = -\frac{1}{2} q(x) \partial^\nu \Pi(x) \tag{4}$$

with auxiliary field $q(x)$ defined as [10, 11]

$$q(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} f(x, p). \tag{5}$$

This ensures also that the quasiparticle spectrum can be obtained from $E(x) = \delta T^{00}(x)/\delta f(x, k)$.

The term $g^{\mu\nu} B[\Pi(x)]$ in $T^{\mu\nu}$, which can be interpreted as potential energy contribution, accounts for the fact that the quasiparticle properties are influenced by the background field created by all other excitations in the medium. Assuming that the space-time dependence of the functional B is solely determined by $\Pi(x)$, which in turn depends on x only via the auxiliary field, it follows that

$$B[\Pi(q(x))] = \frac{1}{2} \int\limits_0^{q(x)} \Pi(q')dq' - \frac{1}{2} q(x) \Pi(q(x)). \tag{6}$$

The auxiliary field depends on x both explicitly via $f(x, p)$ and implicitly via $\Pi(x)$ in $E(x)$. Thus, E itself is a functional of the distribution function via $\Pi(q(x))$ [12, 13, 14]. In thermal equilibrium, this approach recovers the quasiparticle model [15, 16, 17] defined through

$$T_{(0)}^{\mu\nu} = \int \frac{d^3\vec{p}}{(2\pi)^3 E^0} p^\mu p^\nu f^0 \left[E^0\right] + g^{\mu\nu} B^0[\Pi(T)], \tag{7}$$

where $p^\mu = (E^0, \vec{p})$, $E^0 = \sqrt{\vec{p}^2 + \Pi(T)}$ and $f^0[E^0]$ is the equilibrium distribution function. The consistency condition (4) becomes the stationarity condition [18]

$$\frac{\partial B^0[\Pi(T)]}{\partial \Pi(T)} = -\frac{1}{2} \int \frac{d^3\vec{p}}{(2\pi)^3 E^0} f^0 \left[E^0\right], \tag{8}$$

which is necessary to guarantee thermodynamic self-consistency.

3. Small deviations from thermal equilibrium

For small deviations from thermal equilibrium, the distribution function $f(x, p)$ can be expanded around the equilibrium distribution function. Consequently, the energy-momentum tensor can be decomposed as $T^{\mu\nu} = T^{\mu\nu}_{(0)} + \delta T^{\mu\nu}$. Since E is a functional of $f(x, p)$, the equilibrium distribution function, in line with (7), is given by $f^0[E^0]$, where E^0 is the quasiparticle energy in equilibrium. For a system with small deviations from equilibrium one has $E = E^0 + \delta E$. Consequently, deviations in f from $f^0[E^0]$ are associated with the deviation δE as well as with deviations in T, *i.e.* $\delta f^0 \simeq (\partial f / \partial E)|_{E^0} \delta E + (\partial f / \partial T)|_{E^0} \delta T$ for $f = f^0[E^0] + \delta f^0$. Correspondingly, the mean field term B changes from B^0 to $B = B^0 + \delta B$.

The structure of the kinetic equation (1) is such that the full quasiparticle excitation energy E enters the collision term. Thus, Eq. (1) must be linearized around $f^0[E]$, because $\mathcal{C}[f^0[E]] = 0$ according to the principle of detailed balance [4, 13, 14]. In this way, one finds equivalence between kinetic theory approaches and Kubo's formalism, as was shown for some special theories in [10, 11, 19, 20].

Nevertheless, we stress that $f^0[E]$ does not represent the correct equilibrium distribution function for quasiparticles in thermal equilibrium. Deviations in f from $f^0[E]$ are associated with deviations in T only, *i.e.* $\delta f \simeq (\partial f / \partial T)|_E \delta T$ for $f = f^0[E] + \delta f$. It is this δf, which is determined from the kinetic equation in such a way that f is the solution of (1). Thus, it is mandatory to express $\delta T^{\mu\nu}$ in terms of δf. The expansions around $f^0[E^0]$ and $f^0[E]$ are related via $f^0[E] - f^0[E^0] \simeq (\partial f^0[E]/\partial E)|_{E^0} \delta E$. Moreover, as the distribution function has to be unique, one finds the connection between the deviations in both expansions as $\delta f^0 \simeq \delta f + (\partial f^0[E]/\partial E)|_{E^0} \delta E$.

With these definitions, the individual components of $T^{\mu\nu}$ can be studied. Applying in $T^{00} = T^{00}_{(0)} + \delta T^{00}$ the expansions $E = E^0 + \delta E$ and $B = B^0 + \delta B$ in line with $f = f^0[E^0] + \delta f^0$, one finds

$$\delta T^{00} = \int \frac{d^3\vec{p}}{(2\pi)^3} E^0 \delta f^0 \,. \tag{9}$$

To obtain Eq. (9), we have used $\delta E = \delta \Pi / (2E)$ and $\delta B = (\partial B / \partial \Pi) \delta \Pi$ and applied Eqs. (4) and (5). In this way, variations of all quantities which depend on the distribution function are taken into account. Eq. (9) can be reformulated by employing the connection between δf^0 and δf as well as $\delta E \simeq \delta \Pi / (2E^0)$, yielding

$$\delta T^{00} \simeq \int \frac{d^3\vec{p}}{(2\pi)^3} \left(E^0 \delta f + \frac{1}{2} \left. \frac{\partial f^0[E]}{\partial E} \right|_{E^0} \delta \Pi \right) \,. \tag{10}$$

Considering quantum statistics, $f^0[E] = d/(e^{E/T} \pm 1)$ in the local fluid restframe, where d is a degeneracy factor and $+(-)$ applies for fermions (bosons). In this case, one finds with $\delta f \simeq (\partial f^0[E]/\partial T)|_{E^0}\delta T$ that $(\partial f^0[E]/\partial E)|_{E^0}(\delta \Pi/2) \simeq -(T^2/E^0)(\partial \Pi/\partial T^2)\delta f$ and consequently

$$\delta T^{00} \simeq \int \frac{d^3\vec{p}}{(2\pi)^3 E^0} \left((E^0)^2 - T^2 \frac{\partial \Pi}{\partial T^2} \right) \delta f \,. \tag{11}$$

For the spatial components of the energy-momentum tensor the expansion $f = f^0[E] + \delta f$ can be applied. Relating $f^0[E]$ with $f^0[E^0]$, approximating $1/E \simeq 1/E^0$ and using

$$\delta B \simeq -\frac{1}{2} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{f^0 \left[E^0 \right]}{E^0} \delta \Pi \,, \tag{12}$$

where terms of $\mathcal{O}(\delta T^2)$ are omitted, one gets for δT^{ij} in $T^{ij} = T^{ij}_{(0)} + \delta T^{ij}$

$$\delta T^{ij} \simeq \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{p^i p^j}{E^0} \delta f \,. \tag{13}$$

Combining the results for the individual components of the energy-momentum tensor and generalizing to the frame, where matter flows with four-velocity u^μ, one arrives at [11]

$$\delta T^{\mu\nu} = \int \frac{d^3\vec{p}}{(2\pi)^3 E^0} \left(p^\mu p^\nu - u^\mu u^\nu T^2 \frac{\partial \Pi}{\partial T^2} \right) \delta f \,. \tag{14}$$

4. Relaxation time approximation

The deviation δf entering (14) can be determined via the relaxation time approximation to the kinetic equation (1). In this approximation, one assumes that collisions lead to an exponentially fast relaxation towards thermal equilibrium within the relaxation time τ [21]. Accordingly, δf is approximated by

$$\delta f = -\frac{\tau}{E(x)} \mathcal{C}[f(x,p)] \,, \tag{15}$$

i.e. the complexity of the collision term in (1) becomes encoded in a single coefficient τ, which depends on the relevant scattering processes and is, thus, in principle four-momentum dependent. Making use of (1), one finds at leading order that

$$\delta f = -\frac{\tau}{E^0} \left(p^\alpha \partial_\alpha + \frac{1}{2} \partial^\alpha \Pi(T) \frac{\partial}{\partial p^\alpha} \right) f^0 \left[E^0 \right] \,, \tag{16}$$

which becomes

$$\delta f = \frac{\tau}{E^0} f^0 \left[E^0\right] \left(1 \mp d^{-1} f^0 \left[E^0\right]\right) \left(p^\alpha \partial_\alpha \phi + \tfrac{1}{2} \partial^\alpha \Pi(T) \frac{\partial \phi}{\partial p^\alpha}\right) \qquad (17)$$

for $f^0[E^0] = d/(e^\phi \pm 1)$ with $\phi = p^\mu u_\mu(x)/T(x)$.

In the following, the usual decomposition $\partial_\alpha = u_\alpha D + \nabla_\alpha$ with convective derivative $D = u^\rho \partial_\rho$ and spatial gradient operator $\nabla_\alpha = \Delta_{\alpha\beta} \partial^\beta$ is employed. The term $\propto p^\alpha \partial_\alpha \Pi$, which appears in $p^\alpha \partial_\alpha \phi$, vanishes due to the kinematic condition $p^\alpha F_\alpha = 0$. In addition, with the help of the equations of motion in lowest order, $Du_\alpha = \nabla_\alpha P/(\epsilon + P)$ [5], it can be shown that the spatial gradient of the temperature $\nabla_\alpha T$ is exactly canceled in $p^\alpha \partial_\alpha \phi$ via $\nabla_\alpha P/(\epsilon + P) - (\nabla_\alpha T)/T = 0$ since $\epsilon + P = T(\partial P/\partial T)$. Convective derivatives DT stemming from $p^\alpha \partial_\alpha \phi$ and from $\partial^\alpha \Pi (\partial \phi/\partial p^\alpha)/2 = (\partial \Pi/\partial T)(DT/T)/2$ can be converted into spatial gradients of the four-velocity $\nabla_\rho u^\rho$ by making use of the equation of state $\epsilon = \epsilon(P(T))$ and of the equation of energy in lowest order, $D\epsilon = -(\epsilon + P)\nabla_\rho u^\rho$ [5]. In this way, all leading-order gradients of u^μ appearing in δf are taken into account. Together with the identity $p^\alpha p^\sigma \nabla_\alpha u_\sigma = p^\alpha p^\sigma S_{\alpha\sigma}^{\gamma\delta} \partial_\gamma u_\delta/2 + p^\alpha p^\sigma \Delta_{\alpha\sigma} \nabla_\rho u^\rho/3$ and with $(pu) = p^\alpha u_\alpha$, one finds from (17) that

$$\begin{aligned}
\delta f = \; & \frac{\tau}{E^0} f^0 \left[E^0\right] \left(1 \mp d^{-1} f^0 \left[E^0\right]\right) \left\{ \left[\frac{1}{3T} p^\mu p^\alpha \Delta_{\mu\alpha} \right. \right. \\
& \left. \left. + \left(\frac{(pu)^2}{T} - \frac{1}{2}\frac{\partial \Pi}{\partial T}\right)\frac{\partial P}{\partial \epsilon}\right] \nabla_\rho u^\rho + \frac{1}{2T} p^\mu p^\alpha S_{\mu\alpha}^{\gamma\delta} \partial_\gamma u_\delta \right\}, \qquad (18)
\end{aligned}$$

where $+(-)$ applies now for bosons (fermions).

5. Landau–Lifshitz condition and viscosity coefficients

Inserting (18) into (14), $\delta T^{\mu\nu}$ is found to be solely a functional of the correct equilibrium distribution function. With the tensor structure identities

$$\int \frac{d^3 \vec{p}}{(2\pi)^3} \mathcal{F}_1 \left(p^\mu p^\nu - u^\mu u^\nu a\right) = \frac{1}{3} \Delta^{\mu\nu} \int \frac{d^3 \vec{p}}{(2\pi)^3} \mathcal{F}_1 \left(p^2 - (pu)^2\right) \qquad (19)$$

and

$$\int \frac{d^3 \vec{p}}{(2\pi)^3} \mathcal{F}_2 \left(p^\mu p^\nu - u^\mu u^\nu a\right) p^\alpha p^\sigma S_{\alpha\sigma}^{\gamma\delta} = \frac{1}{10} S^{\mu\nu\gamma\delta} \int \frac{d^3 \vec{p}}{(2\pi)^3} \mathcal{F}_2 \, p^\alpha p^\beta p^\sigma p^\tau S_{\alpha\beta\sigma\tau}$$
$$(20)$$

for

$$
\begin{aligned}
\mathcal{F}_1 = {}& f^0 \left[E^0\right] \left(1 \mp d^{-1} f^0 \left[E^0\right]\right) \frac{\tau}{(E^0)^2} \left\{ \left[\left(\frac{pu}{T}\right)^2 - \frac{1}{2T} \frac{\partial \Pi}{\partial T} \right] T \frac{\partial P}{\partial \epsilon} \right. \\
& \left. + \frac{1}{3T} \left(p^2 - (pu)^2\right) \right\} ,
\end{aligned}
\tag{21}
$$

$$
\mathcal{F}_2 = \frac{1}{2T} f^0 \left[E^0\right] \left(1 \mp d^{-1} f^0 \left[E^0\right]\right) \frac{\tau}{(E^0)^2} ,
\tag{22}
$$

where $a = T^2(\partial \Pi / \partial T^2)$ and $p^2 = p^\alpha p_\alpha$, Eq. (14) may be written as

$$
\begin{aligned}
\delta T^{\mu\nu} = {}& \frac{1}{3} \Delta^{\mu\nu} \nabla_\rho u^\rho \int \frac{d^3\vec{p}}{(2\pi)^3} \mathcal{F}_1 \left(p^2 - (pu)^2\right) \\
& + \frac{1}{10} S^{\mu\nu\gamma\delta} \partial_\gamma u_\delta \int \frac{d^3\vec{p}}{(2\pi)^3} \mathcal{F}_2 \frac{4}{3} \left(p^2 - (pu)^2\right)^2 ,
\end{aligned}
\tag{23}
$$

where we have used that $p^\alpha p^\beta p^\sigma p^\tau S_{\alpha\beta\sigma\tau} = 4(p^2 - (pu)^2)^2/3$.

To make the decomposition of $f(x,p)$ unique, the Landau–Lifshitz condition $u_\mu u_\nu \delta T^{\mu\nu} = 0$ can be imposed in addition. This condition may be formulated as

$$
\begin{aligned}
0 = {}& \int \frac{d^3\vec{p}}{(2\pi)^3} \mathcal{F}_1 \left((pu)^2 - a\right) \nabla_\rho u^\rho \\
& + \frac{2}{15} u_\mu u_\nu S^{\mu\nu\gamma\delta} \partial_\gamma u_\delta \int \frac{d^3\vec{p}}{(2\pi)^3} \mathcal{F}_2 \left(p^2 - (pu)^2\right)^2 .
\end{aligned}
\tag{24}
$$

By definition, the second term vanishes and, therefore, likewise $\delta T^{\mu\nu} = \delta T^{\mu\nu} + u_\alpha u_\beta \delta T^{\alpha\beta} X \Delta^{\mu\nu}$ can be considered, where X must be momentum independent, yielding

$$
\begin{aligned}
\delta T^{\mu\nu} = {}& \frac{1}{3} \Delta^{\mu\nu} \nabla_\rho u^\rho \int \frac{d^3\vec{p}}{(2\pi)^3} \mathcal{F}_1 \left\{ \left(p^2 - (pu)^2\right) + 3X \left((pu)^2 - a\right) \right\} \\
& + \frac{2}{15} S^{\mu\nu\gamma\delta} \partial_\gamma u_\delta \int \frac{d^3\vec{p}}{(2\pi)^3} \mathcal{F}_2 \left(p^2 - (pu)^2\right)^2 .
\end{aligned}
\tag{25}
$$

The shear and bulk viscosity coefficients can be obtained by comparing $\delta T^{\mu\nu}$ in (25) with the definition of $T^{\mu\nu}_{(1)}$, cf. [8, 22, 23]. The corresponding shear viscosity reads

$$
\eta = \frac{1}{15T} \int \frac{d^3\vec{p}}{(2\pi)^3} f^0 \left[E^0\right] \left(1 \mp d^{-1} f^0 \left[E^0\right]\right) \frac{\tau}{(E^0)^2} \left(p^2 - (pu)^2\right)^2 .
\tag{26}
$$

The bulk viscosity is obtained as

$$\zeta = \frac{1}{3T} \int \frac{d^3\vec{p}}{(2\pi)^3} f^0\left[E^0\right]\left(1 \mp d^{-1}f^0\left[E^0\right]\right) \frac{\tau}{(E^0)^2} \left\{ \left((pu)^2 - a\right)\frac{\partial P}{\partial \epsilon} \right.$$

$$\left. + \frac{1}{3}\left(p^2 - (pu)^2\right)\right\}\left\{\left(p^2 - (pu)^2\right) + 3X\left((pu)^2 - a\right)\right\}, \qquad (27)$$

where $\nabla_\rho u^\rho = \partial_\rho u^\rho$ was used. Choosing, in particular, $X = \partial P/\partial \epsilon$, one finally arrives at

$$\zeta = \frac{1}{T} \int \frac{d^3\vec{p}}{(2\pi)^3} f^0\left[E^0\right]\left(1 \mp d^{-1}f^0\left[E^0\right]\right) \frac{\tau}{(E^0)^2}$$

$$\times \left\{\left[(pu)^2 - a\right]\frac{\partial P}{\partial \epsilon} + \frac{1}{3}\left[p^2 - (pu)^2\right]\right\}^2. \qquad (28)$$

6. Discussion and conclusions

The viscosity coefficients derived above depend on the distribution function and quasiparticle energy in thermal equilibrium. They deviate from previous results, *cf. e.g.* [9], due to the medium-modified dispersion relation and the contribution related to $\partial \Pi/\partial T$.

In the local fluid rest frame, the factor in parentheses of Eq. (28) may be written as $\left[(E^0)^2 - a\right]\partial P/\partial \epsilon - \vec{p}^2/3 = \vec{p}^2\left(\partial P/\partial \epsilon - 1/3\right) + \partial P/\partial \epsilon\left(\Pi - a\right)$, such that $\zeta = 0$ when $\partial P/\partial \epsilon = 1/3$ and $(\Pi - a)$ vanishes. Thus, the terms $(\partial P/\partial \epsilon - 1/3)$ and $(\Pi - a)$ account for the deviation of the system from conformal invariance.

To explicitly quantify ζ and η, one needs to specify the effective quasiparticle mass $\Pi(T)$. In a gluon plasma, $\Pi(T)$ can be obtained such that corresponding lattice QCD results [24, 25] for the equation of state are reproduced within the quasiparticle model. This, together with a realistic perturbative-QCD inspired expression for τ [26], allows to calculate ζ and η [23]. A direct comparison with available lattice QCD results for the transport coefficients [27] shows very good agreement [23]. This indicates that naive expectations, that systems of weakly interacting quasiparticles should exhibit large transport coefficients, are not necessarily correct.

A remarkable feature is that the ratio of ζ/η as a function of the conformality measure, $\Delta v_s^2 = 1/3 - \partial P/\partial \epsilon \geq 0$, displays a linear (quadratic) behavior at large (small) values of Δv_s^2 [28]. Such dependencies have been predicted for strongly (weakly) coupled systems [29, 30, 31].

We conclude, that within a quasiparticle approach the equilibrium thermodynamics of the gluon plasma can be transferred, via kinetic theory principles, to quantities which describe deviations from local equilibrium in agreement with general principle predictions.

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