CHIRAL WAVES IN QUARK MATTER* **

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A mini-review of non-uniform phases in quark matter is presented, with particular attention to the pion condensation, also known as chiral density waves or chiral spirals. The phase diagram of strongly-interacting matter may involve such a phase, placed on the *quarkyonic island* between the baryonic phase and the chirally-restored quark-gluon plasma.

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1. Introduction

The goal of this talk is to present a short "historical" review of various aspects of non-uniform phases in quark matter, which at present are being actively investigated in a modern framework of the *quarkyonic* medium.

As is well known, the pion emerges as a pseudo-Goldstone boson of the spontaneously broken chiral symmetry. The attractive P-wave interaction with fermions (nucleons, quarks) opens the possibility of the pion condensation in nuclear matter. The phenomenon was first explored in the work by Migdal [1, 2], where the particle-hole and Δ resonance effects of Fig. 1 were included in the pion self-energy non-relativistic calculation. Numerous works followed the issue of the pion condensation in nuclear matter (for reviews and references see [3,4,5,6]). A recent status is reported in [7], where

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the appearance of the pion condensation is expected at ~ 1.5 -2 nuclear saturation density for the symmetric and neutron matter, however, the results depend in a quite sensitive way on the parameters of the NN interactions.



Fig.1. The particle-hole and Δ excitations in the pion self-energy in nuclear medium.

We mention above the "ancient" works, as the questions concerning nonuniform phases in quark matter are very much related to the old issue of the pion condensation. As a matter of fact, a similar phenomenon, linked to particle-hole superconductivity in strong spin-exchange fields has been known in condensed matter for a long time [8, 9], leading to Alternating Layer Spin structures (see, *e.g.* [10] and references therein).

2. Pion condensation in a relativistic approach

In 1979 Dautry and Nyman [11] found an analytic solution of the relativistic problem of the pion condensation, using the σ -model Lagrangian

$$L = \bar{\psi} \left(i \partial \!\!\!/ - g(\sigma + i\tau \cdot \boldsymbol{\pi}) \right) \psi + \dots , \qquad (1)$$

where ψ was the Dirac nucleon field (we change it to quarks), ellipses denote the meson kinetic and interaction terms, g is a coupling constant, finally the sigma and the pion mean fields assume the periodic ansatz

$$\sigma = f \cos[q(z - z_0)], \qquad \pi^0 = f \sin[q(z - z_0)], \qquad \pi^{\pm} = 0, \qquad (2)$$

with f = 93 MeV being the pion decay constant. The wave vector \vec{q} , here pointing in the z-direction, is a parameter, and z_0 is arbitrary. The Dirac spectrum of (1) has the form [11]

$$\epsilon(k) = \pm \sqrt{M^2 + k^2 + q^2/4 \pm \sqrt{M^2 q^2 + q \cdot k^2}},$$
(3)

where \vec{k} is the quark momentum, the outer \pm sign denotes the positive and negative energy solutions, while the \pm sign inside the square root defines the branch. The spectrum (3) for various fixed values of \vec{k} , is visualized in Fig. 2. We note that with the increasing value of q the branches split, with one increasing, and the other decreasing in energy. Thus, Fermi-sea energy may be lowered when the lower branch is occupied. Similarly, the ansatz for the charged pion condensation has the form [11]

$$\pi_1 = f \cos[q(z - z_0)], \qquad \pi_2 = f \sin[q(z - z_0)], \qquad \pi_3 = \sigma = 0, \quad (4)$$

which is obtained with the chiral rotation $\exp\left(i\frac{\pi}{4}\gamma_5\tau_1\right)$ from (2), therefore yields the same spectrum and thermodynamic features, however, offers no interesting magnetic properties (see Sec. 4). We note here some resemblance to the issue of *disoriented chiral condensates* (for a review and reference see, *e.g.* [12]): forms (2), (4) may be viewed as "periodic disoriented chiral condensates".

To understand the dynamical mechanism behind the formation of the pion condensate in more general terms, we may carry out the chiral rotation $\psi = e^{-i\gamma_5/(2f)\boldsymbol{\tau}\cdot\boldsymbol{\phi}}q$ on Lagrangian (1), which yields

$$\mathcal{L} = \bar{q} \left(i \partial \!\!\!/ - \frac{1}{2f} \gamma_5 \gamma_\mu (\partial^\mu \phi) \cdot \boldsymbol{\tau} - M \right) q + \dots , \qquad (5)$$

with M = gf denoting the constituent quark mass due to the spontaneous breaking of the chiral symmetry. Attraction occurs for fermions (quarks) with appropriately correlated spin $\Sigma^i = \gamma_5 \gamma^0 \gamma^i$ and flavor whenever the pion field is nonuniform. For the ansatz (2) we obtain attraction for $|u \downarrow\rangle$ and $|d \uparrow\rangle$ states (the decreasing positive-energy branch in Fig. 2), while repulsion is found for $|u \uparrow\rangle$ and $|d \downarrow\rangle$ (the rising positive-energy branch). This is reminiscent of the well-known hedgehog form found in chiral bags [13] and chiral solitons [14, 15], where the valence quarks occupy the correlated spin-flavor state $q_h = 1/\sqrt{2}(|u \downarrow\rangle - |d \uparrow\rangle)$.



Fig. 2. The Dirac spectrum (3) in the periodic chiral field ansatz (2), plotted for sample values of the quark momentum \vec{k} . We note the appearance of two branches for the positive-energy spectrum. The lower branch is for $u \downarrow$ and $d \uparrow$ states, while the upper one is for $u \uparrow$ and $d \downarrow$ states.

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The pion condensation is driven by the Fermi sea, favoring increased values of q. At the same time, the meson kinetic terms tend to suppress q (for the σ -model they give the contribution $\frac{1}{2}f^2q^2$ to the energy density). Hence the occurrence of the pion-condensed state (state with $q \neq 0$) is a rather subtle dynamical issue, here involving the Fermi energy and the value of the coupling constant g (or, equivalently, the quark mass M = gf).

3. Pion condensation in quark matter

In an early series of papers Kutschera, Kotlorz, and the present author [16, 17, 18] explored various aspects of the pion condensation in quark matter. We would like to recall a figure and a quote from one of these papers [17]. Figure 3 shows the result of the phase-diagram calculation in the σ -model. We note the appearance of the new phase (C), where (neutral) pion condensation appears — a spatially nonuniform phase with fields (2).



Fig. 3. The phase diagram with the quark pion-condensed phase (C), the normal quark phase with broken chiral symmetry (N), the restored phase (R), and the baryonic phase (reprinted from [17]).

We have also provided a percolation argument which may support the validity of using the quark gas at larger baryochemical potentials μ [17]:

We can see that there is room for the new phase only if (at a given temperature) "declustering" occurs at a lower density than the chiral restoration. This situation may be viewed as follows: at low densities we have isolated hadrons. As the density is increased, the "bags" start to overlap, and the quarks can percolate. This is a geometrical effect, and it is hard to imagine why it should occur at the same density as the chiral restoration, which

is a dynamical effect. Thus, it is possible that there exists a quark gas phase with broken chiral symmetry ... if this happens, then the system develops a "pion condensed" phase.

Essentially, this was the same simple argument as brought up recently in [19]. Another early work on nonuniform phase on chiral quark models, "standing wave ground state ..." was reported in [20]. Large- N_c arguments for finite density QCD were subsequently made in [21], while the possibility of the particle-hole pairing (the Overhauser effect) was studied in [22,23].

The pion condensation effect may occur in the Nambu–Jona-Lasinio model as well, but it becomes quite sensitive to the details of the model (regularization, parameters) [24]. Essentially, compared to the σ -model, the NJL model replaces the meson kinetic term $\frac{1}{2}q^2f^2$ with a more general function $K(q) = \frac{1}{2}q^2f^2 + \mathcal{O}(q^4)$. Since q is not small, the formally subleading terms are relevant.

A more detailed study of the pion condensation in the NJL model was first made by Sadzikowski and the author in [25]. Several other calculations followed, confirming the possibility of "chiral spiral" [26] a.k.a. "dual chiral density wave" [27] in the Gross–Neveu and NJL models. Other NJL analyses were reported in [28, 29, 30, 31], also in the presence of the superconducting phase [32, 33, 34, 35]. Bringoltz [36] pointed out the possibility of the effect in strongly-coupled QCD on the lattice. More recently, in the framework of quarkyonic matter [37] inspired with the Polyakov-loop arguments [38, 39, 40, 41], the chiral spirals and their implication for the phase diagram have been actively investigated [42, 43, 44, 45].

4. Magnetization

A very intriguing aspect of the neutral pion-condensed phase is its magnetization property. For the neutral pion condensation ansatz the interaction term in the Dirac Hamiltonian can be written as

$$-\frac{1}{2}\vec{\Sigma}\cdot\vec{q}\tau_3\,,\qquad \Sigma^i=\gamma_5\gamma^0\gamma^i=\left(\begin{array}{cc}\sigma^i&0\\0&\sigma^i\end{array}\right)\,,\qquad(6)$$

i.e., attraction occurs for $|u \downarrow\rangle$ and $|d\uparrow\rangle$, and repulsion for $|u\uparrow\rangle$ and $|d\downarrow\rangle$ states. By definition, the magnetization is equal to $\mathcal{M} = g(\mu_u s_u + \mu_d s_d)$, where g = 2 and $\mu_u = -2\mu_d$, hence

$$\mathcal{M} = 2\left[\mu_u \frac{1}{2}(n_{u\uparrow} - n_{u\downarrow}) + \mu_d \frac{1}{2}(n_{d\uparrow} - n_{d\downarrow})\right].$$
(7)

In flavor-symmetric matter $n_{u\uparrow} = n_{d\downarrow}$, $n_{d\uparrow} = n_{u\downarrow}$, therefore one finds $\mathcal{M} = 3\mu_d(n_{d\uparrow} - n_{d\downarrow})$. Because the $u \downarrow /d \uparrow$ branch is filled more than the $u \uparrow /d \downarrow$ branch (*cf.* Fig. 2), $n_{d\uparrow} > n_{d\downarrow}$, and permanent magnetization occurs. At the mean-field level one can show from stationarity that [3]

$$s_{u/d} = \mp \frac{1}{2} f^2 q \,. \tag{8}$$

Hence, appearance of the chiral wave in the system is equivalent to net magnetization. Kotlorz and Kutschera [46] applied the above effect to obtain large magnetic fields of stars with a pion-condensed quark core, up to $\sim 10^{15} \,\mathrm{G}$.

Conversely, an external magnetic field induces chiral waves. Recently, interesting studies of this effect in dense quark matter were made in [47,48, 49, 50, 51, 52], and also for the color superconducting phases in [53].

5. One-loop physics and validity of the mean-field

A quite remarkable feature of the spectrum (3) is that the expressions for baryon and energy densities (at T = 0) are also analytic [16, 17, 18], which allows for an exact study of the one-quark-loop [54] and also the one-mesonloop [55] contributions to the effective action for the case of a nonuniform periodic background of Eq. (2) (see also [56]). This made possible a study of the convergence radius in the gradient and the heat-kernel expansions, providing interesting formal features. In particular, for the fermion loop the gradient expansion works for q < 2m, while for the expansion in ∂U , with U denoting the chiral field, the effective expansion variable is $q^2/(4m^2 + q^2)$. For the mean-field approach to be valid, the wave vector q, corresponding to the size of the gradient of the field, should not be too large.

6. What is the lowest state?

The fundamental question of what the phase structure of strongly interacting matter is clearly poses difficulties, as it involves dynamics of strong interactions in a multi-scale system of dense medium. Certainly, finding lower-energy solutions than a uniform state shows that the uniform state is unstable. Another issue, however, is if the simple ansatz of Eq. (2), or another possible solution we may find, is indeed the lowest state of the system in certain thermodynamic conditions. It should be noted here that on general grounds one-dimensional structures are unstable with respect to thermodynamic fluctuations [57]. Hence the chiral wave, with a one-dimensional order parameter q, is thermodynamically unstable and there must exist yet lower energy states. Another class of solutions with energy even lower than the chiral wave are the periodic systems of domain-wall solitons, investigated by Nickel and Bubbala [58, 59, 60, 61] by implanting solutions from the Gross–Neveu model [62, 63] to the 3 + 1 dimensional case. Unlike the chiral-wave case, where the baryon density is uniform, the periodic domain-wall solutions have a nonuniform baryon density, thus baryon clusterization may be studied in this case. At low densities one finds isolated baryon slabs, while at increasing density they start to overlap and finally dissolve. The solution has an interesting phase diagram, with the critical end-point coinciding with the Lifshitz point [60].

We also comment that the inclusion of the current quark mass, or a pion mass, in chiral waves is not trivial if self-consistency of the solution is to be preserved. For instance, in the σ model one of the Euler–Lagrange equations has the form

$$\Box \sigma = g \left\langle \bar{\psi} \psi \right\rangle + 2V' \left(\sigma^2 + \pi^2 \right) \sigma + f m_\pi^2, \tag{9}$$

with V' denoting the derivative of the chirally-symmetric Mexican Hat potential with respect to its argument. The constant term coming from the explicit breaking of the chiral symmetry, fm_{π}^2 , breaks the ansatz (2), hence for nonzero m_{π} the Dautry–Nyman solution is not consistent. A study by Maedan [64], where an expansion in m_{π} is applied, shows that the chiral waves survive the nonzero- m_{π} corrections.

At the same time, we know that the inclusion of a nonzero pion mass has significant effects on the phase diagram. In particular, the location of the critical point is sensitive to m_{π} . This fact has been pointed out for the uniform solution [65] and for the periodic domain-wall solution.

7. Conclusion

We have a mounting evidence that quark matter likes to form spatially non-uniform structures. Apart for the discussed solutions, we also have Skyrmion crystals [66, 67, 68, 69], not discussed here. Also, crystalline structures may form in the high-density color superconducting phases [70]. On the other hand, at low densities we know that baryons are the proper degrees of freedom. With all this in mind it is natural to expect that the transition from the baryonic phase to the (uniform) quark-gluon plasma may occur via non-uniform phases. This is somewhat reminiscent of the formation of nuclear pasta [71] at baryon densities below nuclear saturation density.

Nonuniformity of the medium poses a challenge, as analytic methods are of limited application; known example discussed above concern certain periodic systems. Also, important details (existence of new phases, nature of phase transitions) depend sensitively on the model parameters whose values are difficult to assess in the medium. Our knowledge, or rather the spectrum of possibilities for the structure of dense medium comes largely from effective quark models (for a review see, e.g., [72]), with parameters set from the properties in the vacuum sector. Thus, the uncertainties are large.

The pion attraction plays a dominant dynamical role in the physics of strong interactions and, as stressed in Sect. 2, it drives the system towards chiral waves, making the uniform phase unstable in the domain indicated in Fig. 3. We expect strong spin-isospin correlations in this phase, leading to interesting magnetic effects.

We have also argued throughout this talk that the following terms are "historically" linked:

pion condensation \sim particle-hole pairing \sim Alternating Layer Spin \sim chiral density waves \sim chiral spirals.

Last word on these issues, certainly, has not been said. The modern quarkyonic interpretation of the constituent quark matter, with the Polyakov line and large- N_c arguments offers, along the quest of exploring the QCD phase diagram, the ground for studying nonuniform phases.

REFERENCES

- [1] A.B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2209 (1971).
- [2] A.B. Migdal, *Phys. Rev. Lett.* **31**, 257 (1973).
- [3] G. Baym, E. Flowers, *Nucl. Phys.* A222, 29 (1974).
- [4] G.E. Brown, W. Weise, *Phys. Rep.* 27, 1 (1976).
- [5] G. Baym, D. Campbell, *Mesons in Nuclei. Vol. 3*, edited by M. Rho, D. Wilkinson, North Holland, Amsterdam 1979.
- [6] T. Kunihiro et al., Prog. Theor. Phys. Suppl. 112, 307 (1993).
- [7] A. Akmal, V. Pandharipande, *Phys. Rev.* C56, 2261 (1997)
 [arXiv:nucl-th/9705013v2].
- [8] A.I. Larkin, Y.N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 47, 1136 (1964).
- [9] P. Fulde, R.A. Ferrell, *Phys. Rev.* A550, 135 (1964).
- [10] T. Tatsumi, Prog. Theor. Phys. 63, 1252 (1980).
- [11] F. Dautry, E. Nyman, *Nucl. Phys.* A**319**, 323 (1979).
- B. Mohanty, J. Serreau, *Phys. Rep.* 414, 263 (2005)
 [arXiv:hep-ph/0504154v2].
- [13] A. Chodos, C.B. Thorn, *Phys. Rev.* **D12**, 2733 (1975).
- [14] M.C. Birse, M.K. Banerjee, *Phys. Lett.* **B136**, 284 (1984).
- [15] S. Kahana, G. Ripka, V. Soni, Nucl. Phys. A415, 351 (1984).
- [16] M. Kutschera, W. Broniowski, A. Kotlorz, *Phys. Lett.* **B237**, 159 (1990).
- [17] W. Broniowski, A. Kotlorz, M. Kutschera, Acta Phys. Pol. B 22, 145 (1991).

- [18] M. Kutschera, W. Broniowski, A. Kotlorz, *Nucl. Phys.* A516, 566 (1990).
- [19] P. Castorina, R. Gavai, H. Satz, *Eur. Phys. J.* C69, 169 (2010) [arXiv:1003.6078v1 [hep-ph]].
- [20] D. Deryagin, D.Y. Grigoriev, V. Rubakov, Int. J. Mod. Phys. A7, 659 (1992).
- [21] E. Shuster, D. Son, Nucl. Phys. B573, 434 (2000)
 [arXiv:hep-ph/9905448v2].
- [22] B.Y. Park et al., Phys. Rev. D62, 034015 (2000) [arXiv:hep-ph/9910347v1].
- [23] R. Rapp, E.V. Shuryak, I. Zahed, *Phys. Rev.* D63, 034008 (2001)
 [arXiv:hep-ph/0008207v1].
- [24] W. Broniowski, M. Kutschera, *Phys. Lett.* **B242**, 133 (1990).
- [25] M. Sadzikowski, W. Broniowski, *Phys. Lett.* B488, 63 (2000)
 [arXiv:hep-ph/0003282v1].
- [26] V. Schon, M. Thies, *Phys. Rev.* D62, 096002 (2000) [arXiv:hep-th/0003195v3].
- [27] E. Nakano, T. Tatsumi, *Phys. Rev.* D71, 114006 (2005)
 [arXiv:hep-ph/0411350v5].
- [28] R. Casalbuoni *et al.*, *Phys. Lett.* B627, 89 (2005)
 [arXiv:hep-ph/0507247v3].
- [29] T.L. Partyka, M. Sadzikowski, J. Phys. G 36, 025004 (2009)
 [arXiv:0811.4616v2 [hep-ph]].
- [30] G. Basar, G.V. Dunne, M. Thies, *Phys. Rev.* D79, 105012 (2009) [arXiv:0903.1868v1 [hep-th]].
- [31] D. Ebert et al., Phys. Rev. D84, 025004 (2011) [arXiv:1102.4079v3 [hep-ph]].
- [32] M. Sadzikowski, *Phys. Lett.* **B553**, 45 (2003) [arXiv:hep-ph/0210065v2].
- [33] M. Sadzikowski, *Phys. Lett.* B642, 238 (2006) [arXiv:hep-ph/0609186v1].
- [34] M. Inui, H. Kohyama, A. Niegawa, arXiv:0709.2204v1 [hep-ph].
- [35] T.L. Partyka, Mod. Phys. Lett. A26, 543 (2011) [arXiv:1005.2667v2 [hep-ph]].
- [36] B. Bringoltz, J. High Energy Phys. 0703, 016 (2007) [arXiv:hep-lat/0612010v1].
- [37] L. McLerran, R.D. Pisarski, *Nucl. Phys.* A796, 83 (2007)
 [arXiv:0706.2191v3 [hep-ph]].
- [38] K. Fukushima, *Phys. Lett.* **B591**, 277 (2004) [arXiv:hep-ph/0310121v3].
- [39] E. Megias, E. Ruiz Arriola, L. Salcedo, arXiv:hep-ph/0410053v1.
- [40] E. Megias, E. Ruiz Arriola, L. Salcedo, *Phys. Rev.* D74, 065005 (2006)
 [arXiv:hep-ph/0412308v3].
- [41] K. Fukushima, *Phys. Rev.* D77, 114028 (2008) [arXiv:0803.3318v3 [hep-ph]].
- [42] T. Kojo et al., Nucl. Phys. A843, 37 (2010) [arXiv:0912.3800v2 [hep-ph]].

- [43] T.L. Partyka, M. Sadzikowski, Acta Phys. Pol. B 42, 1305 (2011) [arXiv:1011.0921v2 [hep-ph]].
- [44] S. Carignano, D. Nickel, M. Buballa, *Phys. Rev.* D82, 054009 (2010)
 [arXiv:1007.1397v1 [hep-ph]].
- [45] T. Kojo et al., arXiv:1107.2124v1 [hep-ph].
- [46] A. Kotlorz, M. Kutschera, Acta Phys. Pol. B 25, 859 (1994).
- [47] K. Takahashi, J. Phys. G 34, 653 (2007).
- [48] I. Frolov, V. Zhukovsky, K. Klimenko, *Phys. Rev.* D82, 076002 (2010) [arXiv:1007.2984v3 [hep-ph]].
- [49] G. Basar, G.V. Dunne, D.E. Kharzeev, *Phys. Rev. Lett.* 104, 232301 (2010)
 [arXiv:1003.3464v1 [hep-ph]].
- [50] T. Tatsumi, arXiv:1107.0807v2 [hep-ph].
- [51] I. Frolov, Ph.D. Thesis (2011).
- [52] A. Rabhi, C. Providencia, *Phys. Rev.* C83, 055801 (2011)
 [arXiv:1104.1512v1 [nucl-th]].
- [53] E.J. Ferrer, V. de la Incera, *Phys. Rev.* D76, 045011 (2007) [arXiv:nucl-th/0703034v2].
- [54] W. Broniowski, M. Kutschera, *Phys. Lett.* **B234**, 449 (1990).
- [55] W. Broniowski, M. Kutschera, *Phys. Rev.* D41, 3800 (1990).
- [56] M. Kato, W. Bentz, K. Tanaka, *Phys. Rev.* C45, 2445 (1992).
- [57] G. Baym, B. Friman, G. Grinstein, Nucl. Phys. B210, 193 (1982).
- [58] D. Nickel, M. Buballa, *Phys. Rev.* D79, 054009 (2009) [arXiv:0811.2400v2 [hep-ph]].
- [59] D. Nickel, *Phys. Rev. Lett.* **103**, 072301 (2009) [arXiv:0902.1778v2 [hep-ph]].
- [60] D. Nickel, *Phys. Rev.* D80, 074025 (2009) [arXiv:0906.5295v1 [hep-ph]].
- [61] M. Buballa, D. Nickel, Acta Phys. Pol. B Proc. Suppl. 3, 523 (2010)
 [arXiv:0911.2333v1 [hep-ph]].
- [62] O. Schnetz, M. Thies, K. Urlichs, Ann. Phys. 314, 425 (2004)
 [arXiv:hep-th/0402014v4].
- [63] O. Schnetz, M. Thies, K. Urlichs, Ann. Phys. 321, 2604 (2006)
 [arXiv:hep-th/0511206v2].
- [64] S. Maedan, Prog. Theor. Phys. 123, 285 (2010) [arXiv:0908.0594v4 [hep-ph]].
- [65] O. Scavenius et al., Phys. Rev. C64, 045202 (2001)
 [arXiv:nucl-th/0007030v1].
- [66] I.R. Klebanov, Nucl. Phys. **B262**, 133 (1985).
- [67] A.S. Goldhaber, N. Manton, *Phys. Lett.* **B198**, 231 (1987).
- [68] L. Castillejo et al., Nucl. Phys. A501, 801 (1989).
- [69] M. Kugler, S. Shtrikman, *Phys. Lett.* **B208**, 491 (1988).
- [70] M.G. Alford, J.A. Bowers, K. Rajagopal, *Phys. Rev.* D63, 074016 (2001)
 [arXiv:hep-ph/0008208v3].
- [71] C.J. Pethick, D.G. Ravenhall, Annu. Rev. Nucl. Part. Sci. 45, 429 (1995).
- [72] B.J. Schaefer, J.M. Pawlowski, J. Wambach, *Phys. Rev.* D76, 074023 (2007) [arXiv:0704.3234v2 [hep-ph]].