TWO-FLAVOR QCD AT FINITE TEMPERATURE AND CHEMICAL POTENTIAL FROM DYSON–SCHWINGER EQUATIONS*

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We summarize recent results obtained in the Dyson–Schwinger formalism to study the chiral and deconfinement phase transitions of QCD at finite temperature and chemical potential. We compare the quenched SU(2) and SU(3) gauge theories and find a clearer distinction between second and first order transitions as compared to previous studies. For the full theory with two degenerate quark flavors we find coinciding crossover transition lines for the chiral and deconfinement transition at finite chemical potential. These lines merge together at large chemical potential and end in a critical endpoint followed by a first order coexistence region. Our results suggest that there is no critical endpoint in the region $\mu/T < 1$.

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1. Introduction

The behavior of quantum chromodynamics at large temperatures and densities received a lot of attention over the past years and is an ongoing research program from both, theoretical and experimental side. At vanishing

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chemical potential lattice QCD has shown the existence of a crossover from a region, where chiral symmetry is broken and quarks are confined inside hadrons to a quark-gluon plasma phase with (approximately) restored chiral symmetry and deconfined quarks. At finite chemical potential, however, lattice calculations for light quarks are severely limited by the sign problem making it very difficult to extract information from QCD. Consequently, the Polyakov loop extended Nambu–Jona-Lasinio (PNJL) and the Polyakov loop extended quark-meson (PQM) models have so far been the main source of information at large chemical potential. In general, these models favor a scenario, where the chiral crossover turns into a first order phase transition at a critical endpoint joined by the confinement/deconfinement transition line, see *e.g.* [1, 2, 3, 4]. At large chemical potentials and relatively small temperatures, however, there may also exist new phases as *e.g.* inhomogeneous [5, 6] or quarkyonic [7] phases.

An alternative direct approach to non-perturbative QCD without the sign problem is the framework of Dyson–Schwinger equations (DSEs) [8,9,10] and the functional renormalisation group [11,12]. QCD with two degenerate quark flavors has been studied in Ref. [10] by solving the coupled system of quark and gluon DSEs using quenched lattice data for the gluon propagator as input. Within this truncation scheme the behavior of the chiral and deconfinement transitions at finite chemical potential have been investigated using the first calculation of the dressed Polyakov loop in this region of the QCD phase diagram. In this proceedings contribution we give an overview of the employed truncation scheme and summarize the corresponding results.

2. Order parameters for chiral symmetry breaking and confinement

The central object of our study is the in-medium quark propagator

$$S^{-1}(p) = i\vec{\gamma}\vec{p}A\left(\omega_n, \vec{p}^2\right) + i\gamma_4\left(\omega_n + i\mu\right)C\left(\omega_n, \vec{p}^2\right) + B\left(\omega_n, \vec{p}^2\right) . \qquad (1)$$

Here μ is the quark chemical potential and $\omega_n = \pi T(2n+1)$ are the Matsubara modes in the imaginary time formalism with temperature T. The functions A, C and B dress the vector and scalar parts of the propagator and are determined selfconsistently from the DSE of the quark propagator.

A possible order parameter for chiral symmetry breaking is the quark condensate

$$\langle\bar{\psi}\psi\rangle = \mathrm{Tr}[S] = Z_2 Z_m T \sum_n \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}_D\left[S(p)\right] . \qquad (2)$$

In the presence of explicit quark masses, the condensate is divergent with $m\Lambda^2$ and $m^2\Lambda$, but since these terms do not depend on temperature and chemical potential, the condensate can still be used as an order parameter for the regularized theory. We work with approximately physical quark masses and, therefore, expect to find a crossover at small chemical potentials. To define the pseudo-critical temperature in this case we use the susceptibility

$$\chi = \frac{\partial \left\langle \bar{\psi}\psi \right\rangle}{\partial m} \tag{3}$$

and determine its maximum to find T_c. Again, the divergent terms in the condensate only lead to an offset in χ, without changing its maximum. The quark condensate in the chiral limit has been determined in Ref. [13], where critical scaling beyond the mean field level at the second order chiral phase transition has been studied.

In [14, 15, 16] the dressed Polyakov loop has been proposed as an order parameter for centre symmetry breaking, *i.e.* confinement. This quantity is defined by

$$\Sigma_{\pm 1} = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{\mp i\varphi} \left\langle \bar{\psi}\psi \right\rangle_\varphi \,, \tag{4}$$

where $\varphi \in [0, 2\pi[$ is a parameter for U(1)-valued boundary conditions of the quark fields: $\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$. The physical boundary condition is $\varphi = \pi$. The quantity $\Sigma_{\pm 1}$ contains all loops of connections winding once around the Euclidean time direction in positive or negative direction. Among these is the ordinary Polyakov loop together with all kinds of loops which are not straight in the time direction but contain detours. At finite chemical potential Σ_{+1} and Σ_{-1} are not equal and correspond to the dressed Polyakov loop and its conjugate. Clearly, the dressed Polyakov loop can be determined from the quark-DSE equipped with the generalized U(1)-valued boundary conditions [8]. Since the deconfinement transition will be a crossover in full QCD, we use the maximum of $\frac{\partial \Sigma_{\pm 1}}{\partial m}$ to define the pseudo-critical temperature, similar to the chiral transition.

3. Dyson–Schwinger equations

Fig. 1 displays the DSE for the quark propagator. The quark self-energy depends on the dressed gluon propagator and the dressed quark-gluon vertex, which are needed to solve the equation self-consistently.

For the dressed gluon propagator we use quenched and temperature dependent lattice data as input into for the Yang–Mills part of the gluon DSE and add the quark loop as depicted in Fig. 2. This approximation neglects unquenching effects in the Yang–Mills part of the DSE which may affect the transition temperatures on the 5–10 MeV level [10]. As an additional

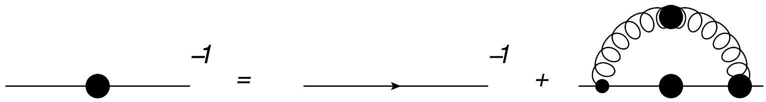

Fig. 1. The Dyson–Schwinger equation for the quark propagator. Dots denote dressed objects.

approximation we treat the quark loop semi-perturbatively by taking bare quarks but a dressed vertex. This allows us to use the hard-thermal loop expression multiplied by the vertex dressing function. The HTL approximation is well justified above the critical temperature, where quark dressing effects are small, but needs to be corrected in the chiral broken phase. A calculation with a fully dressed quark loop is work in progress. Since there are neither lattice nor DSE-results for the temperature dependent quark-gluon vertex we rely on an ansatz, built along its Ward-identity, see [9, 10] for details.

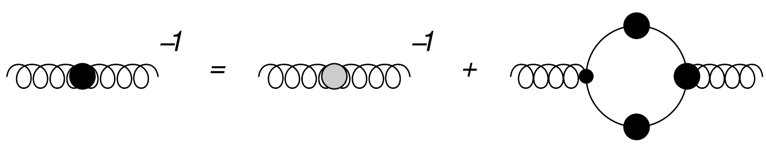

Fig. 2. The truncated gluon DSE. The black dot denotes the full, unquenched propagator, while the grey dot denotes the quenched propagator.

4. Results

4.1. Quenched QCD

As already mentioned above, the phase transition of quenched QCD (*i.e.* without the quark loop contributions in the Yang–Mills sector) can be determined from the quark DSE using quenched lattice data for the temperature dependent gluon propagator as input. Of course, the quality of the results depends then on the statistic and systematic error of the lattice. Starting from the pioneering work of Ref. [17], these have been improved in [9] and analyzed in more detail in recent works [18, 19, 20, 21]. In general, however, it seems fair to say that, in particular, systematic errors due to volume and discretization artifacts at small momenta are not yet well under control. This is particularly true in the vicinity of the critical temperature. Consequently, it proved difficult to distinguish the order of the phase transition between the two-color and three-color cases investigated in Ref. [9]. Here we present updated and improved results for the quark condensate and the dressed Polyakov loop using the high-statistics lattice data of Ref. [21] as input, which are also carried out on a much finer temperature grid than the ones of [9].

Fig. 3 shows how the dressed Polyakov loop and the quark condensate change with temperature in the cases of two and three colors. For the normalization of the temperature scale we use the transition temperatures which have been determined from the Polyakov loop on the lattice and compare with our results obtained from the quark DSE. Indeed, both order parameters show a rapid change at T_c, signaling the (approximate) restoration of chiral symmetry and breaking of centre symmetry at the very same temperature in agreement with the critical temperature determined from the lattice. With the finer temperature grid and the better statistics compared to [9], the behavior of the dressed Polyakov loop is now clearly distinguishable between the SU(2) and SU(3) cases, pointing towards a second order phase transition for SU(2) and a weak first order for SU(3), again in agreement with the expectations. The situation is less clear for the chiral transition, although also here we observe a steeper fall for the SU(3)-case. The behavior of the quark condensate for temperatures below the critical one, *i.e.* the rise with temperature combined with the sharp drop at T_c has also been seen in quenched lattice calculations [22]. Nevertheless, it may very well be that the quantitative aspects of this rise are subject to the systematic uncertainties of the lattice gluon data [19]. These uncertainties are also reflected in the 'noisy' behavior of the quark condensate. Nevertheless, it is remarkable that below T_c the dressed Polyakov loop is consistent with zero, signaling conserved centre symmetry.

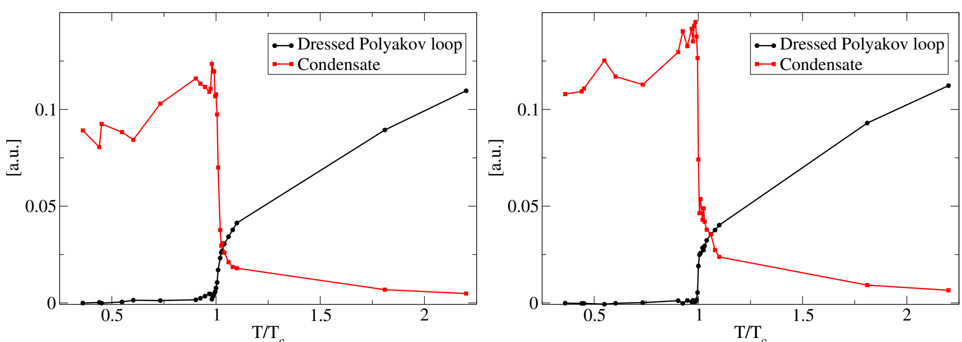

Fig. 3. Dressed Polyakov loop and quark condensate for SU(2) (left) and SU(3) (right).

4.2. Unquenched QCD at finite temperature and chemical potential

We now include two flavors of quarks via the quark loop as explained above. The effect of the matter sector on the gluon is a reduction of the dressing functions, which leads to a reduced interaction strength in the quark self-energy, and therefore to a smaller critical temperature. In Fig. 4 we show the evolution of the order parameters at $\mu = 0$. What we find is a crossover for both the condensate and the dressed Polyakov loop. The value for the pseudo-critical temperature is $T_c^{N_f=2} = 180 \pm 5$ MeV from the quark condensate and $T_c^{N_f=2} = 195 \pm 5$ MeV from the dressed Polyakov loop. The difference in these numbers can be attributed to the crossover nature of the transition.

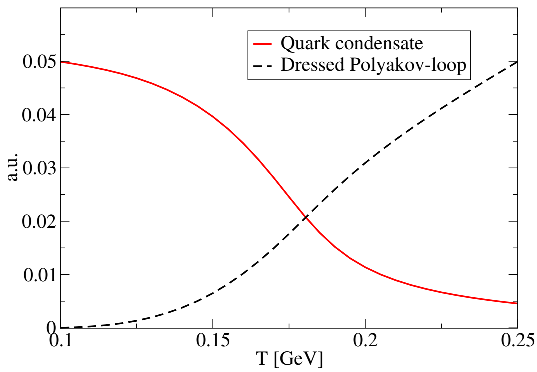

Fig. 4. Dressed Polyakov loop and quark condensate in two flavor QCD as function of temperature at zero chemical potential.

When we go to $\mu > 0$ the condensate for neither periodic ($\varphi = 0$) nor anti-periodic ($\varphi = \pi$) boundary conditions becomes complex. This leads to a difference in Σ_{+1} and Σ_{-1}, *i.e.* in the dressed Polyakov loop and its conjugate. In Fig. 5 the resulting phase diagram of two flavor QCD is shown. For the chiral transition we observe a crossover up to relatively large values of the chemical potential, where we find a critical endpoint at $(T_{\rm EP}, \mu_{\rm EP}) \approx (95, 280)$ MeV. Since $\mu_{\rm EP}/T_{\rm EP} \approx 3 \gg 1$, this result suggests that the CEP is outside the reach of lattice QCD. For the confinement/deconfinement transition we observe that the critical temperature extracted from the dressed Polyakov loop and its conjugate is nearly equal, and close to that extracted from the quark condensate. As the chemical potential is increased the crossover becomes steeper and the two transition lines come closer together, meeting at around $\mu \approx 200$ MeV.

Both results, the CEP at large μ and the coinciding phase transitions agree well with results from the PQM model [3] beyond mean field, where the matter back-reaction on the Yang–Mills sector is also taken into account.

We should note here that at the chemical potentials, where we find the CEP our truncation scheme becomes less reliable, since the influence of baryons is neglected. It may, therefore, be advised to rephrase our results as an exclusion of the CEP in the $\mu/T < 1$ region. This is consistent with longstanding predictions from investigations of the curvature of the chiral critical surface in the Columbia plot [23] and also with recent lattice results on the curvature of the chiral and deconfinement crossover lines at small chemical potential [24].

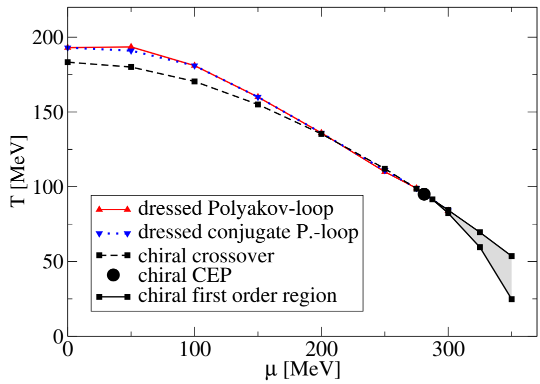

Fig. 5. The phase boundary for chiral symmetry and confinement at real chemical potential. The solid lines above the CEP denote the spinodals which mark the area of coexistence of chiral symmetric and broken solutions of the DSE.

5. Conclusion

We have presented a truncation scheme for the Dyson–Schwinger equations of QCD, where we take data from a lattice calculation for the temperature dependent quenched gluon, and introduce the quark loop for studies of unquenched QCD, namely at finite chemical potential.

Within this truncation we investigated the behavior of the quark condensate as an order parameter for chiral symmetry breaking, and of the dressed Polyakov loop as an order parameter for confinement. In the quenched case at $\mu = 0$ we found that the order parameters reproduce the lattice input, hinting at a second order phase transition for SU(2) and a first order phase transition for SU(3). At finite density we found that thermal fluctuations from the matter sector lead to a critical endpoint at large densities while chiral and deconfinement transitions coincide. Our results serve as a basis for further studies of hot and dense QCD.

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