

CHIRAL RESTORATION PHASE TRANSITION WITHIN THE QUARKYONIC MATTER*

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We overview a possible mechanism for confining but chirally symmetric matter at low temperatures and large densities. As a new development, we employ a diffused quark Fermi surface and show that such diffusion does not destroy possible existence of a confining but chirally symmetric matter at low temperatures and large density.

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1. Introduction

What happens with confinement and chiral symmetry dynamical breaking in the low temperature matter at large density, *i.e.*, which phase will take place just next to the liquid nuclear matter? It was a general belief in the past that deconfinement and chiral restoration phase transitions (crossovers) should coincide. A rationale for such an expectation was the view that the hadron mass generation in the confining mode proceeds through the chiral symmetry breaking in the vacuum and that the hadron mass is practically completely due to the quark condensate of the vacuum. Consequently, beyond the chiral restoration line hadrons cannot exist and the QCD matter should be in the plasma form.

In the vacuum we know from the 't Hooft anomaly matching conditions that indeed in the confining mode chiral symmetry must be realized in the Nambu–Goldstone mode [1]. The essence of the argument is that in the vacuum the anomaly can be saturated only via the massless Goldstone particle associated with the chiral symmetry dynamical breaking. However, in the two-flavor baryonic medium the anomaly can be trivially saturated with the baryon–baryon hole massless excitations and the existence of the massless pion is *a priori* not required.

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Another argument was that, according to Casher [2], chiral symmetry breaking is required for quarks to be confined. Then, naively, hadrons with nonzero mass cannot exist in a world with unbroken chiral symmetry. However, the Casher argument is not general and can be easily bypassed [3]. Recent lattice simulations have convincingly demonstrated that in the world without the low-lying eigenmodes of the Dirac operator (*i.e.*, with the artificially restored chiral symmetry) hadrons still exist and confinement persists [4].

In the large N_c world, confinement survives in a cold matter up to arbitrary large density [5]. In this case nothing can screen the confining gluonic field and gluodynamics is the same as in the vacuum. The allowed excitation modes are of the color singlet nature and it is possible to define the quarkyonic matter as a dense matter with confinement. Then a key question is at which density and how could a dense cold matter be deconfined.

At $T = 0$ deconfinement could happen through the Debye screening of the confining gluon field: A gluon creates the quark–quark hole pair that again annihilates into a gluon. If this vacuum polarization diagram is finite, then at some density confining gluons will be completely screened and the deconfinement would appear. However, in the confining mode the energy of the colored quark–quark hole pair is infinite. The allowed excitations in the confining mode are the color singlet excitations like the baryon–baryon hole pairs, *etc.* These excitation modes cannot screen the colored gluonic confining field. In this sense, the $T = 0$ physics is very different from the high temperature physics where deconfinement (screening) proceeds via the incoherent thermal gluonic loops.

One could expect that the deconfinement in a dense medium should happen due to per-location of baryons. Such a reasoning is too naive, however, as the per-location does not yet imply the screening of the confining gluonic field.

Lattice simulations suggest that for the $N_c = 2$ QCD at low temperatures deconfinement happens at densities of the order of 50 times the nuclear matter density [6]. Since the $N_c = 3$ world is just between two known cases ($N_c = 2$; $N_c = \infty$), we expect that at $N_c = 3$ the deconfinement at low temperatures happens at extremely large densities, much larger than can be achieved in laboratories or in neutron stars.

By definition, the quarkyonic matter is a dense cold matter with confinement. What happens with chiral symmetry breaking in a very dense cold matter with confinement? Is it possible to have a chiral symmetry restoration phase transition in a mode with confinement both below and above the chiral phase transition?

We cannot solve QCD at large density and the only way to address this interesting question is to study such a possibility within a confining and chirally symmetric model. Obviously, such model must provide dynamical chiral symmetry breaking in the vacuum. The simplest possible model that satisfies all requirements is the model of Ref. [7]. It is assumed within the model that there is the linear instantaneous confining potential of the Coulomb type. Such a potential is indeed observed in Coulomb gauge QCD lattice simulations [8] or within the variational approach [9]. Then the chiral symmetry breaking is obtained from the solution of the gap equation. Given the quark Green function derived from the gap equation, one is able to solve the Bethe–Salpeter equation for mesons or the corresponding equations for baryons. An important aspect of this model is that it explicitly demonstrates the effective restoration of chiral symmetry in hadrons with large angular momenta [10,11]. This means that the mass generation mechanism in these hadrons is not through the chiral condensate of the vacuum. Then, it is clear, that there is a chance to obtain within this setup a chirally symmetric but confining matter.

The latter question was addressed in Ref. [12]. Assuming a liquid phase, *i.e.*, that the translational and rotational symmetries are intact (as it is in the nuclear matter), as well as a rigid quark Fermi surface in a dense confining matter, one indeed obtains a chiral restoration phase transition within the confining matter. However, if such a chirally symmetric confining phase exists at low temperatures, relevant degrees of freedom near the Fermi surface are baryons. Quarks interact inside baryons. Consequently, the quark distribution function near the “Fermi surface” must be smooth. In this paper, we report our findings for such a diffused quark “Fermi surface” [13]. Our main conclusion is that for any reasonable diffusion there always exists such a critical “Fermi momentum” of quarks at which the chiral restoration phase transition persists.

2. The model in a vacuum

We use the $SU(2)_L \times SU(2)_R \times U(1)_A \times U(1)_V$ symmetric Hamiltonian with the instantaneous linear Coulomb-like inter-quark potential. This model was intensively used in the past to study chiral symmetry breaking, chiral properties of hadrons, *etc.* [7]. The model can be considered as a straightforward 3+1 dim generalization of the 1+1 dim ’t Hooft model [14]. An important aspect of this model is that at large spins J it exhibits the effective restoration of chiral symmetry in hadrons, *i.e.*, their mass is practically unrelated with the spontaneous breaking of chiral symmetry in the vacuum [10,11].

The self-energy of quarks

$$\Sigma(\vec{p}) = A_p + (\vec{\gamma}\vec{\hat{p}})(B_p - p) \quad (1)$$

consists of the Lorentz-scalar chiral symmetry breaking part A_p and the chirally symmetric part $(\vec{\gamma}\vec{\hat{p}})(B_p - p)$. The unknown functions A_p and B_p are obtained from the gap equation. The functions A_p and B_p , as well as the self-energy of a quark, contain an infrared divergence. Consequently, the single quark energy is infinite. At the same time, this infrared divergence cancels exactly in all possible color-singlet quantities, like the quark condensate, hadron mass, *etc.*, which is a manifestation of confinement. While the model Hamiltonian is manifestly chirally symmetric (it does not contain any mass term), the self-interaction of quarks produces chiral symmetry breaking via the non-zero Lorentz-scalar self-energy A_p . This is how both confinement and chiral symmetry breaking are guaranteed.

3. Effect of a dense medium at $T = 0$

It is practically impossible to solve exactly the model in a dense matter. Indeed, that would imply to solve it first for a single baryon; then to obtain a baryon–baryon interaction; given this interaction to construct a nuclear matter and then slowly to increase its density. Obviously, it is a formidable problem. In order to proceed and get some insight, one needs justifiable simplifications.

In the large N_c limit, the nucleon is infinitely heavy, translational invariance is broken and a many-nucleon system is certainly in a crystal phase. Whether a (dense) nuclear matter will be a liquid or a crystal at $N_c = 3$ is a subject to dynamical calculations. Such microscopical calculations cannot be performed for any “realistic” model in 3+1 dimensions with confinement and (broken) chiral symmetry. However, in the real world $N_c = 3$ we do know that the nuclear matter is in a liquid phase; both translational and rotational invariances are intact. We then assume a liquid phase with manifest translational and rotational invariances in a dense quarkyonic matter.

We treat the system in the mean field approximation and assume first a simple valence quark distribution function, like for the noninteracting quarks, see Fig. 1. Given this quark distribution function we solve the gap equation and at some critical Fermi momentum obtain the chiral restoration phase transition, see Fig. 2.

In a dense matter at $T = 0$, the most important physics that leads to restoration of chiral symmetry is the Pauli blocking by valence quarks of the positive energy levels required for the very existence of the quark condensate. This is similar to the chiral restoration in the Nambu and Jona-Lasinio

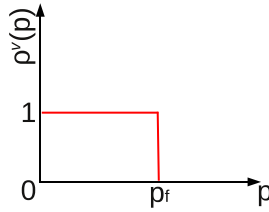


Fig. 1. Valence quark distribution for a rigid quark Fermi surface.

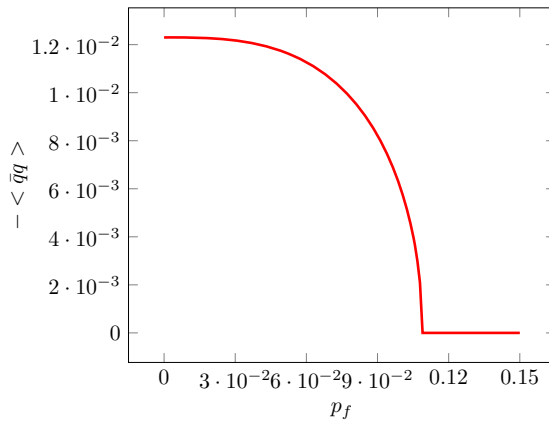


Fig. 2. Quark condensate in units of $\sigma^{3/2}$ as a function of the Fermi momentum, which is units of $\sqrt{\sigma}$.

model. At sufficiently large Fermi momentum the gap equation does not admit a nontrivial solution with broken chiral symmetry. Consequently, the chiral symmetry breaking Lorentz-scalar part A_p of the quark self-energy vanishes and the chiral symmetry gets restored. However, the chirally symmetric part of the quark self-energy does not vanish and is still infrared divergent, like in the vacuum. This means that even with restored chiral symmetry the single quark energy is infinite and the quark is confined. This infrared divergence cancels exactly in all color singlet hadronic modes that remain finite and well defined.

In this respect, the model is radically different from the non-confining NJL model. In the latter a dense matter is a Fermi gas of free quarks. In the Nambu–Goldstone mode these quarks are massive. In the Wigner–Weyl mode they are massless. In our case, physical degrees of freedom that can be excited are color singlet hadrons. In the Wigner–Weyl mode these are the chirally symmetric hadrons.

4. Diffusion of the quark Fermi surface

In reality, valence quarks near the Fermi surface interact and cluster into the color singlet baryons. This interaction in general would lead to a diffusion of the rigid Fermi surface for quarks. Some levels above the “Fermi momentum” must be occupied with some probability as well as some levels below the “Fermi momentum” with some probability must be empty.

In principle, the quark distribution function near the diffused Fermi surface could be obtained self-consistently from the full solution of the problem. It is a formidable task and such a program cannot be performed. With the present state of the arts it is difficult to obtain a microscopic insight into the dynamics of the diffusion. However, it is clear that the realistic distribution function will be smooth, of the form in Fig. 3.

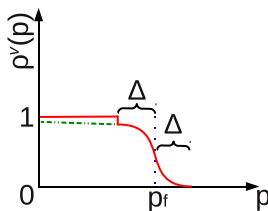


Fig. 3. Valence quark distribution for a diffused quark Fermi surface.

We parameterize a smooth valence quark distribution function by

$$\rho^v(p) = \Theta(-p + p_f - \Delta) + \Theta(p - p_f + \Delta) \frac{1}{e^{(p-p_f)/\Delta} + 1} \quad (2)$$

and solve the gap equation for different p_f and diffusion width Δ .

For each fixed diffusion width Δ there always exists such critical “Fermi momentum” at which the chiral restoration phase transition does take place. This can be seen from Fig. 4, where a line of “critical Fermi momenta” is depicted. The area above this critical line corresponds to the chirally symmetric phase, while all points below the critical line represent a matter with broken chiral symmetry.

This can be easily understood. At all momenta $p \ll p_f$ the Pauli blocking in Fig. 3 is the same as for the rigid quark Fermi surface. At momenta just below the p_f the effect of the Pauli blocking is weaker than for the rigid Fermi surface. However, this is compensated by additional Pauli blocking of the levels that are just above the Fermi momentum for the rigid quark distribution.

This does imply that in a sufficiently dense strongly interacting matter in the confining mode, assuming that it is a liquid phase, the chiral restoration does take place. The mass generation mechanism in such chirally symmetric but confining liquid is not related to the chiral symmetry breaking.

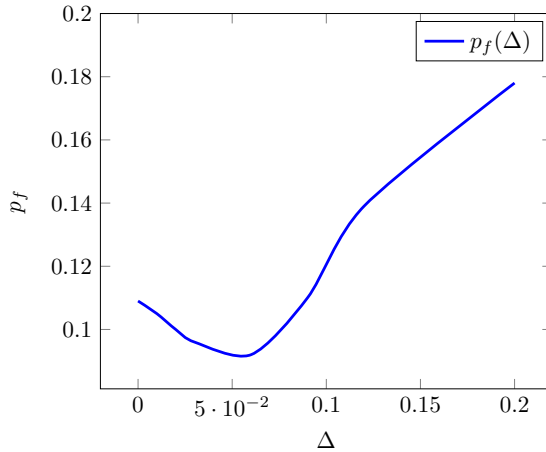


Fig. 4. Critical line that separates the quarkyonic matter with broken and restored chiral symmetry.

Another explicit illustration of chiral symmetry of a dense matter above the chiral restoration phase transition are properties of hadronic excitations. In the Nambu–Goldstone mode of chiral symmetry there must be a massless excitation mode that is associated with the massless pion. At the same time, energies of all other mesons must be finite. In particular, there must be a finite splitting of the excitations with quantum numbers $I, J^{\text{PC}} = 1, 0^{-+}$ and $I, J^{\text{PC}} = 0, 0^{++}$, that will be referred as the pion and the σ -meson, respectively, according to the standard nomenclature. In contrast, these excitations must be exactly degenerate in the Wigner–Weyl mode of chiral symmetry and form the $(1/2, 1/2)_a$ representation of the $\text{SU}(2)_L \times \text{SU}(2)_R$ chiral group.

To obtain the quark–antiquark bound states we solve the homogeneous Bethe–Salpeter equation in the rest frame. In the Wigner–Weyl mode, *i.e.*, when dynamical quark mass and chiral angle vanish, $M(p) = 0$; $\varphi_p = 0$, the Bethe–Salpeter equations for the $1, 0^{-+}$ and $0, 0^{++}$ bound states become identical and consequently energies of these states coincide.

In Fig. 5, we show masses of both pseudo-scalar and scalar modes for different “Fermi momenta” p_f and diffusion widths Δ . For each Δ there is a critical $p_f^{\text{cr}}(\Delta)$ at which the chiral restoration phase transition takes place. Below this $p_f^{\text{cr}}(\Delta)$ there is a massless pion and a massive σ -meson. Above the critical p_f both the pion and the σ -meson are massive and exactly degenerate.

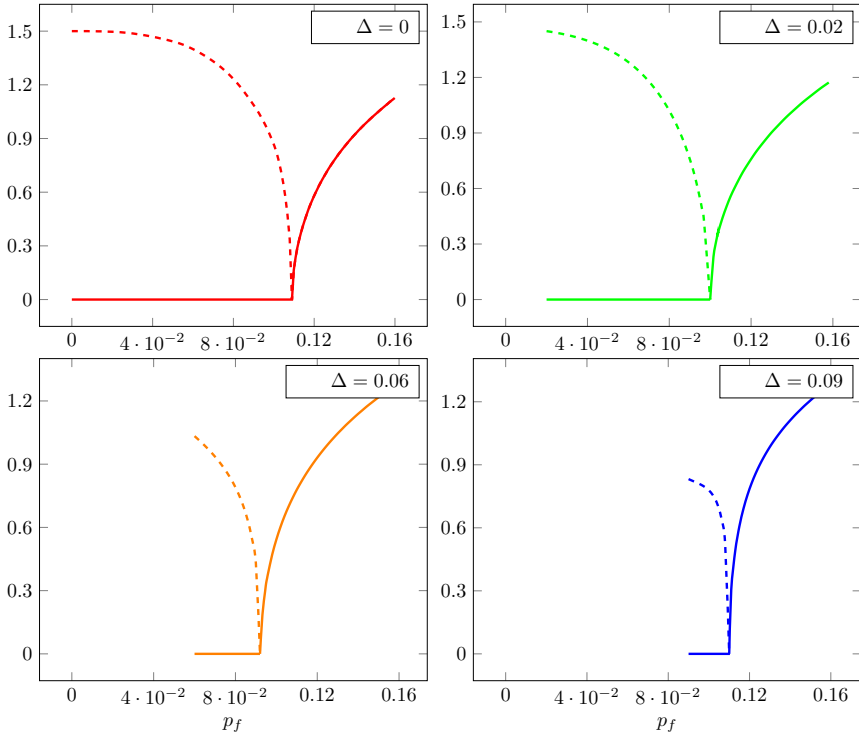


Fig. 5. Masses of the pseudoscalar (solid) and scalar (dashed) mesons in units of $\sqrt{\sigma}$ as functions of the “Fermi momentum” p_f and of the diffusion width Δ (in units of $\sqrt{\sigma}$).

5. Conclusions

In the confining mode, the valence quarks interact and near the Fermi surface cluster into the color singlet baryons. This implies that there cannot be a rigid quark Fermi surface and the valence quark distribution function near the Fermi surface must be smooth. Assuming a liquid phase, *i.e.*, unbroken translational and rotational invariances, we parameterize such diffused “Fermi surface” by a simplest possible function and solve the corresponding gap and Bethe–Salpeter equations. By this, we verify whether a chiral phase transition, previously observed for a rigid quark Fermi surface, survives or not. It turns out that for any reasonable diffusion width there always exists such a “Fermi momentum” that the chiral restoration phase transition does take place. This reconfirms our previous conclusions about possible existence of the confining but chirally symmetric phase. Below the phase transition the elementary excitation modes of a matter are hadrons with broken chiral symmetry, while above the phase transition such excitations are chirally symmetric hadrons.

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