UNIVERSAL PROPERTIES OF MOMENTS OF NET BARYON NUMBER FLUCTUATIONS*

Frithjof Karsch

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA
and
Fakultät für Physik, Universität Bielefeld, 33615 Bielefeld, Germany

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We discuss universal properties of higher order cumulants of net baryon number fluctuations and point out their relevance for the analysis of freeze-out and critical conditions in heavy ion collisions at LHC and RHIC. We focus on a discussion of universal properties of sixth order cumulants and compare with calculations performed in the Polyakov loop extended Quark Meson model.

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1. Introduction

Higher order cumulants, of conserved charges are considered to be central observables in the search for the critical point that has been suggested to exist in the QCD phase diagram at some non-zero value of the baryon chemical potential [1]. The search for this critical point is one of the main motivations for the ongoing low energy runs at RHIC. First results on net baryon number fluctuations and their cumulants up to fourth order have been reported recently by the STAR Collaboration [2]. It has been pointed out that these experimental findings are in reasonable agreement with hadron resonance gas (HRG) model calculations [3]. Although small discrepancies between experimental results and HRG model calculations have been found when analyzing in more detail the shape of net proton number distributions [4], this may raise the question whether the thermal conditions at freeze-out are sensitive to critical behavior or not.

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Thermal properties of higher order cumulants of conserved charge fluctuations can also be analyzed in equilibrium thermodynamics of QCD, *e.g.* by performing lattice QCD calculations. The early calculations of net quark number fluctuations [5,6,7] and their extension to fluctuations of conserved charges [8], although performed on rather coarse lattices, demonstrated quite convincingly that ratios of cumulants of baryon number, electric charge or strangeness fluctuations, are quite sensitive probes for detecting critical behavior in QCD. They are sensitive to universal scaling properties at vanishing as well as non-vanishing baryon chemical potential (μ_B) and directly reflect the internal degrees of freedom that carry the corresponding conserved charge [7,9] in a thermal medium. Ratios of cumulants of conserved charges change rapidly in the crossover region corresponding to the chiral transition in QCD and reflect the change from hadronic to partonic degrees of freedom [10]. Higher order cumulants become increasingly sensitive to critical behavior as they are obtained from high order derivatives of the QCD partition function and thus enhance singular contributions over regular terms in the partition function. Scaling properties of higher order cumulants that can be deduced from critical behavior of strongly interacting matter as it is described by QCD are, of course, quite different from predictions based on HRG model calculations, which are insensitive to any form of critical behavior. If freeze-out happens close to any critical point or line in the QCD phase diagram, one thus should expect to observe deviations from HRG model results that show up more prominently in higher order cumulants.

In this paper, we focus on a discussion of the structure of sixth order cumulants of the fluctuations of net baryon number. After a discussion of the QCD phase diagram in the next section we turn to a discussion of universal properties of higher order cumulants in Sec. 3. In Sec. 4 we discuss how these general universal properties show up in QCD motivated model calculations. We conclude in Sec. 5.

2. The chiral phase transition, crossover and freeze-out

In the limit of vanishing light quark masses, strongly interacting matter, as described by QCD, will undergo a thermal phase transition at a critical temperature T_c. In QCD with 2 massless (u, d) quarks the chiral $SU(2)_L \times SU(2)_R$ symmetry, which is isomorphic to O(4), gets restored at this temperature. If the breaking of the axial $U(1)_A$ symmetry of QCD does not get strongly reduced at the same time and its breaking thus remains 'substantial', the transition will be second order, belonging to the universality class of 3-dimensional O(4) symmetric spin models [11]. Recent studies of scaling properties of the chiral condensate support this scenario [12] and also suggest [13] that the chiral phase transition stays second order for small, but non-zero values of the baryon chemical potential (μ_B). As this second order phase transition line does not seem to reach zero temperature at a finite value of μ_B, it is expected to either end in a tri-critical point at temperature $T_{\rm tri}$ and baryon chemical potential $\mu_{\rm tri}$ [1] (see Fig. 1), or continue to persist up to infinite temperature.

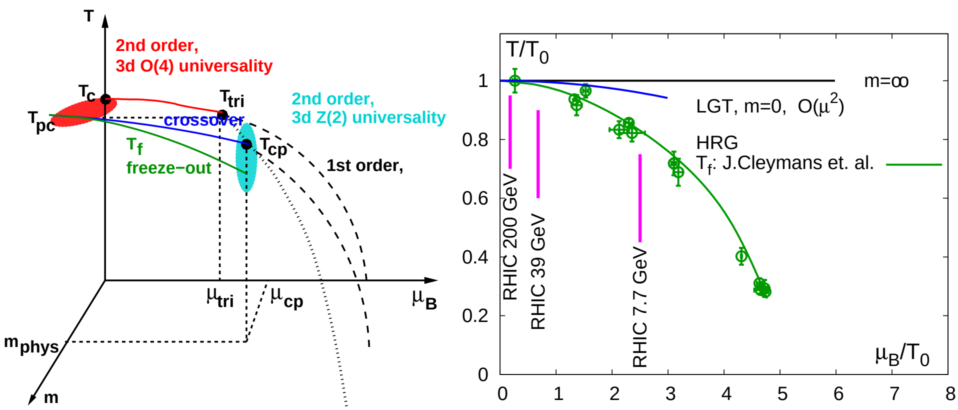

Fig. 1. Phase diagram of QCD in the space of temperature, baryon chemical potential and light quark mass (left) and the freeze-out line determined from a comparison of ratios of particle yields measured at RHIC and hadron resonance gas model calculations (right). Also shown in the right-hand figure are results for the crossover transition line, calculated in lattice QCD to leading order in the square of the baryon chemical potential [13].

For any non-zero value of the light quark masses, however, the chiral phase transition will only be a crossover transition, characterized by pseudo-critical temperatures $T_{\rm pc}(\mu_B)$, which reduce to the critical temperature in the chiral limit. In the vicinity of $T_{\rm pc}(\mu_B)$ thermodynamic observables, obtained as derivatives of the free energy, will show universal scaling behavior for small but non-zero values of the light quark masses. Whether the regime of physical light quark mass values is close enough to the chiral limit so that bulk thermodynamic observables remain sensitive to these universal scaling properties is an important question to clarify. Current lattice studies of the quark mass dependence of scaling relations at $\mu_B = 0$ suggest that this is the case [12]. It thus seems that at least at vanishing baryon chemical potential studies of thermal properties of the QCD medium with its physical quark masses should be sensitive to the existence of a phase transition in the massless (chiral) limit of QCD[1].

[1] We note, however, that current lattice studies of universal scaling properties have been performed on rather coarse lattices and need to be verified through calculations on finer lattices.

Another line in the QCD phase diagram, the chemical freeze-out line $T_{\rm f}(\mu_B)$, is well established experimentally. It is characterized by values of the temperature and baryon chemical potential at which hadrons start to form in the expanding dense medium generated in a heavy ion collision. The parametrization of this line has been obtained by comparing ratios of different hadron yields with calculations in a HRG model [14]. Empirically, one finds that for small values of the chemical potential the freeze-out temperature $T_{\rm f}$ and estimates for the pseudo-critical temperatures $T_{\rm pc}$ in QCD with physical light quark masses are quite close to each other. However, differences between these temperatures as well as the chiral phase transition temperature $T_{\rm c}(\mu_B)$ seem to increase with increasing values of μ_B. A calculation of the latter, using universal scaling properties of the chiral order parameter yields a curvature of the phase transition line [13], that is significantly smaller than the curvature of the freeze-out line obtained from a parametrization of experimental data [14] (see Fig. 1 (right)). It thus seems that with increasing baryon chemical potential freeze-out in heavy ion collisions happens *further away* from the QCD chiral critical line in the QCD phase diagram as well as from the crossover line that characterizes the QCD transition at physical values of the quark masses. Nonetheless, the hope is that $T_{\rm f}(\mu_B)$ stays close enough to the (not yet established) QCD critical point at $T_{\rm cp}$ (see Fig. 1 (left)) so that experimental studies of hadron properties at the time of freeze-out remain sensitive to critical behavior in the vicinity of this second order phase transition point.

Critical behavior goes along with large fluctuations, *i.e.*, large correlation lengths, of thermodynamic response functions that couple to the relevant thermal control parameters (T, μ_B). As far as studies of critical behavior with observables sensitive to *equilibrium thermodynamics* at $T_{\rm f}$ is concerned, the situation in the vicinity of the QCD critical point at $(T_{\rm cp}, \mu_{\rm cp})$ as well as the chiral phase transition at $T_{\rm c}(\mu_B = 0)$ (shaded areas in Fig. 1) is very similar. Experimentally, one tests whether freeze-out happens close to a critical point by checking whether scaling properties that leave their imprint in the non-analytic structure of certain thermodynamic observables are detectable at the time of chemical freeze-out. The basic tool to do so is the analysis of event-by-event fluctuations [15].

As indicated above, a comparison of experimental results on the freeze-out line and lattice QCD results on the chiral phase transition line suggests that evidence for critical behavior may more easily be established in studies of higher order cumulants close to or at vanishing μ_B than at larger values of μ_B. The analysis of higher order cumulants now performed in lattice calculations at vanishing baryon chemical potential thus are of direct relevance for studies at the LHC as well as the highest beam energies at RHIC, where leading order corrections in μ_B/T, that can be introduced using Taylor expansions, are still small [16, 17].

3. Universal properties of higher order cumulants

Close to the chiral limit and at temperatures near the chiral phase transition temperature $T_{\rm c}$, higher order derivatives of the free energy density (f) with respect to temperature or chemical potential are increasingly sensitive to the non-analytic (singular) contribution to the free energy density. The free energy density may be represented in terms of singular ($f_{\rm s}$) and regular ($f_{\rm r}$) contributions

$$f(T,\mu_q,m_q) = f_{\rm s}(T,\mu_q,m_q) + f_{\rm r}(T,\mu_q,m_q)\,. \tag{1}$$

In addition to the dependence on temperature T, we also introduced here an explicit dependence on the light quark chemical potential, $\mu_q = \mu_B/3$. In the vicinity of any point $(T_{\rm c},\mu_{\rm c})$ on the chiral phase transition line the singular part of the free energy may be written as

$$\frac{f_{\rm s}(T,\mu_q,h)}{T^4} = A h^{1+1/\delta} f_f(z)\,, \qquad z \equiv \frac{t}{h^{1/\beta\delta}}\,, \tag{2}$$

where β and δ are critical exponents of the 3-dimensional O(4) spin model [18] and

$$t \;\equiv\; \frac{1}{t_0}\left(\frac{T-T_{\rm c}}{T_{\rm c}} + \kappa_q\left(\left(\frac{\mu_q}{T}\right)^2 - \left(\frac{\mu_{\rm c}}{T_{\rm c}}\right)^2\right)\right)\,, \qquad h \equiv \frac{1}{h_0}\frac{m_q}{T_{\rm c}} \tag{3}$$

with non-universal scale parameters $t_0,\ h_0$. We also suppressed any possible dependence of the non-universal parameters $T_{\rm c},\ t_0,\ h_0,\kappa$ on $\mu_{\rm c}$.

The net baryon number fluctuations and the corresponding cumulants are obtained from Eq. (1) by taking derivatives with respect to $\hat{\mu}_q = \mu_q/T$

$$\chi_n^B = -\frac{\partial^n f/T^4}{\partial \hat{\mu}_B^n}\,. \tag{4}$$

From Eq. (2) it is apparent that, in the vicinity of the critical temperature, the susceptibilities χ_n^B show a strong dependence on the explicit symmetry breaking term, the quark mass

$$\chi_n^B \sim \begin{cases} -(2\kappa_q)^{n/2} h^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z)\,, & \text{for } \mu_{\rm c}/T = 0\,, \text{ and } n \text{ even} \\ -(2\kappa_q)^n \left(\frac{\mu_{\rm c}}{T}\right)^n h^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z)\,, & \text{for } \mu_q/T > 0\,, \end{cases} \tag{5}$$

where the critical exponent α is related to $\beta,\ \delta$ through a hyper-scaling relation $2-\alpha = \beta\delta(1+1/\delta)$. As $\alpha < 0$ for the 3d, O(N) universality class, the first cumulants that diverge in the chiral limit are χ_6^B at $\mu_{\rm c}/T = 0$ or χ_3^B at $\mu_{\rm c}/T > 0$. The behavior of both susceptibilities is controlled by the same universal scaling function $f_f^{(3)}(z)$. Relative to the strength of the singularity at $\mu_c/T = 0$ the amplitude of the singular term contributing to χ_3^B, however, is suppressed by a factor $(\mu_c/T)^3$.

The universal scaling function $f_f(z)$, which characterizes the singular behavior of models in the universality class of 3-dimensional, O(4) symmetric models, and its derivatives have been analyzed recently [18]. We show in Fig. 2 (left) the third derivative of $f_f(z)$ and the resulting scaling behavior of the singular part of the 6th order cumulant of net baryon number fluctuations (Fig. 2 (right)). We note that χ_6^B diverges to $\pm\infty$ when approaching T_c from below/above. This reflects the behavior of $f_f^{(3)}(z)$ in the limit $z \to \pm\infty$. In the chiral limit one obtains for $\mu_B/T = 0$

$$\chi_6^B \sim -(2\kappa_q)^3 \left|\frac{T-T_c}{T_c}\right|^{-1-\alpha} f_\pm^{(3)}\,, \tag{6}$$

where $f_\pm^{(3)} = \lim_{z\to\pm\infty} |z|^{1+\alpha} f_f^{(3)}(z)$. The behavior of the universal scaling functions $f^{(3)}(z)$ for large $|z|$ thus controls the change of sign in χ_6^B in the vicinity of $t \simeq 0$. Whether such a change of sign indeed occurs in QCD and, if so, where exactly this change of sign occurs at non-zero values of the quark mass (non-zero h), depends on the relative magnitude of the regular and singular contributions to the free energy. This also controls the ratio of maximal and minimal value of χ_6^B. In fact, in the chiral limit this ratio is a universal number and deviations from it can be used to quantify the influence of regular terms relative to the singular contributions. As can be seen in Fig. 2,

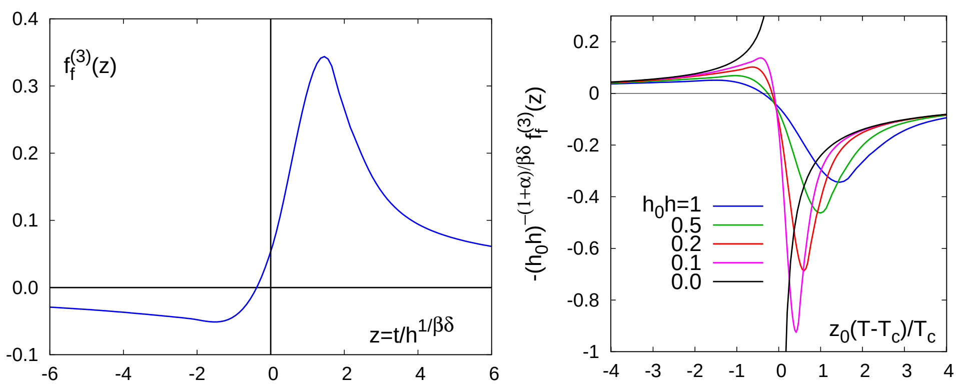

Fig. 2. The third derivative of the scaling function controlling the singular part of the free energy [18] in theories belonging to the 3d, O(4) universality class (left) and its contribution to third or higher order cumulants (see Eq. (2)) (right) [19]. Here h_0 and $z_0 = h_0^{1/\beta\delta}/t_0$ are non-universal scale parameters.

the maximum and minimum of χ_6^B both diverge in the chiral limit. The maximum at $T_{\max} < T_c$ is substantially shallower than the deep minimum at $T_{\min} > T_c$. In the chiral limit, the ratio of the cumulant χ_6^B at these two extrema takes on a universal value,

$$\lim_{h \to 0} \frac{\chi_6^B(T_{\min})}{\chi_6^B(T_{\max})} \simeq -6.7\,. \tag{7}$$

4. A comparison with model calculations

The generic structure deduced from the universal O(4) scaling of cumulants indeed is seen in the calculation of higher order cumulants in lattice QCD [8] as well as in model calculations [19]. We show in Fig. 3 results from an analysis performed in the Polyakov loop extended quark meson (PQM) model. Figure 3 (left) shows results for χ_6^B at $\mu_B = 0$. The expected change in sign is clearly visible, indicating that the singular contributions at the physical value of the pion mass indeed dominate over the regular terms, although the ratio of minimal to maximal value of χ_6^B is only about -1.5, *i.e.* it is about a factor four smaller than the universal asymptotic value given in Eq. (7).

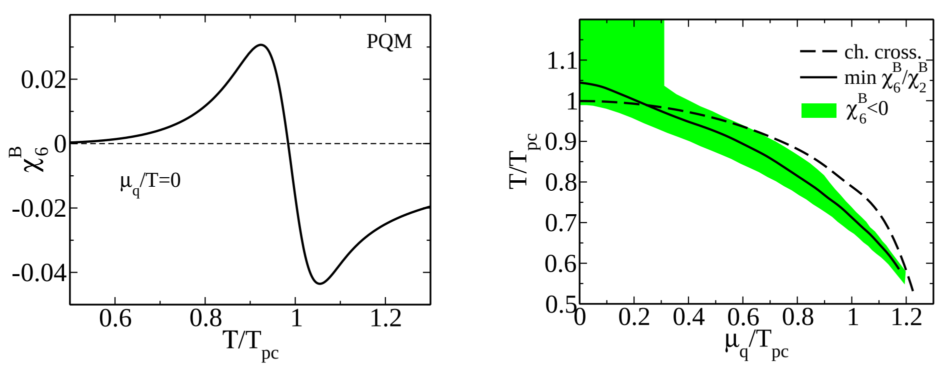

Fig. 3. The sixth order cumulant of net baryon number fluctuations calculated in the PQM model at $\mu_B/T = 0$ (left) and the region in which this cumulant is negative for $\mu_B/T \geq 0$ (right). The solid line indicates the location of the minimum in χ_6^B/χ_2^B and the dashed line gives the crossover transition line determined from the maximum in the chiral susceptibility.

In Fig. 3 (right) we show the region in the $T-\mu$ plane in which χ_6^B stays negative for $\mu_B > 0$. For small values of μ_B/T this regime extends to infinite temperature, reflecting the structure of χ_6^B at $\mu_B = 0$. For larger μ_B/T, however, this regime is restricted to a narrow band in the vicinity of the crossover temperature. This too can be understood in terms of contributions from the singular part of the free energy. In sub-leading order χ_6^B receives contributions from χ_8^B that are proportional to $(\mu_B/T)^2$. The eighth order cumulant is more singular than χ_6^B and approaches $+\infty$ for $T \to T_c^+$. Also at non-zero values of the quark mass (h) this contribution thus is expected to dominate for sufficiently large μ_B/T. This makes the region, in which χ_6^B can be negative, shrink to a small interval around the crossover transition line. The structure of χ_8^B is illustrated in Fig. 4. It is apparent that χ_8^B calculated in the PQM model (left) resembles closely the structure expected to be found from the singular part of 3d, O(4) symmetric models. In particular, we note that the ratio of the minimal value of χ_8^B to the maximum on the high temperature side is quite similar to the universal value expected from an analysis of the corresponding extrema of the O(4) scaling function shown in Fig. 4 (right). This suggests that the regular contributions are indeed negligible in high order cumulants.

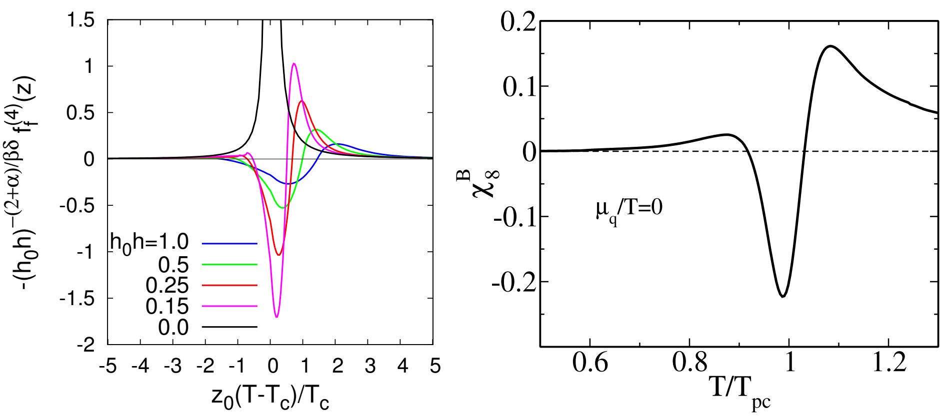

Fig. 4. The eighth order cumulant of net baryon number fluctuations calculated in the PQM model at $\mu_B/T = 0$ (right) and universal scaling of this cumulant obtained from an analysis of 3d, O(4) symmetric spin models (left).

5. Conclusions

We have discussed the general structure of higher order cumulants of net baryon number fluctuations resulting from universal scaling properties of these cumulants that dominate their behavior in the chiral limit. We discussed in detail the structure of the sixth order cumulant at zero and non-zero values of the baryon chemical potential and showed how the appearance of a region of negative χ_6^B and its structure can be understood in terms of universal scaling properties in the vicinity of a chiral phase transition line. A similar discussion can also be given for lower order cumulants, *e.g.* χ_3^B

or χ_4^B. They are finite at $\mu_B/T = 0$, but will receive singular contributions from $f_f^{(n)}(z)$, $n \geq 3$, at $\mu_B/T > 0$. This too will lead to regions of negative cumulants in the vicinity of the crossover temperature.

We have shown that these properties are robust features of cumulants that also manifest themselves in model calculations, although at physical values of the pion mass they are influenced by non-singular, non-universal contributions. Whether these features of higher order cumulants become detectable in heavy ion experiments crucially depends on the location of the freeze-out temperature in the QCD phase diagram in comparison to the chiral crossover line.

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