HYDRODYNAMIC EVOLUTION OF FLUCTUATIONS IN HOT QUARK MATTER*

Igor Mishustin

Frankfurt Institute for Advanced Studies
J.W. Goethe Universität, Frankfurt am Main, Germany
and
National Research Center "Kurchatov Institute", Moscow, Russia

(Received January 2, 2012)

Observable manifestations of the chiral/deconfinement phase transition in relativistic heavy-ion collisions may be strongly affected by the fast expansion of the produced quark-gluon plasma. We study this effect within the linear sigma model with constituent quarks, which predicts a chiral phase transition for a static system in thermal equilibrium. We derive coupled equations for the hydrodynamic variables and the order-parameter field, so-called chiral fluid dynamics. Stability of the chiral fluid in the static and expanding backgrounds is investigated by considering the evolution of fluctuations with respect to the mean-field solution. The effects of supercooling and reheating in the case of the first order phase transition are studied for the Bjorken-like background.

DOI:10.5506/APhysPolBSupp.5.791
PACS numbers: 25.75.q, 47.75.+f, 11.30.Qc, 24.60.Ky

1. Introduction

The investigation of the phase diagram of strongly interacting matter, has remained in the focus of theoretical and experimental studies for more than two decades. In particular, searching for the critical point and onset of the deconfinement phase transition in relativistic heavy-ion collisions is an active research area. Most previous studies of phase transition signatures were based on the equilibrium concepts. On the other hand, a relativistic heavy-ion collision is a very fast process and one should expect that the phase transition may be strongly affected by the dynamics. Non-equilibrium effects associated with the chiral/deconfinement phase transition have been studied within several macroscopic approaches [1, 2, 3, 4, 5, 6, 7, 8, 9].

* Talk presented at HIC for FAIR Workshop and XXVIII Max Born Symposium "Three Days on Quarkyonic Island", Wrocław, Poland, May 19–21, 2011.

In the region of the phase diagram, where the deconfinement phase transition is of first order, an extra time-scale appears, which is associated with the nucleation process [10]. In this paper, we are particularly interested in the competition between the instability associated with the first order phase transition and expansion of the deconfined fluid produced in relativistic heavy-ion collisions. Only when the phase transition timescale is short with respect to the hydrodynamic timescale, we can assume a two-phase equilibrium and the Equation of State given by the Maxwell construction [11,12,13,14]. However, if this is not the case, the fluid can pass through a thermodynamically unstable region of the Equation of State, leading to large-scale fluctuations, which themselves can be used as a signal of the phase transition. Generally, a fast expansion of the fluid leads to a suppression of the nucleation process in favor of the spinodal decomposition [4, 7]. To investigate this effect, below we study the stability of fluctuations around a hydrostatic state and Bjorken-type expanding state. By comparing the results, we will be able to identify these features of the phase transition which are affected by the fast dynamics.

This paper is based on theoretical developments and extended calculations presented in Refs. [15, 16]. For our analysis, we use a generalized hydrodynamic approach, namely, so-called Chiral Fluid Dynamics (CDF), where the fluid evolution is coupled with the dynamics of the chiral order parameter. This model was first proposed in Ref. [17] and further developed in several subsequent works [16, 18, 19, 20, 21]. It is derived by assuming that microscopic and macroscopic degrees of freedom are clearly separated. Then, the coarse-grained macroscopic dynamics can be described by a reduced number of variables, which are called the gross variables.

2. Derivation of chiral fluid dynamics

As the low-energy effective theory of QCD, we adapt the linear sigma model with constituent quarks [22] whose qualitative features (chiral symmetry, universality class, phase transition structure) are thought to coincide with the QCD [23, 24]. More recently, the thermodynamics of this model was studied on the mean-field level [25], as well as including the field fluctuations [26, 27]. Following the previous works [17, 18], we describe the coarse-grained dynamics of the quark degrees of freedom with the hydrodynamic variables, coupled to the order parameter field σ via its equation of motion.

The Lagrangian of the linear σ model is

$$\mathcal{L} = \bar{q}\left(i\gamma^{\mu}\partial_{\mu} - g\left(\sigma + i\gamma_5\vec{\tau}\vec{\pi}\right)\right)q + \tfrac{1}{2}\left[(\partial_{\mu}\sigma)^2 + (\partial_{\mu}\vec{\pi})^2\right] - V(\sigma, \pi)\,, \qquad (1)$$

where q is the quark field, σ and $\vec{\pi}$ are the chiral fields. The “Mexican Hat” potential V is given by

$$V(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}\left(\sigma^2 + (\vec{\pi})^2 - v^2\right)^2 - H\sigma\,, \tag{2}$$

where the parameters λ, v and H are calculated by using the pion mass $m_\pi = 138$ MeV, the pion decay constant $f_\pi = 93$ MeV, and assumed sigma mass $m_\sigma = 600$ MeV: $H = f_\pi m_\pi^2$, $v^2 = f_\pi^2 - m_\pi^2/\lambda^2$ and $\lambda^2 = (m_\sigma^2 - m_\pi^2)/(2f_\pi^2)$ [25].

The coupling constant g is usually chosen so as to reproduce the one-third of the vacuum nucleon mass, that is, $g = 3.3$. Then, the chiral phase transition at the vanishing chemical potential is shown to be of the crossover type. But the phase diagram has a critical (end) point at a finite baryon chemical potential, where the order of the phase transition changes to the first order [25]. In this paper, we limit our consideration to the vanishing chemical potential only. In this case, the effect of the first order phase transition can be studied by changing the magnitude of the coupling constant g, as was proposed in Ref. [5]. Below, we consider the cases of crossover ($g = 3.3$), second order ($g = 3.6$, $T_\mathrm{c} = 140$ MeV) and first order ($g = 4.5$, $T_\mathrm{c} = 128.5$ MeV), phase transitions.

For the sake of simplicity, the pion field is disregarded in the rest of the paper. We consider an idealized situation, where the quark degrees of freedom have already achieved the local thermal equilibrium and can be approximately described as an ideal fluid, characterized by the energy density ε, the pressure P and the four-velocity u^μ. On the other hand, the sigma field behaves as a classical external field (the chiral order parameter) acting on quarks through the mass term $M = g\sigma$. Then, the energy momentum tensor of the system can be represented as

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{fluid}} + T^{\mu\nu}_{\mathrm{field}}\,. \tag{3}$$

The energy-momentum tensor of an ideal fluid is generally represented as

$$T^{\mu\nu}_{\mathrm{fluid}} = (\varepsilon + P)u^\mu u^\nu - P g^{\mu\nu}\,, \tag{4}$$

where ε and P are the energy density and pressure of the fluid in the rest frame.

These quantities are calculated via the thermodynamic potential of the quark sector in the mean-field approximation

$$\begin{aligned}\Omega(M) \;=\; -\nu_q T \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\Bigg\{ &\ln\left[1 + \exp\left(\frac{\mu - E_p}{T}\right)\right] \\ &+ \ln\left[1 + \exp\left(\frac{-\mu - E_p}{T}\right)\right]\Bigg\}\,,\end{aligned} \tag{5}$$

where T, μ and E_p are temperature, quark chemical potential and single-particle energy $\sqrt{\boldsymbol{p}^2+M^2}$ with M being the constituent quark mass, respectively. The degeneracy factor $\nu_q = 2N_cN_f = 12$, where $N_c = 3$ is the number of colors and $N_f = 2$ is the number of flavors. Strictly speaking, the above expressions are valid only when M is a space-time independent constant determined by the minimum of the total thermodynamic potential

$$\Omega_{\rm tot}(\sigma,\pi) = \Omega(M) + V(\sigma,\vec{\pi})\,. \tag{6}$$

However, in our exploratory study below, we assume that $M = g\sigma(x)$ even when the $\sigma(x)$ is varying in space and time according to the equation of motion obtained from the Lagrangian

$$\partial^2\sigma + \lambda\left(\sigma^2-\sigma_0^2\right)\sigma - H = -g\bar{q}q\,. \tag{7}$$

Further on, we replace the term $\bar{q}q$ in the r.h.s. by the thermal expectation value, that is, the quark scalar density

$$\langle\bar{q}q\rangle \equiv \rho_{\rm s}(x) = 2\nu_q\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{M}{E_p}f(E_p)\,, \tag{8}$$

where we have introduced the Fermi distribution function, $f(E)=\left(e^{\frac{E}{T}}+1\right)^{-1}$. As was already mentioned above, in the following calculations we consider only the case of $\mu = 0$.

After separating the quark contribution, the energy-momentum tensor of the σ field is obtained from the meson part of Lagrangian (1)

$$T^{\mu\nu}_{\rm field} = (\partial^\mu\sigma)(\partial^\nu\sigma) - g^{\mu\nu}\left[\tfrac{1}{2}(\partial_\mu\sigma)^2 - V(\sigma,0)\right]\,. \tag{9}$$

Now the continuity equation for the total energy-momentum tensor (3), $\partial_\nu T^{\mu\nu} = 0$, can be written as

$$\partial_\nu T^{\mu\nu}_{\rm fluid} = -\partial_\nu T^{\mu\nu}_{\rm field} \equiv S^\mu\,, \tag{10}$$

where the source term is given by

$$S^\mu = -\left[\partial^2\sigma + \lambda^2\left(\sigma^2-\sigma_0^2\right)\sigma - H\right]\partial^\mu\sigma = g\rho_{\rm s}\partial^\mu\sigma\,. \tag{11}$$

This term gives rise to the coupling between the ideal fluid and the evolution of the chiral order parameter.

3. Stability analysis in a static background

In this section, we investigate the stability of the chiral fluid with respect to perturbations of the static background characterized by the chiral order parameter σ_0, temperature T and four-velocity $u_0^\mu = (1,0,0,0)$. Obviously, when σ_0 is initially chosen on the slope or at the maximum of the thermodynamic potential, at later times it will roll down to reach the minimum of the potential. Without any analysis we know that such a state is unstable. Such situation corresponds to the spinodal decomposition. Thus in the following, we discuss the stability only around the local minima of the thermodynamic potential and σ_0 is chosen to be the solution of the gap equation (7).

Let us introduce the plan-wave perturbations of these quantities in the x direction around the hydrostatic state

$$\delta\sigma(x) = \delta\sigma(\omega,k)e^{i\omega t - ikx}\,, \tag{12}$$

$$\delta T(x) = \delta T(\omega,k)e^{i\omega t - ikx}\,, \tag{13}$$

$$\delta u^1(x) = \delta u^1(\omega,k)e^{i\omega t - ikx}\,. \tag{14}$$

Then, the perturbed fluid characteristics $F = (\varepsilon, P, \rho_{\rm s})$ can be expressed as

$$F(\sigma,T) = F(\sigma_0,T) + \left(\frac{\partial F(\sigma_0,T)}{\partial T}\right)_{\sigma_0}\delta T(x) + \left(\frac{\partial F(\sigma_0,T)}{\partial\tilde{\sigma}}\right)_T\delta\sigma(x)\,. \tag{15}$$

By linearizing the fluid dynamical equations (10) and the equation of motion (7) for these perturbations, we obtain the following matrix equation

$$\mathcal{A}X = 0\,, \tag{16}$$

where $X^T = (\delta T(\omega,k), \delta u^x(\omega,k), \delta\sigma(\omega,k))$ and $\mathcal{A}$ is given in Ref. [15]. The dispersion relations for perturbations are obtained by solving the equation

$$\det[\mathcal{A}] = 0\,. \tag{17}$$

When there is no coupling between the quark fluid and the chiral order parameter, that is, $g = 0$, the dispersion relations are given by

$$\omega^2 = \left(\frac{\partial P}{\partial\varepsilon}\right)_{\sigma_0} k^2\,, \tag{18}$$

$$\omega^2 = k^2 + \lambda^2\left(3\sigma_0^2 - v^2\right)\,. \tag{19}$$

The physical interpretation of these equations is as follows: the first solution describes the sound wave in the quark fluid, while the second one gives the dispersion relation for the sigma field fluctuations. The sound velocity,

$c_s^2 = (\partial P/\partial\varepsilon)_{\sigma_0}$, can be negative at the first order phase transition. In this case, we obtain solutions with $\omega^2 < 0$ or $\omega = \pm i|\omega|$, *i.e.* one of the solutions has negative imaginary part. Such perturbations will grow exponentially, signaling the instability of the homogeneous static state.

In general, for a finite g, Eq. (17) has four different solutions. However, they are symmetric with respect to the axis of $\omega = 0$. Thus we consider only the positive branches of Re ω and Im ω. The solution passing through the origin, $(\omega, k) = (0, 0)$ (analogous to Eq. (18)), is associated with the propagation of sound waves and is called the sound branch. The other solution, which has a mass gap at $k = 0$, corresponds to the propagation of the sigma field fluctuations and is called the sigma branch. In the vacuum, the mass gap corresponds to the sigma meson mass,$m_\sigma \approx \sqrt{2}\lambda f_\pi$, see Eq. (19).

In the case of the crossover transition, $g = 3.3$, there is no imaginary part and two real branches correspond to the sound and sigma excitations, modified by the interaction. In this case, the two branches are always separated from each other and never intersect.

The behavior becomes more complex for $g = 4.5$. Figures 1 and 2 show, respectively, the real and imaginary parts of the dispersion relation for different temperatures. They correspond to the four points A ($T = 128.5$ MeV, broken phase), B (T=132.1 MeV), D (T=122.7 MeV) and E (T=128.5 MeV, restored phase) on the (S-shaped) dependence $\sigma_0(T)$ (see details in Ref. [15]). In these calculations, we did not apply the Maxwell construction at $T = T_c = 128.5$ MeV but consider also metastable states. In the temperature interval between 122.7 MeV and 132.1 MeV, there exist three solutions for the sigma field, corresponding to two local minima of the thermodynamic potential, and one maximum between these minima. For the background states which have higher (lower) temperature than the point B (D), we have only one local minimum and the two branches of the dispersion relation are separated and have no imaginary part, which is shown for the points A and E in Fig. 1.

The states between points B and D correspond to a local maximum of the thermodynamic potential and the perturbations around these states are trivially unstable (spinodal instability). A non-trivial behavior can be observed for the metastable states. Although these states are located in the local minima of the thermodynamic potential, c_s^2 becomes negative in this region of temperatures that induces instability through the coupling to the hydrodynamic modes. To illustrate this point, the real and imaginary parts of the dispersion relation for the points B and D are shown in Figs. 1 and 2. One can see that at low k the sound and sigma branches degenerate into a single branch (Fig. 1) which has a positive imaginary part (Fig. 2). It is interesting to note that for $g = 4.5$, both branches of the dispersion relation become unstable at small k, *i.e.* when either $c_s^2 < 0$ (sound waves) or $m_\sigma^2 < 0$ (sigma waves).

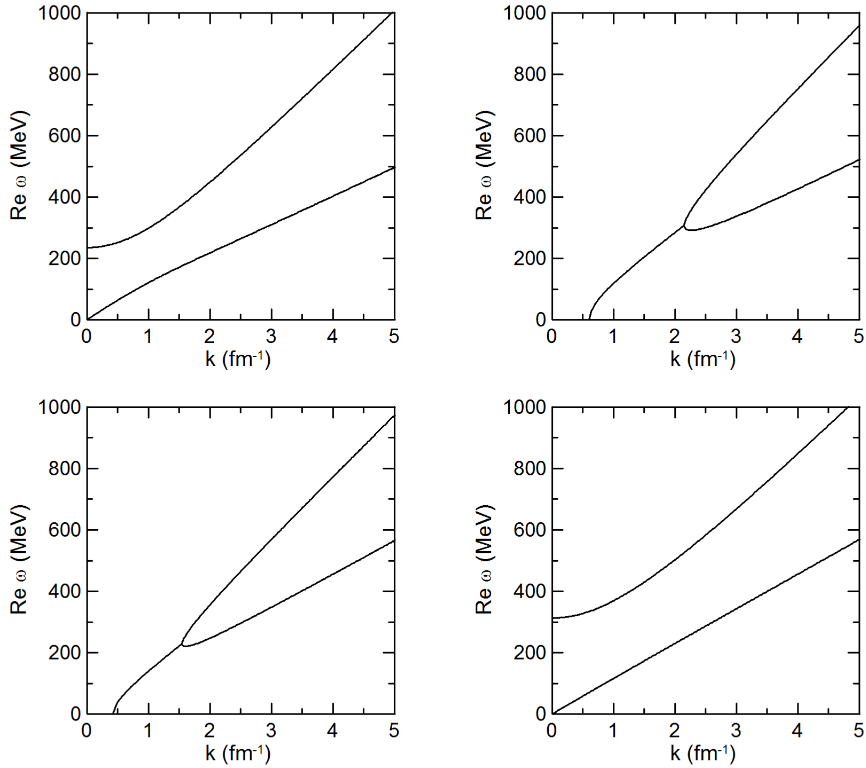

Fig. 1. Real parts of the dispersion relation for $g = 4.5$ at the point A (left up), B (right up), D (left down) and E (right down), respectively (see details in the text).

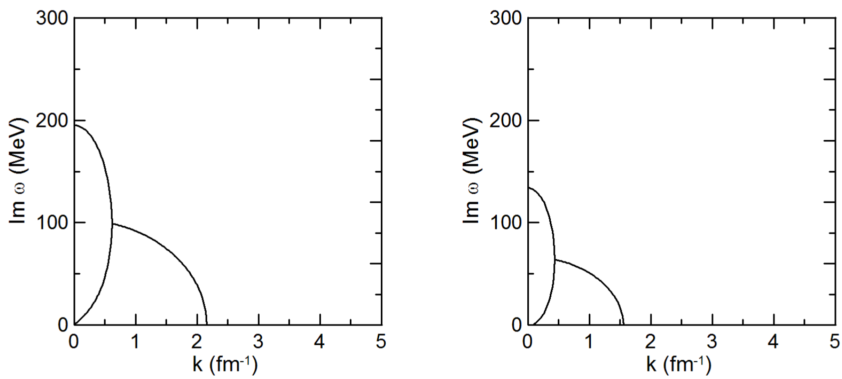

Fig. 2. Imaginary parts of the dispersion relation for $g = 4.5$ at the point B (left) and D (right), respectively. The imaginary parts at the points A and E vanish.

The appearance of the growing modes of density fluctuations is very well known in the physics of first order phase transitions. They appear when a homogeneous system is suddenly quenched into the spinodal region of the phase diagram, defined by the condition $c_{\rm s}^2 < 0$. Usually the instability region is limited by sufficiently small wave numbers, $k < k_{\rm max}$, as is clearly seen in Fig. 2. The value of $k_{\rm max}$ is determined by the non-locality scale of the particle interaction, which is given by m_σ in our calculations.

4. Phase transitions in Bjorken-like background

Up to now the dissipative effects, such as the viscosity of the fluid and damping of the field fluctuations, have been neglected. However, the damping due to the interaction with the quark fluid should lead to the relaxation of the chiral fields. This can be taken into account by introducing a friction term in the equation of motion for the chiral field. Due to these terms, the energy of the chiral field fluctuations dissipates into the quark fluid. To fulfill the dissipation–fluctuation theorem the field fluctuations are included as a noise term on the r.h.s. of the equation of motion

$$\partial_\mu \partial^\mu \sigma + \frac{\partial \Omega_{\rm tot}}{\partial \sigma} + \eta u_\mu \partial^\mu \sigma = \xi \,, \tag{20}$$

which contains the damping coefficient η and the stochastic noise field ξ. For consistency, the dynamics of the quark fluid should be described now by the Eq. (10) with the source term

$$S^\mu = (g\rho_{\rm s} + \eta u_\nu \partial^\nu)\partial^\mu \sigma \,, \tag{21}$$

which accounts for the energy-momentum exchange between the fluid and the field. Details of numerical implementation of this approach are given in Ref. [16]. For illustration, we present here only a few examples of our analysis.

The dynamics of fluctuations was studied for an expanding ellipsoidal fireball (corresponding to semi-peripheral collisions of Au nuclei) with Bjorken-type velocity field in z direction. The time evolution of the average value of the sigma field and its fluctuations are shown in Fig. 3 (upper panel), while Fig. 3 (lower panel) shows the time evolution of the average temperature and its fluctuations. The average is taken over the central sphere with radius $r = 3$ fm. Especially interesting behavior is observed for the scenario with the first order phase transition. When the transition temperature is reached after $t = 5$ fm, large parts of the system are still in the chirally symmetric phase as the average value of the sigma field is still around 10 MeV. These large deviations of the sigma field from its equilibrium value is the nonequilibrium effect of supercooling. Due to the barrier separating

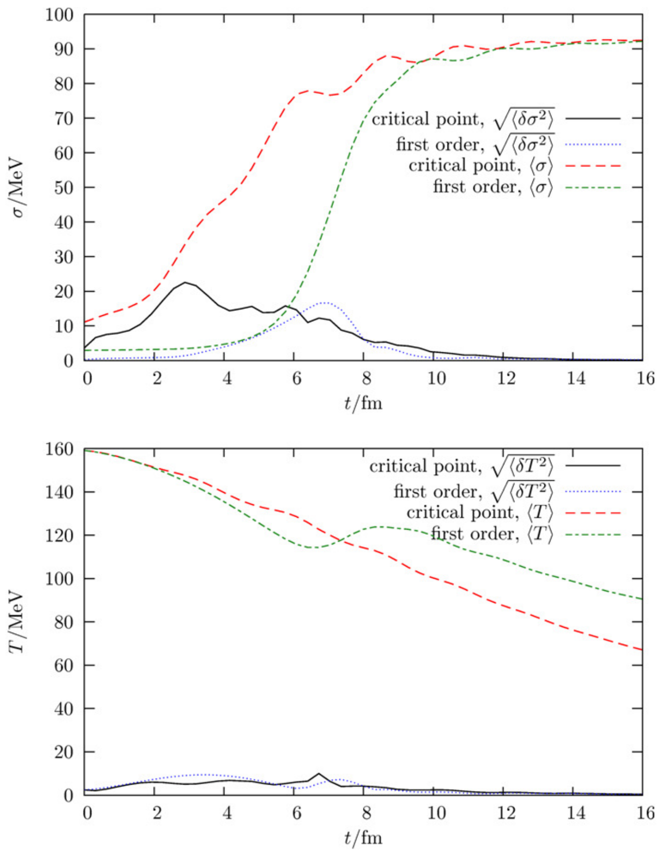

Fig. 3. The average values and the variances of the fluctuations of the sigma field (upper panel) and the average values and the variances of the fluctuations of the temperature (lower panel) for two phase transition scenarios.

the two minima of the thermodynamic potential, the sigma field is trapped in the chirally symmetric state even at $T < T_c$, until the barrier disappears. Then the sigma field rolls down into a lower minimum corresponding to the broken phase. The released potential energy is transformed effectively into kinetic energy of field oscillations, which leads to the dissipation of energy into the fluid via η-term in the source term (21). We can clearly observe the reheating effect at the first order phase transition. Between $t = 7$ fm and $t = 9$ fm the system in the central region is reheated from temperature below T_c to slightly above T_c, followed by a subsequent cooling. This explains the delayed relaxation of the average sigma field in the scenario with a first order phase transition compared to the critical point scenario. Moreover, during the relaxation and reheating process the fluctuations in the sigma field are enhanced between $t = 5$ fm and $t = 8$ fm.

5. Conclusions

Our main conclusion is that the chiral/deconfinement phase transitions in relativistic heavy-ion collisions will most likely proceed out of equilibrium. Therefore, dynamical effects are expected to play a crucial role. Most important among them are the following:

- Second order phase transitions (with CEP) are too weak to produce significant observable effects.
- Non-equilibrium effects in a 1st order transition (spinodal decomposition, strong fluctuations of order parameter) may help to identify the phase transition.
- If large QGP droplets are produced in the 1st order phase transition they will show up in large non-statistical multiplicity fluctuations.

I thank T. Koide, G. Danicol, G. Torrieri, Marlene Nahrgang and M. Bleicher for many fruitful discussions in the course of our work on this project. I would like to acknowledge support provided by the Helmholtz International Center for FAIR within the framework of the LOEWE program launched by the State of Hesse, and grant NSH-215.2012.2 (Russia).

REFERENCES

[1] K. Rajagopal, F. Wilczek, *Nucl. Phys.* **B404**, 577 (1993).

[2] L.P. Csernai, I.N. Mishustin, *Phys. Rev. Lett.* **74**, 5005 (1995).

[3] J. Randrup, *Phys. Rev.* **D55**, 1188 (1997).

[4] I.N. Mishustin, *Phys. Rev. Lett.* **82**, 4779 (1999); *Nucl. Phys.* **A681**, 56 (2001).

[5] O. Scavenius, A. Dumitru, *Phys. Rev. Lett.* **83**, 4697 (1999).

[6] O. Scavenius *et al.*, *Phys. Rev.* **D63**, 116003 (2001).

[7] J. Randrup, *Phys. Rev. Lett.* **92**, 122301 (2004); *Phys. Rev.* **C82**, 034902 (2010).

[8] G. Torrieri, B. Tomasik, I. Mishustin, *Phys. Rev.* **C77**, 034903 (2008).

[9] A. Bessa, E.S. Fraga, B.W. Mintz, *Phys. Rev.* **D79**, 034012 (2009).

[10] L.P. Csernai, J.I. Kapusta, *Phys. Rev.* **D46**, 1379 (1992); also talk, J. Kapusta, ETD-HIC conference, Montreal, 2007, `http://www.physics.mcgill.ca/etd-hic/`

[11] U.W. Heinz, P.F. Kolb, *Nucl. Phys.* **A702**, 269 (2002).

[12] D. Teaney, J. Lauret, E.V. Shuryak, *Phys. Rev. Lett.* **86**, 4783 (2001) [arXiv:nucl-th/0110037].

[13] P.F. Kolb *et al.*, *Nucl. Phys.* **A696**, 197 (2001).

[14] V. Merdeev, L.M. Satarov, I.N. Mishustin, *Phys. Rev.* **C84**, 014907 (2011).

[15] T. Koide, G.S. Danicol, G. Torrieri, I.N. Mishustin, Dynamics and Stability of Chiral Fluid, paper in preparation.

[16] M. Nahrgang, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962 [nucl-ph].

[17] I.N. Mishustin, O. Scavenius, *Phys. Rev. Lett.* **83**, 3134 (1999).

[18] K. Paech, H. Stoecker, A. Dumitru, *Phys. Rev.* **C68**, 044907 (2003).

[19] Y. Lallouet, D. Davesne, C. Pujol, *Phys. Rev.* **C67**, 057901 (2003).

[20] K. Paech, S. Pratt, *Phys. Rev.* **C74**, 014901 (2006).

[21] C.E. Aguiar, E.S. Fraga, T. Kodama, *J. Phys.* **G32**, 179 (2006) [arXiv:nucl-th/0306041].

[22] M. Gell-Mann, M. Levy, *Nuovo Cimento* **16**, 705 (1960).

[23] T.D. Lee, G.C. Wick, *Phys. Rev.* **D9**, 2291 (1974).

[24] R.D. Pisarski, F. Wilczek, *Phys. Rev.* **D29**, 338 (1984).

[25] O. Scavenius, Á. Mócsy, I.N. Mishustin, D.H. Rischke, *Phys. Rev.* **C64**, 045202 (2001).

[26] A. Mocsy, I.N. Mishustin, P.J. Ellis, *Phys. Rev.* **C70**, 015204 (2004).

[27] E.S. Bowman, J.I. Kapusta, *Phys. Rev.* **C79**, 015202 (2009).