FLUCTUATIONS OF CONSERVED CHARGES IN THE POLYAKOV LOOP EXTENDED QUARK-MESON MODEL AT FINITE BARYON DENSITY*

V. Skokov

Physics Department, Brookhaven National Laboratory
Upton, NY 11973, USA

(Received January 2, 2012)

In this paper, we review the properties of baryon number fluctuations close to the crossover transition at zero and finite baryon densities. We argue on a phenomenological importance of high order cumulants at zero baryon density. Main properties of the cumulants are illustrated beyond a mean-field approximations in an effective low energy model of QCD.

DOI:10.5506/APhysPolBSupp.5.877
PACS numbers: 24.85.+p, 21.65.–f, 25.75.–q, 24.60.–k

1. Introduction

From theory side, the thermodynamics of hot nuclear matter has been explored numerically in the first principle lattice QCD (LQCD) calculations [1, 2, 3]. Complementary experimental studies of the hot and dense matter created in heavy-ion collisions have been also carried out [4]. The major goal of these investigations is to reveal the structure of the QCD phase diagram and properties of nuclear matter in extreme conditions.

The LQCD results show that, at the physical pion mass, hot nuclear matter exhibits residual properties of both dynamical chiral symmetry breaking and confinement at finite temperature. In the LQCD calculations, it has been demonstrated that the transition between hadrons and quark/gluon degrees of freedom is of crossover type. At finite baryon densities, progress in LQCD calculations have been impeded by the sign problem. Nevertheless, several attempts to sidestep the sign problem have been performed [5,6,7,8]. In some of the studies indications of the expected critical end point (CEP) at finite values of the baryon chemical potential have been revealed [8].

* Talk presented at HIC for FAIR Workshop and XXVIII Max Born Symposium "Three Days on Quarkyonic Island", Wrocław, Poland, May 19–21, 2011.

Experimentally, one would expect a drastic modification of measured fluctuations of conserved charges [9, 10, 11], *e.g.* the baryon number, if

(i) a significant part of a fireball created in a collision enters and lives long enough in the critical region of the CEP to develop critical fluctuations;

(ii) the CEP is located close to the chemical freeze-out[1] to guarantee survival of the fluctuations.

The latter, the proximity of the freeze-out to the CEP, is, however, not known *a priori*. At present, from LQCD, we have very limited information about the phase diagram of nuclear matter:

(1) the temperature of the crossover at zero baryon chemical potential $T_{\rm pc}$ has been calculated [1, 3];

(2) the curvature of the transition line at zero baryon chemical potential κ has been also found [12].

These results infer that the temperature of the transition at zero baryon chemical potential is very close to the chemical freeze-out and that the phase transition line and the chemical freeze-out line separate at non-zero chemical potential.

When comparing theoretical expectation to the experimental data, an additional complication arises from conservation laws, *e.g.* baryon number conservation. Theoretically, the singular behaviour of the fluctuations is predicted in the *grand canonical* formulation of thermodynamics. Consequently, in the experiment, one is required to simulate the grand canonical ensemble, *i.e.* to study fluctuations in a restricted phase space. This is achieved by performing cuts in rapidity and transfers momentum of the detected particles. Ideally, the cuts should be fixed in such a way as to minimize the effects of the conservation laws. In practice, one, however, has to find a compromise between this and statistical significance of experimental measurements. As was shown by transport model calculations, this tight compromise is very challenging or even impossible in the low energy collisions [17]. If the CEP is located at large baryon chemical potential and small temperature, it can be studied only in low energy collisions. However, at low energies the conservation laws become increasingly important and even may "fake" non-monotonic behaviour of the fluctuations expected in critical dynamics.

[1] The chemical freeze-out line is determined from an analysis of experimental hadron yields by the statistical Hadron Resonance Gas (HRG) model (see Ref. [13] for a review). On the one hand, the HRG model provides a good description of relative particle yields and the low order cumulants [14, 15]. On the other hand, the thermodynamics of the model is in a very good agreement with LQCD at $\mu_B = 0$ and temperature below the crossover temperature (see *e.g.* [16]).

These considerations have motivated us to study properties of the baryon number fluctuations in high-energy heavy-ion collisions or equivalently at zero or small baryon chemical potential [18], where, as we mentioned earlier, the chemical freeze-out line obtained from the analysis of experimental particle yields and the crossover temperature calculated in LQCD is very close to each other. As we will argue in the main body of this article, this proximity leads to a significant modifications of the higher order cumulants of the baryon number fluctuations. An experimental observation of these effects may provide experimental evidence in favor of the transition, as well as constraints on the relative position of the chemical freeze-out line and the phase transition line at small chemical potential, including a piece of experimental information on the phase transition temperature.

In this article, we show results obtained in the framework of the Polyakov loop-extended quark-meson (PQM) model [19, 20, 21]. This model can reproduce essential properties of the QCD thermodynamics obtained in the LQCD already within the mean-field approximation. However, to correctly account for the critical behavior and scaling properties near the chiral phase transition one needs to go beyond the mean-field approximation and include critical fluctuations and non-perturbative dynamics. This can be achieved by using the methods based on the functional renormalization group (FRG). The details on the FRG treatment of the PQM model are given in the Appendix.

2. Results and discussion

Fluctuations of conserved charges are quantified by the cumulants $\chi_n^{B,Q}$ [22]. For the grand canonical ensemble, the cumulants are proportional to the generalized susceptibilities, derivatives of the pressure $p = T^4\hat{p}$ with respect to the corresponding chemical potential $\mu_{B,Q} = T\hat{\mu}_{B,Q}$

$$\chi_n^{B,Q} = \frac{\partial^n}{\partial \hat{\mu}_{B,Q}^n}\hat{p}\,. \tag{1}$$

The cumulants of conserved charges are sensitive probes of the chiral phase transition. Potentially, they are useful for determining the position, the order, and in the case of a second-order phase transition the universality class of the corresponding phase transition. The net baryon number density n_B is discontinuous at a first-order transition, whereas the susceptibility χ_2^B and higher cumulants diverge at the critical end point [10] and at the spinodal lines of a first-order chiral phase transition in non-equilibrium [23]. In the chiral limit and at non-zero chemical potential, all generalized susceptibilities χ_n^B with $n > 2$ diverge at the $O(4)$ chiral critical line [22], while at vanishing chemical potential this divergence is present only for higher order cumulants with $n \geq 6$. Since this fact has some importance for further discussion, let us consider it in details.

LQCD calculations have indicated that QCD in the chiral limit belongs to the three-dimensional $O(4)$ universality class [2]. Thus, based on the universality argument we expect that properties of QCD close to the chiral limit will be governed by $O(4)$ dynamics. In the chiral limit of $O(N)$ model, the singular contribution to the thermodynamic potential is proportional to

$$\Omega^{\rm sing} \propto t^{2-\alpha}\,, \tag{2}$$

where α is the critical exponent of the specific heat and the scaling variable t is given by $t = T/T_{\rm c} - 1 + \kappa(\mu/T)^2$. For the three dimensional $O(N)$ universality class with $N > 1$, the exponent α is negative $0 < \alpha \leq -1$. From Eqs. (1) and (2), one can find the most singular contribution to the $(2n)$th cumulant of the conserved charge conjugated to the chemical potential μ

$$\chi_{2n}^{\rm sing} \propto (2-\alpha)(2-\alpha-1)\cdots(2-\alpha-n+1)t^{2-\alpha-n}\,. \tag{3}$$

From Eq. (3) it follows, that at zero chemical potential, $\chi_6 \propto t^{-1-\alpha}$ is divergent and changes sign at the transition for any N, except for the case of the spherical model $N \to \infty$, where $\alpha_{N\to\infty} = -1$ and the singular contribution to χ_6 is t-independent. If the chiral symmetry is explicitly broken by a finite pion mass, the cumulants do not diverge at the transition. Still the underlying non-analyticity is reflected in the cumulants [18].

As it was demonstrated for zero chemical potential in LQCD and for non-zero chemical potential in model calculations, close to the phase transition the sixth order cumulant χ_6 decreases and becomes negative [18] opposite to the HRG model predictions of manifestly positive χ_6. Since the fourth order cumulant is unaffected by chiral critical phenomena at zero chemical potential, it remains positive for small chemical potentials, as in the HRG model. However, at larger densities, it picks up a non-trivial contribution from higher order $\mu = 0$ cumulants, as can be seen from the Taylor expansion

$$\chi_4^B(\hat{\mu}) = \chi_4^B(0) + \tfrac{1}{2}\chi_6^B(0)\hat{\mu}^2 + O\left(\hat{\mu}^4\right)\,. \tag{4}$$

The zeros order term in this expansion is always positive, while, as we argued above, χ_6^B is negative close to the transition. Consequently, at non-zero potential the second term can overcome the first one, leading to negative χ_4^B. Thus, the fourth order cumulant and consequently the kurtosis (χ_4^B/χ_2^B) may become negative close to the crossover transition, even if the critical end point does not exist in the T–μ_B plane [18].

In Fig. 1, we show the dependence of the ratios $R_4^B = \chi_4^B/\chi_2^B$ and $R_6^B = \chi_6^B/\chi_2^B$ on temperature for different chemical potential in the FRG PQM model. At small temperatures $T \ll m_N$, baryon number fluctuations are dominated by the massive three-quark states with the baryon charge $|q_B| = 1$

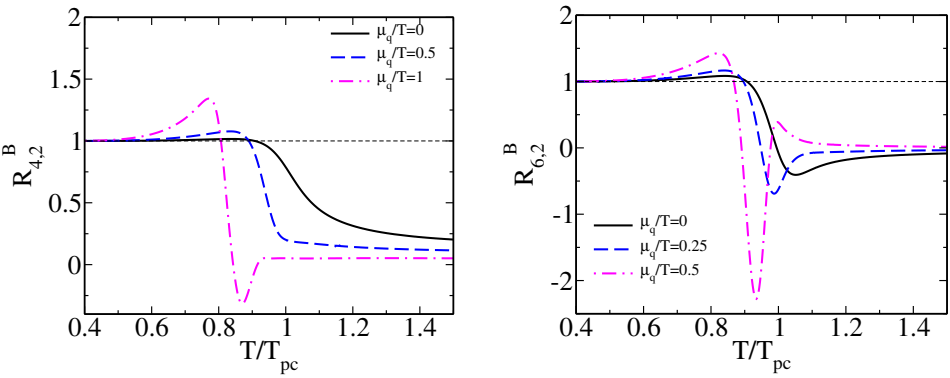

Fig. 1. Scaling of the non-analytic contributions to χ_4^B (left) and χ_6^B (right) arising from second and third derivatives of the singular part of the free energy. Shown are results for different values of the symmetry breaking parameter $h_0 h = m_q/T_c$; h_0 and $z_0 = h_0^{1/\beta\delta}/t_0$ are non-universal scale parameters. Note that for $h_0 h = 1$ the abscissa is the scaling variable z. The corresponding curve thus directly shows the $O(4)$ scaling function.

and the effective thermodynamic potential in the Boltzmann approximation is given by

$$\Omega(T \to 0) = -f(T)\cosh(\mu_B/T) + g(T)\,, \tag{5}$$

where $f(T)$ and $g(T)$ are independent of the chemical potential and not important for the current discussion. From Eq. (5), we obtain $R_4^B(T \to 0) = R_6^B(T \to 0) = 1$ in an agreement with Fig. 1. On the other hand, at high temperatures, thermodynamics is governed by almost massless single quark states. In this case, the effective thermodynamical potential in the Stefan–Boltzmann limit

$$\Omega(T \gg T_{\rm pc}) = -\frac{N_c N_f}{6} T^4 \left[\frac{7\pi^2}{60} + \left(\frac{\mu_B}{3T}\right)^2 + \frac{1}{2\pi^2}\left(\frac{\mu_B}{3T}\right)^4\right] \tag{6}$$

provides the following results for the ratios at zero baryon chemical potential $R_4^B(T \gg T_{\rm pc}) = 2/(3\pi^2)$ and $R_6^B(T \gg T_{\rm pc}) = 0$. The model calculations in Fig. 1 show that the change from the low temperature to high temperature regime happens in vicinity of the crossover $T/T_{\rm pc} \approx 1$. The ratio R_6^B becomes negative close to the transition at zero chemical potential, while R_4^B is negative only for high chemical potentials in accordance with our expectations.

In Fig. 2, we show negative region of the sixth order cumulant for baryon and electric charge fluctuations $\chi_6^{B,Q}$. In the case of the electric charge fluctuations, it is essential to perform calculations beyond the mean-field approximation taking into account charged pion contributions to χ_n^Q. In Fig. 1, we

show the range of negative χ_6^Q, which is similar to that found for χ_6^B. Thus, negative fluctuations of the sixth order moments of net baryon as well as net electric charge fluctuations can be attributed to the crossover. Consequently, the experimental observation of negative sixth order cumulants in heavy ion collisions at RHIC and LHC would indicate that the chemical freeze-out takes place in the vicinity of the chiral crossover transition [18].

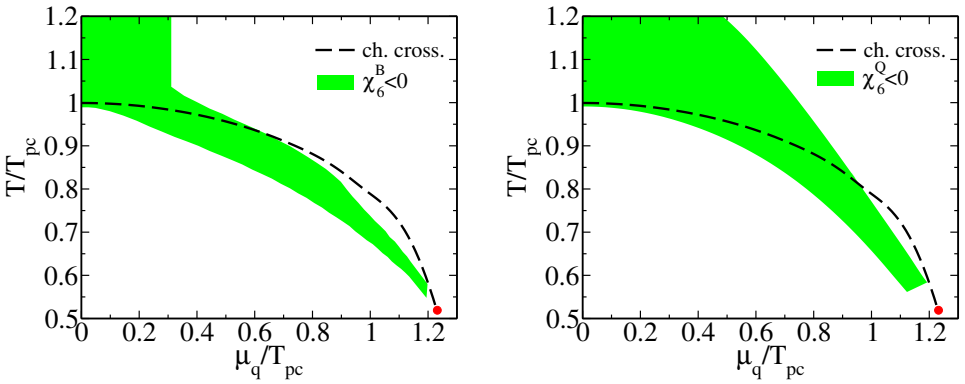

Fig. 2. Scaling of the non-analytic contributions to χ_4^B (left) and χ_6^B (right) arising from second and third derivatives of the singular part of the free energy. Shown are results for different values of the symmetry breaking parameter $h_0 h = m_q/T_c$; h_0 and $z_0 = h_0^{1/\beta\delta}/t_0$ are non-universal scale parameters. Note that for $h_0 h = 1$ the abscissa is the scaling variable z. The corresponding curve thus directly shows the $O(4)$ scaling function.

3. Conclusion

In this paper, we review results of Refs. [18, 24, 25]. We show that the fluctuations of the conserved charges are potentially capable of revealing the structure of the phase diagram. The properties of the fluctuations are quite general. They follow either from thermodynamic consistency or the universal critical properties and thus can be directly extended to QCD.

We show that already at zero chemical potential, higher order moments deviate from their hadron resonance gas values and may become negative close to the crossover transition. Since, according to lattice QCD calculations, the transition temperature is close to the chemical freeze-out line, the higher order cumulants play an important role for heavy-ion phenomenology. If measured experimentally, negative higher order cumulants may serve as an indication of the transition and reveal the relative position of the transition and chemical freeze-out line.

This paper is mainly based on results of Ref. [18] done in collaboration with B. Friman, F. Karsch and K. Redlich, to whom I am very grateful for numerous discussions, comments concerning the talk, and, finally, for fruitful collaboration. This manuscript has been authorized under Contract No. DE-AC02-98H10886 with the U.S. Department of Energy.

Appendix

Functional renormalization group for the PQM model

A formulation of the thermodynamics of the PQM model close to the transition requires the use of non-perturbative methods. In an analysis of electric charge fluctuations, it is of particular importance to account for pion fluctuations beyond the mean-field approximation. Here, we employ the functional renormalization group (FRG) to compute the thermodynamic potential in the PQM model. This method involves an infrared regularization of the fluctuations at a sliding momentum scale k, resulting in a scale-dependent effective action Γ_k (see Ref. [26] for a review). The Polyakov loop is treated as a background field, which is introduced self-consistently on the mean-field level while the quark and meson fields, fluctuations are accounted for by solving the FRG flow equations.

We follow the procedure used in Ref. [24] in the formulation of the flow equation for the scale-dependent grand canonical potential density, $\Omega_k = T\Gamma_k/V$, for the quark and meson subsystems at finite temperature and for a non-vanishing electric charge chemical potential.

The thermodynamic potential is obtained by solving the flow equation

$$\begin{aligned}\partial_k \Omega_k\left(\Phi,\bar{\Phi};T,\mu\right) = \frac{k^4}{12\pi^2}\Bigg\{ & \frac{1}{E_\pi}\left[1+2n_B(E_\pi;T)\right] \\ & + \frac{1}{E_\pi}\left[1+2n_B(E_\pi-\mu_\pi;T)\right] + \frac{1}{E_\pi}\left[1+2n_B(E_\pi+\mu_\pi;T)\right] \\ & + \frac{1}{E_\sigma}\left[1+2n_B(E_\sigma;T)\right] - \sum_{f=u,d}\frac{4N_c}{E_q}\left[1-N\left(\Phi,\bar{\Phi};T,\mu_f\right)-\bar{N}\left(\Phi,\bar{\Phi};T,\mu_f\right)\right]\Bigg\}\,.\end{aligned} \tag{7}$$

Here $n_B(E;T)$ is the bosonic distribution function

$$n_B(E;T) = \frac{1}{\exp(E/T)-1}\,, \tag{8}$$

$\mu_\pi = e_\pi \mu_Q$ is the charge pion chemical potential and $e_\pi = 1$ the charge of a π^+. The pion and sigma energies are given by

$$E_\pi = \sqrt{k^2+\overline{\Omega}_k'}\,, \qquad E_\sigma = \sqrt{k^2+\overline{\Omega}_k'+2\rho\,\overline{\Omega}_k''}\,, \tag{9}$$

where the primes denote derivatives with respect to $\rho = (\sigma^2 + \vec{\pi}^2)/2$ of $\overline{\Omega} = \Omega + c\sigma$. The fermion distribution functions $N(\Phi, \bar{\Phi}; T, \mu_f)$ and $\bar{N}(\Phi, \bar{\Phi}; T, \mu_f)$

$$N\left(\Phi, \bar{\Phi}; T, \mu_f\right) = \frac{1 + 2\bar{\Phi}e^{\beta(E_q - \mu_f)} + \Phi e^{2\beta(E_q - \mu_f)}}{1 + 3\Phi e^{2\beta(E_q - \mu_f)} + 3\bar{\Phi}e^{\beta(E_q - \mu_f)} + e^{3\beta(E_q - \mu_f)}}\,, \tag{10}$$

$$\bar{N}\left(\Phi, \bar{\Phi}; T, \mu_f\right) = N\left(\bar{\Phi}, \Phi; T, -\mu_f\right)\,, \tag{11}$$

are modified because of the coupling to the gluon field. Finally, the quark energy reads

$$E_q = \sqrt{k^2 + 2g^2\rho} \tag{12}$$

and the quark chemical potentials are defined by

$$\mu_u = \tfrac{1}{3}\mu_B + e_u\mu_Q\,, \qquad \mu_d = \tfrac{1}{3}\mu_B + e_d\mu_Q \tag{13}$$

with $e_u = 2/3$ and $e_d = -1/3$.

The flow equation (7) is solved numerically with an ultraviolet cutoff $\Lambda = 1.2$ GeV using the polynomial method (see Ref. [24] and references therein), truncated at order $N = 3$.

In this scheme, the stationarity condition

$$\left.\frac{d\Omega_k}{d\sigma}\right|_{\sigma=\sigma_k} = \left.\frac{d\overline{\Omega}_k}{d\sigma}\right|_{\sigma=\sigma_k} - c = 0 \tag{14}$$

is implemented in the flow equation. The initial conditions for the flow are chosen to reproduce the following vacuum properties: the physical pion mass $m_\pi = 138$ MeV, the pion decay constant $f_\pi = 93$ MeV, the sigma mass $m_\sigma = 600$ MeV, and the constituent quark mass $m_q = 300$ MeV at the scale $k \to 0$. The symmetry breaking term, $c = m_\pi^2 f_\pi$, is treated as an external field which does not flow. The flow of the Yukawa coupling g is neglected in our studies.

The thermodynamic potential (15) does not contain contributions of thermal modes with momenta larger than the cutoff Λ. In order to obtain the correct high-temperature behavior of the thermodynamic functions, we supplement the FRG potential with the contribution of the high-momentum states [24], by including the flow of quarks interacting with the Polyakov loop for momenta $k > \Lambda$.

By solving Eq. (7) one obtains the thermodynamic potential $\Omega_{k\to 0}(\sigma = \sigma_{k\to 0}, \Phi, \bar{\Phi}; T, \mu) = a_0$ as a function of the Polyakov loop variables Φ and $\bar{\Phi}$. In the PQM model, the full thermodynamic potential $\Omega(\Phi, \bar{\Phi}; T, \mu)$, including quark, meson and gluon degrees of freedom, is obtained by adding the effective gluon potential $\mathcal{U}(\Phi, \bar{\Phi})$. The potential $\mathcal{U}(\Phi, \bar{\Phi})$ was chosen such

that it reproduces the finite temperature lattice results of SU(3) Yang–Mills theory [20]. Consequently,

$$\Omega\left(\Phi,\bar{\Phi};T,\mu\right)=\Omega_{k\to 0}\left(\Phi,\bar{\Phi};T,\mu\right)+\mathcal{U}\left(\Phi,\bar{\Phi}\right)\,. \tag{15}$$

At a given temperature and chemical potential, the Polyakov loop variables, Φ and $\bar{\Phi}$, are then determined by the stationarity conditions

$$\frac{\partial}{\partial\Phi}\Omega\left(\Phi,\bar{\Phi};T,\mu\right)=0\,,\qquad \frac{\partial}{\partial\bar{\Phi}}\Omega\left(\Phi,\bar{\Phi};T,\mu\right)=0\,. \tag{16}$$

REFERENCES

[1] Y. Aoki *et al.*, *Nature* **443**, 675 (2006).

[2] S. Ejiri *et al.*, *Phys. Rev.* **D80**, 094505 (2009).

[3] A. Bazavov *et al.*, *Phys. Rev.* **D85**, 054503 (2012) [arXiv:1111.1710 [hep-lat]].

[4] See *e.g. J. Phys. G* **35** (2008), No. 10, "Quark Matter", Proceedings, 20th International Conference on Ultra-Relativistic Nucleus–Nucleus Collisions.

[5] P. de Forcrand, O. Philipsen, *Nucl. Phys.* **B642**, 290 (2002).

[6] M. D'Elia, M.P. Lombardo, *Phys. Rev.* **D67**, 014505 (2003).

[7] C.R. Allton *et al.*, *Phys. Rev.* **D66**, 074507 (2002).

[8] Z. Fodor, S.D. Katz, *J. High Energy Phys.* **0404**, 050 (2004).

[9] M.A. Stephanov, K. Rajagopal, E.V. Shuryak, *Phys. Rev.* **D60**, 114028 (1999).

[10] M.A. Stephanov, *Phys. Rev. Lett.* **102**, 032301 (2009).

[11] M.A. Stephanov, *Phys. Rev. Lett.* **107**, 052301 (2011).

[12] O. Kaczmarek *et al.*, *Phys. Rev.* **D83**, 014504 (2011).

[13] P. Braun-Munzinger, K. Redlich, J. Stachel, arXiv:nucl-th/0304013.

[14] P. Braun-Munzinger *et al.*, arXiv:1111.5063 [hep-ph].

[15] P. Braun-Munzinger *et al.*, arXiv:1107.4267 [hep-ph].

[16] S. Borsanyi *et al.*, *J. High Energy Phys.* **1011**, 077 (2010).

[17] T. Schuster *et al.*, arXiv:0903.2911 [hep-ph].

[18] B. Friman, F. Karsch, K. Redlich, V. Skokov, *Eur. Phys. J.* **C71**, 1694 (2011).

[19] K. Fukushima, *Phys. Lett.* **B591**, 277 (2004).

[20] C. Ratti, M.A. Thaler, W. Weise, *Phys. Rev.* **D73**, 014019 (2006).

[21] B.-J. Schaefer, J.M. Pawlowski, J. Wambach, *Phys. Rev.* **D76**, 074023 (2007).

[22] S. Ejiri, F. Karsch, K. Redlich, *Phys. Lett.* **B633**, 275 (2006).

[23] C. Sasaki, B. Friman, K. Redlich, *Phys. Rev. Lett.* **99**, 232301 (2007).
[24] V. Skokov, B. Stokic, B. Friman, K. Redlich, *Phys. Rev.* **C82**, 015206 (2010).
[25] V. Skokov, B. Friman, K. Redlich, *Phys. Rev.* **C83**, 054904 (2011).
[26] J. Berges, N. Tetradis, C. Wetterich, *Phys. Rep.* **363**, 223 (2002).
[27] J. Berges, D.U. Jungnickel, C. Wetterich, *Phys. Rev.* **D59**, 034010 (1999).