WIDTH OF THE QCD TRANSITION IN A POLYAKOV-LOOP DSE MODEL*

D. Horvatić[a], D. Blaschke[b,c], D. Klabučar[a], O. Kaczmarek[d]

[a]Physics Department, Faculty of Science, University of Zagreb
Zagreb 10000, Croatia
[b]Institute for Theoretical Physics, University of Wrocław
50-204 Wrocław, Poland
[c]Bogoliubov Laboratory of Theoretical Physics, JINR Dubna
141980 Dubna, Russia
[d]Department of Physics, Bielefeld University, 33501 Bielefeld, Germany

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We consider the pseudocritical temperatures for the chiral and deconfinement transitions within a Polyakov-loop Dyson–Schwinger equation approach which employs a nonlocal rank-2 separable model for the effective gluon propagator. These pseudocritical temperatures differ by a factor of two when the quark and gluon sectors are considered separately, but get synchronized and become coincident when their coupling is switched on. The coupling of the Polyakov-loop to the chiral quark dynamics narrows the temperature region of the QCD transition in which chiral symmetry and deconfinement is established. We investigate the effect of rescaling the parameter T_0 in the Polyakov-loop (PL) potential on the QCD transition for both the logarithmic and polynomial forms of the potential. While the critical temperatures vary in a similar way, the width of the transition is stronger affected for the logarithmic potential. For this potential the character of the transition changes from crossover to a first order one when $T_0 < 210$ MeV, but it remains crossover in the whole range of relevant T_0 values for the polynomial form.

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1. Introduction

The QCD phase transition between highly excited hadronic matter and the quark-gluon plasma is presently under experimental investigation at ultra-relativistic heavy-ion collider facilities like the Relativistic Heavy-Ion

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Collider (RHIC) at the Brookhaven National Laboratory or the Large Hadron Collider (LHC) at CERN, Geneva. Its theoretical description requires methods to solve QCD at finite temperature in the highly nonperturbative low-energy domain. At present, the only method to obtain *ab initio* solutions of QCD in this domain is lattice QCD (LQCD).

Alternatively, one can start from QCD Dyson–Schwinger equations (DSEs), apply a symmetry-preserving truncation scheme, and solve the resulting equations for the Schwinger functions. In the recent years, this approach has reached a new level of maturity (see, *e.g.*, Refs. [1, 2, 3] for reviews). This ongoing progress in DSEs also includes *ab initio* type of calculations [4, 5] pertaining to PL and related to other functional approaches.

For the purpose of this exploratory approach, we will restrict ourselves to the model, where PL is coupled to the quark sector DSE (PDSE), and to a rank-2 separable form of the effective gluon propagator [6].

In the present work, by analyzing the temperature behavior of the order parameters in the model, the dynamically generated light and strange quark masses as well as the PL variable, we find that the critical temperatures for the chiral and the deconfinement transitions measured by the peaks in the corresponding susceptibilities (defined here as the temperature derivatives of the order parameters) coincide at the per mille level of accuracy. We will discuss the effect of rescaling the critical temperature parameter T_0 of the PL potential [7, 8, 9, 10] once applications with a finite number of quark flavors and a possible chemical potential are considered and how this affects the width of the QCD transition region.

2. Separable PDSE model

2.1. Thermodynamical potential and order parameters

The central quantity for the analysis of the thermodynamical behavior is the thermodynamical potential which in the PDSE approach is a straightforward generalization of the standard CJT functional [11, 12]

$$\Omega(T) = \mathcal{U}(\Phi,\bar{\Phi}) - T\,\mathrm{Tr}_{\vec{p},n,\alpha,f,D}\left[\ln\left\{S_f^{-1}\left(p_n^\alpha,T\right)\right\} - \Sigma_f\left(p_n^\alpha,T\right)\cdot S_f\left(p_n^\alpha,T\right)\right], \tag{1}$$

where the full quark propagator for the flavor $f = u, d, s$,

$$\begin{aligned} S_f^{-1}\left(p_n^\alpha,T\right) &= S_{f,0}^{-1}\left(p_n^\alpha,T\right) + \Sigma_f\left(p_n^\alpha,T\right) \\ &= i\vec{\gamma}\cdot\vec{p}\,A_f\left(\left(p_n^\alpha\right)^2,T\right) + i\gamma_4\omega_n\,C_f\left(\left(p_n^\alpha\right)^2,T\right) + B_f\left(\left(p_n^\alpha\right)^2,T\right)\,, \end{aligned} \tag{2}$$

is defined by the DSE for the quark self-energy Σ, see below. The Polyakov-loop potential is first taken in the form [13]

$$\frac{\mathcal{U}_{\log}\left(\Phi,\bar{\Phi}\right)}{T^4} = -\frac{1}{2}a(T)\bar{\Phi}\Phi + b(T)\ln\left[1-6\bar{\Phi}\Phi+4\left(\bar{\Phi}^3+\Phi^3\right)-3\left(\bar{\Phi}\Phi\right)^2\right] \quad (3)$$

with $a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2$, $b(T) = b_3(T_0/T)^3$. The corresponding parameters $a_0 = 3.51$, $a_1 = -2.47$, $a_2 = 15.22$ and $b_3 = -1.75$ are taken from Ref. [13], where they have been adjusted to fit the pressure obtained in lattice gauge theory simulations of SU(3) Yang–Mills theory. In most of the literature on the PNJL model, the parameter $T_0 = 270$ MeV has been taken over for applications in QCD with N_f quark flavors, while following Ref. [8] its dependence on quark flavors and chemical potential should be invoked. Accordingly, for the case $N_f = 2+1$ discussed in the present work, in [8] the value $T_0 = 187$ MeV is suggested with an error margin of about 30 MeV. Applying the Matsubara formalism of finite temperature field theory, the squared quark 4-momenta are to be replaced by $(p_n^\alpha)^2 = (\omega_n^\alpha)^2 + \vec{p}^{\,2}$, $\omega_n^\alpha = \omega_n + \alpha\phi_3$, where $\omega_n = (2n+1)\pi T$ are the fermionic Matsubara frequencies and the indices $\alpha = -1, 0, +1$ specify the three quark colors and their coupling to the parameter ϕ_3 of the temporal gauge field.

In order to check the sensitivity to various parameterizations of the Polyakov-loop potential, we will also try the polynomial parametrization specified in [14].

For the effective gluon propagator in a Feynman-like gauge, $g^2 D^{\rm eff}_{\mu\nu}(p-q) = \delta_{\mu\nu} D(p^2, q^2, p\cdot q)$, we employ a rank-2 separable ansatz [6]

$$D\left(p^2, q^2, p\cdot q\right) = D_0\mathcal{F}_0\left(p^2\right)\mathcal{F}_0\left(q^2\right) + D_1\mathcal{F}_1\left(p^2\right)\left(p\cdot q\right)\mathcal{F}_1\left(q^2\right)\,, \quad (4)$$

so that the quark propagator amplitudes are given by

$$B_f\left((p_n^\alpha)^2, T\right) = m_f^0 + b_f(T)\mathcal{F}_0\left((p_n^\alpha)^2\right)\,, \quad (5)$$

$$A_f\left((p_n^\alpha)^2, T\right) = 1 + a_f(T)\mathcal{F}_1\left((p_n^\alpha)^2\right)\,, \quad (6)$$

$$C_f\left((p_n^\alpha)^2, T\right) = 1 + c_f(T)\mathcal{F}_1\left((p_n^\alpha)^2\right)\,. \quad (7)$$

In the present work, we will use the functions specified in [15, 16] which satisfy the constraints $\mathcal{F}_0(0) = \mathcal{F}_1(0) = 1$ and $\mathcal{F}_0(\infty) = \mathcal{F}_1(\infty) = 0$. Their functional form can be chosen such that the 4-momentum dependence of the dynamical mass function $M(p) = B(p)/A(p)$ and the wave function renormalization $Z(p) = 1/A(p)$ is in good agreement [17] with LQCD simulations of the quark propagator [18]. Models which employ a rank-1 separable ansatz (see, *e.g.* [19, 20]) result in $A(p) = Z(p) = 1$ and miss an important aspect of quark dynamics in QCD.

The temperature-dependent gap functions $a_f(T)$, $b_f(T)$, $c_f(T)$ and $\phi_3(T)$ are obtained as solutions of the DSE for the quark self energy in rainbow-ladder truncation as in Ref. [6], and correspond to minima of the thermodynamical potential (1). Once the gap equations are solved for different temperatures, one can extract the pseudocritical temperatures for chiral and deconfinement transitions from the peak positions of the temperature derivatives of the corresponding order parameters, the quark mass functions $m_f(T) = [m_f^0 + b(T)]/[1 + a_f(T)]$ and the Polyakov loop $\Phi(T)$, respectively. For a discussion of the quark mass function as an order parameter of the chiral transition see, *e.g.* Refs. [21, 22, 23].

3. Results and discussion

For the numerical calculations, we are using the same parameter set as in Refs. [24, 16, 15], namely $m_u^0 = m_d^0 = m_q^0 = 5.49$ MeV, $m_s^0 = 115$ MeV, $D_0 \Lambda_0^2 = 219$, $D_1 \Lambda_0^4 = 69$, $\Lambda_0 = 0.758$ GeV, $\Lambda_1 = 0.961$ GeV and $p_0 = 0.6$ GeV.

3.1. Order parameters for chiral and deconfinement transition

In Fig. 1 (a), we show the resulting temperature dependence of the derivatives of the quark mass being an order parameter of the chiral phase transition and of the Polyakov loop expectation value as an order parameter of the deconfinement transition. The peak values are attained at the corresponding pseudocritical temperatures for the chiral (T_χ) and deconfinement ($T_{\rm d}$) transitions, respectively. In the right panel of Fig. 1 (a), we show the results when the quark and gluon sectors are uncoupled. In this case, we have in the light quark sector $T_\chi = 128$ MeV, whereas $T_{\rm d} = 270$ MeV according to the parametrization of the PL potential in the pure gauge sector. The value obtained for T_χ is in the typical range found in the DSE approach [25].

At the same time, when coupling the PL potential to the chiral quark sector, the width of the transition region collapses to a tiny temperature interval around $T_{\rm c}$, as is demonstrated in Fig. 1 (b).

Both effects of coupling the chiral quark sector to the PL, the synchronization of the chiral and deconfinement transitions as well as the narrowing of the width of the QCD transition region, are obtained in a similar way for the polynomial PL potential.

The value obtained for the QCD transition temperature, $T_{\rm c} = 195$ MeV (193 MeV) for the logarithmic (polynomial) PL potential, is closer to recent LQCD results than the one obtained in PNJL or rank-1 separable nonlocal PNJL models but unsatisfactory for a quantitative description. Within the framework of the PQM model, it has been suggested [8] to rescale the T_0 parameter of the PL potential depending on the quark flavor content of the

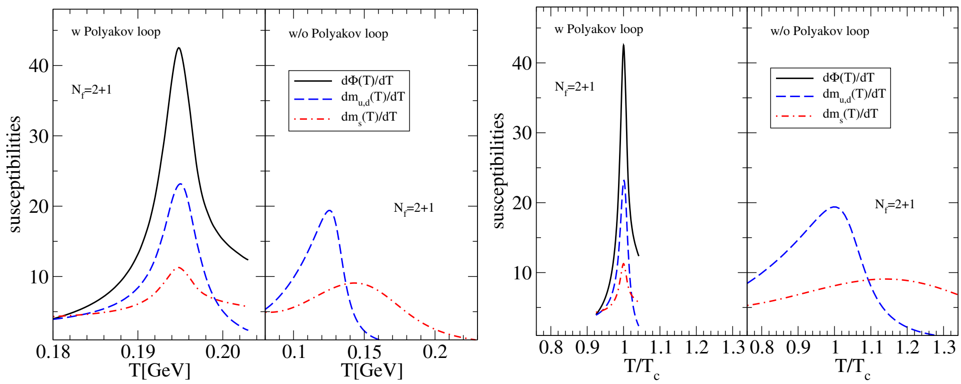

Fig. 1. (a) Quark mass susceptibilities (dashed/blue line: light flavors; dash-dotted/red line: strange flavor) with coupling to the Polyakov loop (left panel) and without it (right panel) as a function of the temperature. Note that without coupling to the Polyakov loop the chiral transition temperature is unrealistically low and the peak value for the light flavors is different from that for the strange one. (b) The same as Fig. 1 (a), but as a function of the scaled temperature T/T_c with $T_c = 195$ MeV (left panel) and $T_c = T_\chi = 128$ MeV (right panel). Without coupling to the Polyakov-loop $T_d = 2.11\ T_c$ is outside the range shown.

system and the chemical potential. In Fig. 2 (a), we show the resulting temperature dependence of the order parameters for chiral symmetry breaking (the normalized mass function $m(T)/m(0)$) and for deconfinement (the PL $\Phi(T)$) for three values of T_0. According to [8], the case $T_0 = 187$ MeV corresponds to $N_f = 2 + 1$ while $T_0 = 270$ MeV is the value for the pure gauge theory, where the deconfinement is a first order phase transition. The coupling to the chiral quark dynamics changes the character of this transition to a crossover. Lowering the T_0 parameter to 187 MeV changes both deconfinement and chiral restoration to strong first order phase transitions! This change of character happens at the critical value $T_0 = 210$ MeV, also shown in Fig. 2 (a).

In Fig. 2 (b), we summarize this finding by showing the dependence of T_c on the T_0 parameter of the PL potentials. For the logarithmic PL potential (3), the positions of first order transitions are characterized by the full dots connected by a solid line, while the crossover transitions are given as open dots connected by a dashed line. Two regions of linear dependence can be identified when using the logarithmic PL potential: $T_c = \text{const} + 0.30\ T_0$ for $T_0 < 210$ MeV and $T_c = \text{const} + 0.40\ T_0$ for $T_0 > 210$ MeV. When using the polynomial form of the PL potential, we find the linear dependence as $T_c = \text{const}{+}0.36\ T_0$. The change in the character of the QCD transition from a crossover for $T_0 > 210$ MeV to a first oder transition for $T_0 < 210$ MeV is

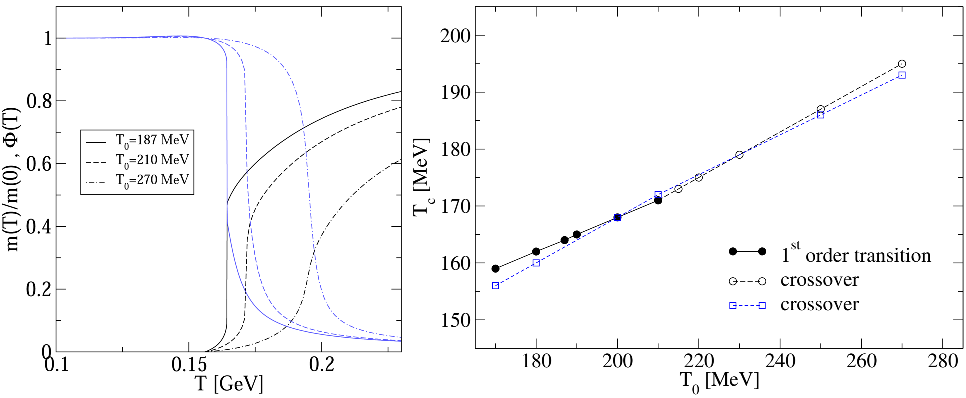

Fig. 2. (a) Temperature dependence of the order parameters for chiral symmetry breaking ($m(T)/m(0)$, grey/blue lines) and for deconfinement ($\Phi(T)$, black lines) for different choices for the parameter T_0 in the Polyakov-loop potential. (b) Pseudocritical temperature for the chiral restoration transition *versus* parameter T_0 of the Polyakov-loop potential in the logarithmic form (3) (black circles) and in the polynomial form (open squares (blue)). For further details, see the text.

accompanied by a sudden change in slope at $T_0 = 210$ MeV. It is remarkable that the T_0- rescaling introduced to account for a quark flavor dependence of the PL potential when applied to the nonlocal separable PDSE model considered here, results in an obvious contradiction with LQCD concerning the character of the QCD transition: while in LQCD for $N_f = 2 + 1$ the finite-T transition is a crossover [26, 27], the application of the suggested reparametrization with the corresponding value $T_0 = 187$ MeV leads in the present model to a first order transition.

On the other hand, for the polynomial PL potential the transition is a crossover for any of the considered values of T_0. Fig. 2 (b) illustrates this by the dashed line connecting the points depicted by squares.

4. Conclusions

The separable PDSE approach provides an essential improvement of the chiral quark dynamics in PNJL models and nonlocal PNJL models which use a rank-1 separable ansatz for the quark interaction kernel, since it provides a running of both, the dynamical quark-mass function and the wave-function renormalization in close agreement with LQCD simulations of the quark propagator. It also provides the strong-coupling aspect of a dynamical confinement mechanism due to the absence of real quark mass poles.

However, the investigation of the temperature dependence of the chiral and deconfinement order parameters characterizing the (pseudo-)critical temperature and the width of the QCD transition reveals also some inadequate aspects of the present level of description of this transition. The critical temperature is too high and the transition region is too narrow when compared with LQCD results. A rescaling of the PL-potential results in a lower value for T_c, in accordance with recent LQCD results, but at the price of a narrowing of the QCD transition region, for the logarithmic PL potential even changing the character of the transition to a first order one, in striking contradiction with LQCD.

We expect that going beyond the rainbow-ladder level by including hadronic fluctuations beyond the mean field [10, 28, 30] will entail an improvement of the approach. As has been demonstrated recently by including π and σ fluctuations in a consistent $1/N_c$ scheme [29, 30], going beyond the mean field will lead to a lowering of the chiral transition temperature. The width of the transition region, however, appears as a sensitive constraint for the choice of an appropriate functional form of the PL potential. Its possible dependence on the inclusion of hadronic correlations deserves a detailed study. We plan to extend our work in this direction.

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