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TRANSPORT PROPERTIES OF THE QGP FROM A VIRIAL EXPANSION*

Stefano Mattiello

Institute for Theoretical Physics, University of Giessen
35392 Giessen, Germany

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In this work, we investigate the transport coefficients, *i.e.* shear and bulk viscosity η and ζ, and heat conductivity κ of the quark-gluon plasma within a virial expansion approach. We derive a realistic Equation of State using a virial expansion approach which allows us to include the interactions between the partons in the deconfined phase. From the interaction, we directly extract the effective coupling $\alpha_{\rm V}$ for the determination of η, ζ and κ. The shear viscosity and the heat conductivity show a pronounced temperature dependence. Furthermore, we find that the bulk viscosity ζ is strongly suppressed. Our results for the ratio η to the entropy density s show a minimum near $T_{\rm c}$, very close to the lowest bound $\eta/s = 1/(4\pi)$ and, furthermore, in line with the experimental value from RHIC as well as with the lattice calculations.

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1. Introduction

Understanding the rich phase structure of quantum chromodynamics for the density-temperature plane is a challenge for theoretical as well as experimental high energy physics. From the experimental point of view, heavy-ion collisions are the tool for such investigations. The experimental findings of the heavy-ion collisions at the Relativistic Ion Collider (RHIC) led to the announcement about the discovery of the nearly perfect fluidity of the strongly-coupled quark-gluon plasma (sQGP) [1, 2]. Ideal hydrodynamics seems to offer a good description for the experimental data for moderate momenta, in particular, of the strong elliptic flow v_2. Nevertheless, ideal hydrodynamics gradually breaks down at larger impact parameter,

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at lower collision energy and away from midrapidity due of strong viscous effects. Therefore, dissipative hydrodynamical calculations including a dynamic evaluation of the transport properties, *i.e.* shear viscosity η, bulk viscosity ζ and heat conductivity κ within a model describing the strong coupling properties of the QGP are mandatory. Recently, we have proposed a generalization of the classical virial expansion approach to calculate the QCD partition function in the partonic phase with an interaction inspired by lattice calculations [3]. We have derived an Equation of State (EoS) for the sQGP that describes well the three-flavor QCD lattice data [4] at nonzero temperature as well as vanishing quark chemical potential ($\mu_q = 0$). In this approach, all thermodynamic quantities are based on an explicit parton interaction in form of a potential. Therfore, it is also the ideal framework for a consistent description of transport properties of the sQGP.

2. Virial expansion

The virial expansion formalism has been developed in a previous work [3], where a detailed derivation of the partition function $Z(T,V)$, of all thermodynamic quantities — such as pressure, entropy density, interaction measure and sound velocity — and of the EoS of the QGP at vanishing and finite μ_q has been presented. We achieve an expansion of $\ln Z$ in powers of the logarithm of the partition function in the Stefan Boltzmann limit $\zeta = \ln Z^{(0)}$ [3]. All quantities can be calculated from the partition function using thermodynamic relations. For the entropy density one obtains

$$s = \frac{\partial P}{\partial T}\,. \tag{1}$$

Following Ref. [3], we use an effective quark–quark potential inspired by a phenomenological model which includes non-perturbative effects from dimension two gluon condensates that reproduce the free energy of quenched QCD very well [5]. The effective potential between the quarks explicitly reads

$$V_1(r,T) = \left(\frac{\pi}{12}\frac{1}{r} + \frac{\mathcal{C}_2}{2N_c T}\right) e^{-M(T)r}\,, \tag{2}$$

where $\mathcal{C}_2$ is the non-perturbative dimension two condensate and $M(T)$ a Debye mass estimated as

$$M(T) = \sqrt{N_c/3 + N_f/6}\; gT = \tilde{g}T\,, \tag{3}$$

where we have neglected any scale dependence in the coupling constant. A comparison with three-flavor lattice QCD calculations with almost physical masses from Ref. [4] shows that a coupling parameter $\tilde{g} = 1.30$ allows for a good description of all thermodynamic quantities in the temperature range from 0.8 to 5 T_c. In Fig. 1 the entropy density s (divided by T^3) is shown as a function of the temperature (expressed in units of the critical temperature T_c) from the virial expansion approach using Eq. (4) (solid line) as well as in the SB limit (dashed line). The symbols denote the lattice calculations from Ref. [4]. Near T_c, the deviation of entropy density within our virial expansion approach from the ideal gas limit are sizeable in the confined phase.

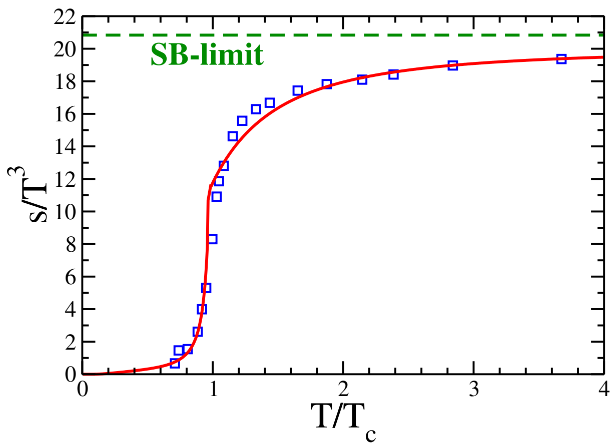

Fig. 1. (Color online) Entropy density s of the QGP as a function of the temperature divided by T^3 from the virial expansion (solid line). For comparison the corresponding SB limit is displayed by the dashed line. The lQCD results (open squares) have been adopted from Ref. [4].

3. Transport properties

In an ultrarelativistic quark-gluon plasma, where the temperature T is much larger than the constituent masses m_i, the transport coefficient can be calculated in the first approximation to the first Enskog order as [6,7]

$$\eta = \frac{4T}{5\sigma_t}\left(1+\frac{1}{20}z^2+O\left(z^4\ln z\right)\right), \tag{4}$$

$$\kappa = \frac{4}{3\sigma_t}\left(1-\frac{1}{4}z^2+O\left(z^4\ln z\right)\right), \tag{5}$$

$$\zeta = \frac{mz^3}{108\sigma_t}\left(1+O\left(z^5\ln z\right)\right), \qquad \text{with } z=\frac{m}{T}. \tag{6}$$

The relevant transport cross section is given by

$$\sigma_{\rm t}(\hat{s}) \equiv \int d\sigma_{\rm el}\ \sin^2\theta_{\rm cm} = \sigma_0\ 4\hat{z}(1+\hat{z})\left[(2\hat{z}+1)\ln(1+1/\hat{z})-2\right] , \qquad (7)$$

with the total cross section $\sigma_0(\hat{s}) = 9\pi\alpha_{\rm V}^2(\hat{s})/2\mu_{\rm scr}^2$. Here $\alpha_{\rm V} = \alpha_{\rm V}(T)$ and $\mu_{\rm scr}$ are the effective temperature-dependent coupling constant and the screening mass, respectively, and $\hat{z} \equiv \mu_{\rm scr}^2/\hat{s}$. For simplicity, we assume σ_0 to be energy independent and neglect its weak logarithmic dependence on $\hat{s}$ in the relevant energy range and set $\hat{s} \approx 17\,T^2$.

In order to calculate a transport cross section with this interaction, the coupling $\alpha_{\rm V}$ has to be extracted from V_1. Following Ref. [8], we define the coupling in the so-called qq-scheme,

$$\alpha_{qq}(r,T) \ \equiv\ -\frac{12}{\pi} r^2 \frac{dV_1(r,T)}{dr} \, . \qquad (8)$$

The coupling $\alpha_{qq}(r,T)$ then exhibits a maximum for fixed temperature at a certain distance denoted by $r_{\rm max}$. By analyzing the size of the maximum at $r_{\rm max}$ we fix the temperature dependent coupling, $\alpha_{\rm V}(T)$, as

$$\alpha_{\rm V}(T) \ \equiv\ \alpha_{qq}(r_{\rm max},T) \, . \qquad (9)$$

To investigate the importance of the different coefficients η, κ and ζ we scale them by the appropriate power of the temperature in order to obtain a dimensionless quantity. Therefore, we show in Fig. 2 the ratios η/T^3 (red line), κ/T^2 (blue line) and ζ/T^3 (green line) as a function of the temperature expressed in units of the critical temperature $T_{\rm c}$. Several features become evident:

(i) For the shear viscosity and the heat conductivity an increase with the temperature is found for $T_{\rm c} \leq T \leq 3.5T_{\rm c}$.

(ii) At higher temperatures $T \geq 4T_{\rm c}$ η/T^3 and κ/T^2 become constant. For the shear viscosity also very different approaches show this functional dependence: the strong quark-gluon plasma from AdS/CFT [10], the quasiparticle approximation with differently modeled quark selfenergy [11, 12] as well as the weak coupling estimate from Ref. [13, 14]. For the heat conductivity that is in line with the results of perturbative calculations [15], where it scales with the second power of the temperature.

(iii) As already mentioned, the bulk viscosity is completely negligible in comparison to the other coefficients. This is in agreement with other calculations and justifies the omission of ζ in several hydrodynamical and transport calculations [16].

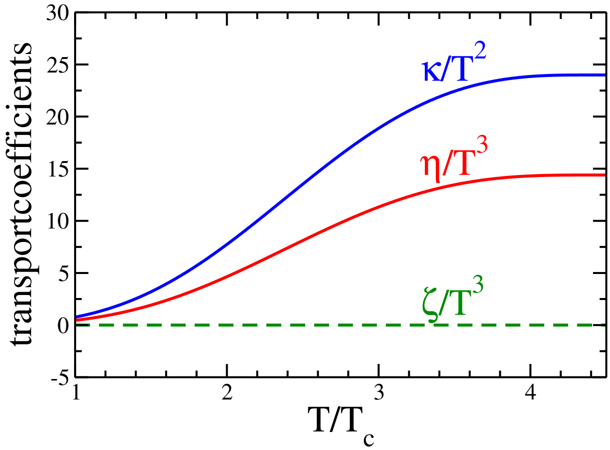

Fig. 2. (Color online) The scaled sheer viscosity η/T^3 (red line), heat conductivity κ/T^2 (blue line) and bulk viscosity ζ/T^3 (green line) as a function of temperature expressed in units of the critical temperature T_{c}.

Nevertheless, it is useful to look more in detail at the bulk viscosity in our model. In Fig. 3 we show ζ as a function of temperature expressed in units of the critical temperature T_{c} (solid/green line). To investigate the role of the temperature dependent transport cross section, we compare it with the results obtained using the constant cross section $\sigma_{\mathrm{t}}^{(0)}$ (dashed/green line).

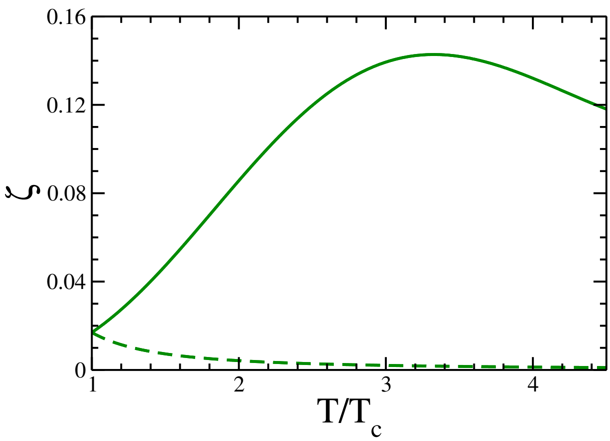

Fig. 3. (Color online) The bulk viscosity ζ (solid line) as a function of temperature expressed in units of the critical temperature T_{c}. In comparison, the bulk viscosity the temperature independent transport cross section $\sigma_{\mathrm{t}}^{(0)}$ also as a function of the scaled temperature.

The calculation with the full cross section shows a different behavior as generally expected, because the suspected enhancement of the bulk viscosity is missed. In contrast, the interplay between cross section and z^3 leads to a broad maximum around $3.2\,T_{\rm c}$. This is clearly a consequence of the temperature dependence of $\sigma_{\rm t}$: namely, using the constant value $\sigma_{\rm t}^{(0)} = \sigma_{\rm t}(T_{\rm c})$ for the transport cross section, we find a decreasing behavior with increasing temperature. This observation suggests the possibility of a maximum of the bulk viscosity at $T_{\rm c}$. In general, it is not surprising that our ultrarelativistic model cannot describe the peak of ζ at the critical temperature. This maximum is a consequence of the hadronic correlations at $T_{\rm c}$ [17], that are not included in this approach.

Finally in Fig. 4, we show the results for specific shear viscosity in comparison to other estimates. In the deconfined region, $T/T_{\rm c} \geq 1$, the solid/red line shows the results for η/s as a function of temperature (in units of the critical temperature $T_{\rm c}$) within the virial expansion approach. Additionally, the experimental point (square) from [18] and the lattice data [19, 20] (triangles and full dots) are shown for comparison. In the confined phase the dotted/red line shows the scaling $\eta/s \propto T^{-4}$ from the chiral perturbation theory [21] combined with the requirement that $\eta/s = 1/4\pi$ at $T_{\rm c}$. Furthermore, the range (0.8–1.5) for η/s from perturbative QCD (pQCD) from Ref. [19] is depicted as a light grey/blue region and the lowest bound is

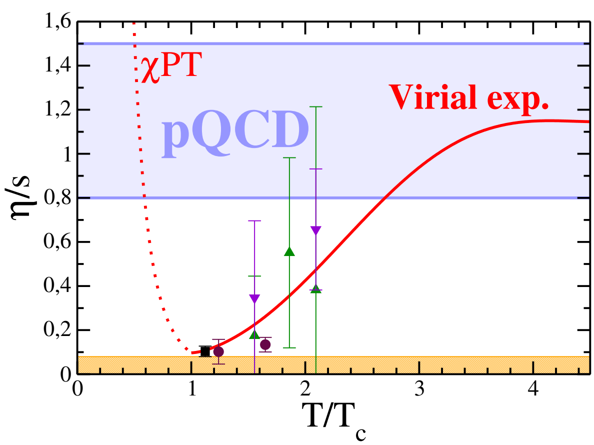

Fig. 4. (Color online) The viscosity/entropy density ratio η/s as a function of temperature expressed in units of the critical temperature $T_{\rm c}$ for $T/T_{\rm c} < 1$ and $T/T_{\rm c} > 1$ from the virial expansion (red solid line) The lattice results are from Ref. [19] (triangles) and from Ref. [20] (full dots). The dotted line, denoted by χPT, stands for the results from the scaling behavior of the chiral perturbation theory.

indicated by the grey/orange area. At T_c our result for $\eta/s \approx 0.1$ is very close to the theoretical bound of $1/(4\pi)$. For temperatures $1.5T_c \leq T \leq 3T_c$ the ratio η/s increases almost linearly until saturation at high temperatures is achieved. Qualitatively, this increasing behavior of the specific viscosity with the temperature is confirmed by lattice calculation. However, the large error bars of the lattice data do not allow for a conclusive comparison. In contrast, the experimental point is reproduced very well by our result. A detailed analysis of the temperature dependence of our results for η/s — using a Taylor expansion around the critical temperature — indicates the existence of a minimum in η/s close to T_c.

4. Conclusion

We have performed an investigation of the transport coefficient in the QGP in a dynamical way within kinetic theory using a generalized virial expansion approach. We find a suppression of the bulk viscosity that justifies the neglecting of ζ in several hydrodynamical calculations. Furthermore, our numerical results give a ratio $\eta/s \approx 0.1$ at the critical temperature T_c, which is very close to the lower theoretical bound of $1/(4\pi)$.

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