RESTORATION OF SINGLET AXIAL SYMMETRY AT FINITE TEMPERATURE*

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To accommodate recent RHIC data on η' multiplicity, we propose a minimal modification of the Witten–Veneziano relation at high temperature. This renders a significant drop of η' mass at high temperature signaling a restoration of the $\mathrm{U}(1)_A$, and the Goldstone character of η'.

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1. Introduction

On the classical level, QCD with N_f massless flavors enjoys a global chiral $\mathrm{U}(N_f)_\mathrm{L} \otimes \mathrm{U}(N_f)_\mathrm{R}$ symmetry. The subgroup with unit determinant, namely the $\mathrm{SU}(N_f)_\mathrm{L} \otimes \mathrm{SU}(N_f)_\mathrm{R}$, is spontaneously broken down to its diagonal part $\mathrm{SU}(N_f)_\mathrm{L+R} = \mathrm{SU}(N_f)_\mathrm{V}$. This gives rise to 8 pseudoscalar Goldstone bosons: πs, Ks and the η.

On the other hand, the non-Abelian anomaly in the $\mathrm{U}(1)_A$ sector,

$$\partial_\mu J^0_{5\mu}(x) = \frac{g_s^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}F_a^{\mu\nu}F_a^{\rho\sigma} \tag{1}$$

explicitly breaks $\mathrm{U}(1)_A$ symmetry. Coupled with non-trivial topology in the field space of QCD, it prevents the spontaneous breaking of $\mathrm{U}(1)_A$, thus generating the extra mass for η'. The simplest example of such contribution to $m_{\eta'}$ is the diamond diagram in Fig. 1.

Nevertheless, the fate of $\mathrm{U}(1)_A$ at $T > 0$ could be drastically changed. New RHIC data from central Au+Au collisions [1] on η' multiplicity shows a drop in its mass by at least 200 MeV inside the fireball

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$$m_{\eta'}(\text{vacuum}) = 958 \text{ MeV} \to m_{\eta'}(\text{fireball}) = 340^{+50+280}_{-60-140} \pm 45 \text{ MeV}\,, \tag{2}$$

thus signaling restoration of the Goldstone character of η'.

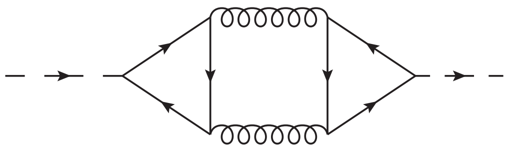

Fig. 1. The lowest order contribution to the excess of the η' mass.

2. Witten–Veneziano relation at finite *T*

On the theoretical side, excess in the η' mass is inferred from the Witten–Veneziano relation (WVR) [2, 3]

$$M_\eta^2 + M_{\eta'}^2 - 2M_K^2 = \frac{2N_f}{f_\pi^2}\chi_{\text{YM}}\,. \tag{3}$$

Due to the flavor content of the pseudoscalars, WVR just says that the excess in the singlet η^0 mass is coming from the glue sector, *i.e.* χ_{YM} is the Yang–Mills topological susceptibility.

Previous studies [4] of WVR indicated, for various lattice forms of χ_{YM}, that η' mass increases as χ_{YM} melts, *in contrast* to the result (2). This is happening as T approaches the chiral restoration temperature T_{ch}, due to the fact that there $f_\pi(T)$ starts decreasing significantly.

2.1. Connection between QCD and YM topological susceptibility

In that light, we propose [5] a minimal modification of WVR first by using Leutwyler–Smilga result [6] in the vacuum

$$\chi_{\text{YM}} = \frac{\chi}{1 + \chi\frac{N_f}{\Sigma m}}\,. \tag{4}$$

Just like the WVR, Leutwyler–Smilga relation connects the quantities from two different theories: χ is the QCD topological susceptibility, and Eq. (4) shows that it approaches χ_{YM} only for large quark masses, whereas χ_{YM} and χ are very different for light quarks. $\Sigma = \langle\bar{q}q\rangle_0$ is the condensate in the chiral limit.

We denote by $\tilde{\chi}$ the whole right-hand side of Eq. (4), and use it at finite temperature. Importance of this relation comes from the fact that χ is driven by the chiral quark condensate in the leading order of expansion in small quark masses

$$\chi = -\frac{\Sigma m}{N_f} + C_m \,, \qquad \frac{N_f}{m} = \sum_f \frac{1}{m_f} \,. \tag{5}$$

This is the Di Vecchia–Veneziano result [6, 7]. The next term in this expansion, C_m, is essential as it keeps $\chi_{\rm YM}$ from blowing up. We fix its value at $T = 0$ by demanding $\tilde{\chi}(0) = \chi_{\rm YM}$.

2.2. Exploring the thermal dependence

For the thermal dependence of χ we use an ansatz

$$\chi(T) = -\frac{\Sigma(T)m}{N_f} + C_m(0)\left[\frac{\Sigma(T)}{\Sigma(0)}\right]^{\delta} .$$

This gives

$$\tilde{\chi}(T) = \frac{\Sigma(T)m}{N_f}\left\{1 - \frac{1}{C_m(0)}\frac{\Sigma(T)m}{N_f}\left[\frac{\Sigma(0)}{\Sigma(T)}\right]^{\delta}\right\} . \tag{6}$$

The interesting window for δ is then $0 < \delta < 1$ since the lower limit gives no thermal dependence for the correction term, and a quadratic one in the last equation for $\tilde{\chi}$. As can be seen in Fig. 2, (5) ensures that f_π remains finite while $\tilde{\chi}$ goes to zero. With $\delta = 1$ there is an enhancement of the η' mass [5], and therefore this is the upper limit of interest.

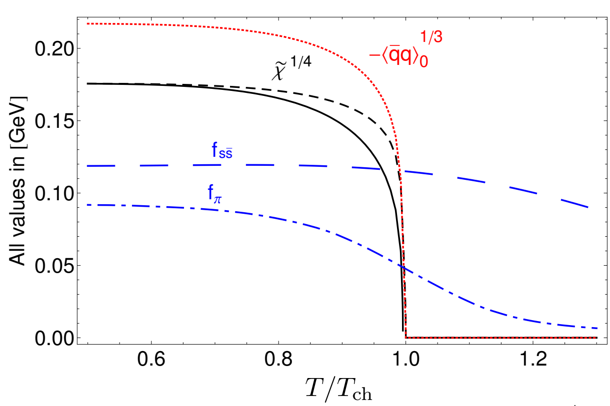

Fig. 2. The relative-temperature dependences, on $T/T_{\rm ch}$, of $\widetilde{\chi}^{1/4}$, $\langle\bar{q}q\rangle_0^{1/3}$, f_π and $f_{s\bar{s}}$. The solid curve depicts $\widetilde{\chi}^{1/4}$ for $\delta = 0$, and the short-dashed curve is $\widetilde{\chi}^{1/4}$ for $\delta = 1$. At $T = 0$, both $\widetilde{\chi}$'s are equal to $\chi_{\rm YM} = (0.1757\,{\rm GeV})^4$, the weighted average [4] of various lattice results for $\chi_{\rm YM}$.

The interesting window for δ is then $0 \leq \delta \leq 1$, since $\delta = 0$ gives the T-independent correction term, while $\delta = 1$ already leads to precursors of the unwanted mass enhancement in the η'–η complex. Therefore, $\delta \sim 1$ is the upper limit of interest, although the result for the T-dependence of the meson masses is quantitatively not much different for $\delta = 1$ than from the case $\delta = 0$, which is depicted in Fig. 3.

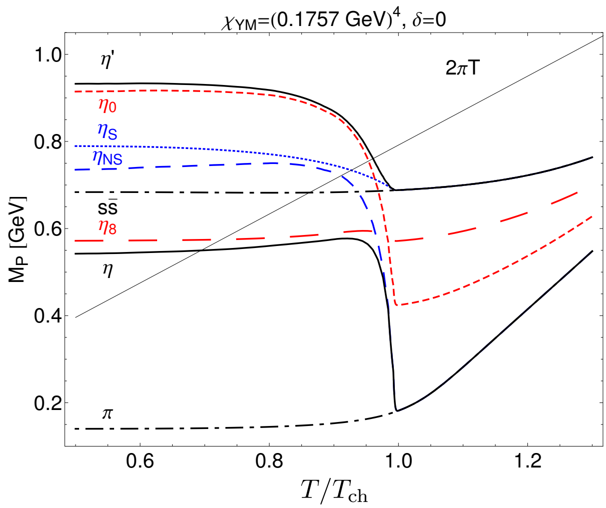

Fig. 3. The relative-temperature dependences, on $T/T_{\rm ch}$, of the pseudoscalar masses for $\delta = 0$.

3. Results and discussion

Mesons are constructed as $q\bar{q}$ bound states via the Bethe–Salpeter equation in the ladder approximation. Dynamical quarks are built up from the Dyson–Schwinger equation in the rainbow approximation. We use the successful rank-2 separable model [8] for the gluon propagator, which was also used in Ref. [4]. Rainbow-ladder approximation is the simplest symmetry preserving truncation, necessary for the correct chiral behavior of the theory.

The bound state approach enables an access only to the non-anomalous part of the meson masses, since the ladder Bethe–Salpeter kernel does not include diagrams like in Fig. 1. Therefore, the anomalous part is inferred from Eq. (3). The strategy is to use flavor mass matrices to extract η and η' masses from the calculated non-anomalous sector. This is presented in detail in Ref. [4].

Our main result is presented in Figs. 3 and 4. The reduction in the η' mass is around 200 MeV, which is in quantitative agreement with RHIC data. This is possible only due to the proposed modification of the WVR relation at finite T. Topological susceptibility of pure glue, $\chi_{\rm YM}$, is just too resistant, with the characteristic melting temperature being $T_{\rm YM} = 260$ MeV (see *e.g.* [9, 10]).

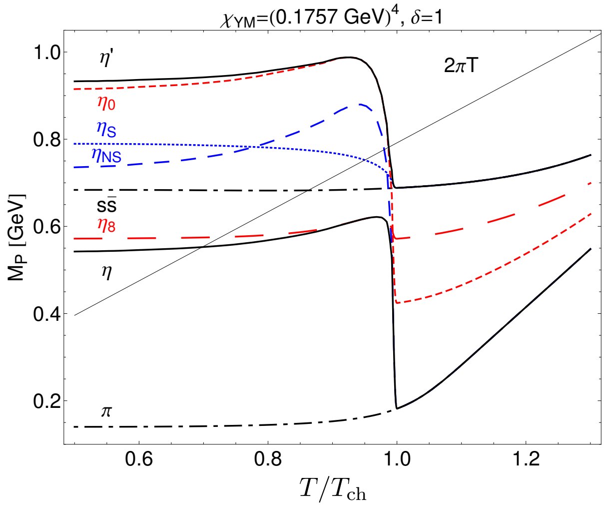

Fig. 4. The relative-temperature dependences, on $T/T_{\rm ch}$, of the pseudoscalar masses for $\delta = 1$.

In contrast, the pseudocritical temperatures for the chiral and deconfinement transitions in the full QCD are lower than $T_{\rm YM}$ by some 100 MeV or more (*e.g.*, see Ref. [11]) due to the presence of the quark degrees of freedom. In that regard, Eq. (5) is essential as it couples chiral restoration to $U(1)_A$ restoration, allowing for $\tilde{\chi}(T)$ to melt away even sooner than $f_\pi(T)$. In the separable model used here we have $T_{\rm ch} = 128$ MeV which is admittedly lower than the so far accepted value around 160–170 MeV. It has been shown [12] that the same model coupled with the gluon degrees of freedom in the form of the Polyakov loop cures this discrepancy.

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