PARTON ENERGY LOSS IN AN UNSTABLE QUARK-GLUON PLASMA*

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The energy loss of a fast parton scattering elastically in a weakly coupled quark-gluon plasma is formulated as an initial value problem. The approach is designed to study an unstable plasma, but it reproduces the well known result in the case of an equilibrium plasma. Contributions to the energy loss due to unstable modes are shown to exponentially grow in time. An unstable two-stream system is considered as an example.

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1. Introduction

When a highly energetic parton travels through the quark-gluon plasma (QGP), it loses its energy due to elastic interactions with plasma constituents. This is the so-called *collisional energy loss* which for the equilibrium QGP is well understood, see the review [1] and the handbook [2]. The quark-gluon plasma produced in relativistic heavy-ion collisions, however, reaches the state of local equilibrium only after a short but finite time

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interval, and during this period the momentum distribution of plasma partons is anisotropic. Collisional energy loss has been computed for such an anisotropic QGP [3]. However, the plasma with anisotropic momentum distribution is unstable due to the chromomagnetic modes (for a review see [4]), and the fact that unstable systems are explicitly time dependent as unstable modes exponentially grow in time has not been taken into account in the study [3].

Our aim is to formulate an approach where the energy loss is found as the solution of an initial value problem. The parton is treated as an energetic classical particle with SU(3) color charge. For the equilibrium plasma we recover the known results but for the unstable plasma the energy loss is shown to strongly depend on time. Our approach to the energy-loss problem is similar to the method used earlier to study the momentum broadening $\hat{q}$ of a fast parton in anisotropic plasma [5]. Analogous methods have also been used to study the spectra of chromodynamic fluctuations of an unstable plasma [6] which, in particular, are responsible for the collision integrals of transport equations [7].

Throughout the paper we use the natural system of units with $c = \hbar = k_B = 1$ and the signature of our metric tensor is $(+,-,-,-)$.

2. General formula

We consider a classical parton which moves across a quark-gluon plasma. Its motion is described by the Wong equations [8]

$$\frac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau)\,, \tag{1}$$

$$\frac{dp^\mu(\tau)}{d\tau} = gQ^a(\tau)\,F_a^{\mu\nu}(x(\tau))\,u_\nu(\tau)\,, \tag{2}$$

$$\frac{dQ_a(\tau)}{d\tau} = -gf^{abc}u_\mu(\tau)\,A_b^\mu(x(\tau))\,Q_c(\tau)\,, \tag{3}$$

where τ, $x^\mu(\tau)$, $u^\mu(\tau)$ and $p^\mu(\tau)$ are, respectively, the parton's proper time, its trajectory, four-velocity and four-momentum; $F_a^{\mu\nu}$ and A_a^μ denote the chromodynamic field strength tensor and four-potential along the parton's trajectory and Q^a is the classical color charge of the parton; g is the coupling constant and $\alpha_s \equiv g^2/4\pi$ is assumed to be small. We also assume that the potential vanishes along the parton's trajectory *i.e.* our gauge condition is $u_\mu(\tau)\,A_a^\mu(x(\tau)) = 0$. Consequently, due to Eq. (3) the classical parton's charge $Q_c(\tau)$ is constant within the chosen gauge.

The energy loss is given directly by Eq. (2) with $\mu = 0$. Using the time $t = \gamma\tau$ instead of the proper time τ and replacing the field strength tensor $F_a^{\mu\nu}$ by the chromoelectric $\boldsymbol{E}_a(t,\boldsymbol{r})$ and chromomagnetic $\boldsymbol{B}_a(t,\boldsymbol{r})$ fields, Eq. (2) gives

$$\frac{dE(t)}{dt} = gQ^a \boldsymbol{E}_a(t,\boldsymbol{r}(t)) \cdot \boldsymbol{v}\,, \tag{4}$$

where $\boldsymbol{v}$ is the parton's velocity. Since we consider a parton which is very energetic, $\boldsymbol{v}$ is assumed to be constant and $\boldsymbol{v}^2 = 1$. Introducing the current generated by the parton $\boldsymbol{j}_a(t,\boldsymbol{r}) = gQ^a\boldsymbol{v}\delta^{(3)}(\boldsymbol{r} - \boldsymbol{v}t)$, Eq. (4) gives

$$\frac{dE(t)}{dt} = \int d^3r \boldsymbol{E}_a(t,\boldsymbol{r}) \cdot \boldsymbol{j}_a(t,\boldsymbol{r})\,. \tag{5}$$

Since we deal with an initial value problem, we apply to the field and current not the usual Fourier transformation but the so-called *one-sided Fourier transformation* defined as

$$f(\omega,\boldsymbol{k}) = \int_0^\infty dt \int d^3r e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} f(t,\boldsymbol{r})\,, \tag{6}$$

with the inverse

$$f(t,\boldsymbol{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} f(\omega,\boldsymbol{k})\,, \tag{7}$$

where the real parameter $\sigma > 0$ is chosen is such a way that the integral over ω is taken along a straight line in the complex $\omega-$plane, parallel to the real axis, above all singularities of $f(\omega,\boldsymbol{k})$. Using Eqs. (6) and (7), Eq. (5) can be rewritten

$$\frac{dE(t)}{dt} = gQ^a \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t}\, \boldsymbol{E}_a(\omega,\boldsymbol{k}) \cdot \boldsymbol{v}\,, \tag{8}$$

where $\bar{\omega} \equiv \boldsymbol{k}\cdot\boldsymbol{v}$.

The next step is to compute the chromoelectric field $\boldsymbol{E}_a$. Applying the one-sided Fourier transformation to the linearized Yang–Mills equations we get the set of equations familiar from electrodynamics

$$\begin{aligned}
ik^i\varepsilon^{ij}(\omega,\boldsymbol{k})E_a^j(\omega,\boldsymbol{k}) &= \rho_a(\omega,\boldsymbol{k})\,, \qquad ik^iB_a^i(\omega,\boldsymbol{k}) = 0\,,\\
i\epsilon^{ijk}k^jE_a^k(\omega,\boldsymbol{k}) &= i\omega B_a^i(\omega,\boldsymbol{k}) + B_{0a}^i(\boldsymbol{k})\,,\\
i\epsilon^{ijk}k^jB_a^k(\omega,\boldsymbol{k}) &= j_a^i(\omega,\boldsymbol{k}) - i\omega\varepsilon^{ij}(\omega,\boldsymbol{k})E_a^j(\omega,\boldsymbol{k}) - D_{0a}^i(\boldsymbol{k})\,,
\end{aligned} \tag{9}$$

where ρ_a is the color-charge density, the fields with the index 0 are initial values; the chromoelectric induction $\boldsymbol{D}_a$ is expressed as $D_a^i(\omega,\boldsymbol{k}) = \varepsilon^{ij}(\omega,\boldsymbol{k})E_a^j(\omega,\boldsymbol{k})$ with $\varepsilon^{ij}(\omega,\boldsymbol{k})$ being chromodielectric tensor which carries all information about the medium. For an anisotropic plasma it equals

$$\varepsilon^{ij}(\omega,\boldsymbol{k}) = \delta^{ij} + \frac{g^2}{2\omega}\int \frac{d^3p}{(2\pi)^3}\frac{v^i}{\omega - \boldsymbol{k}\cdot\boldsymbol{v} + i0^+}\frac{\partial f(\boldsymbol{p})}{\partial p^k}\left[\left(1 - \frac{\boldsymbol{k}\cdot\boldsymbol{v}}{\omega}\right)\delta^{kj} + \frac{k^k v^j}{\omega}\right],$$

where $f(\boldsymbol{p})$ is the momentum distribution of plasma constituents. The color indices a, b are dropped as $\varepsilon(\omega,\boldsymbol{k})$ is a unit matrix in color space.

Although equations (9) strongly resemble those of electrodynamics, the gluon contribution to the color charge density ρ_a and color current $\boldsymbol{j}_a$, which is a genuine non-Abelian effect, is fully incorporated in these equations.

Using Eq. (9), the field $E_a^i(\omega,\boldsymbol{k})$ is found to be

$$E_a^i(\omega,\boldsymbol{k}) = -i\left(\Sigma^{-1}\right)^{ij}(\omega,\boldsymbol{k})\left[\omega j_a^j(\omega,\boldsymbol{k}) + \epsilon^{jkl}k^k B_{0a}^l(\boldsymbol{k}) - \omega D_{0a}^j(\boldsymbol{k})\right], \tag{10}$$

where

$$\Sigma^{ij}(\omega,\boldsymbol{k}) \equiv -\boldsymbol{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\boldsymbol{k}). \tag{11}$$

Substituting the expression (10) into Eq. (8), we get the final formula

$$\frac{dE(t)}{dt} = gQ^a v^i \int\limits_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi i}\int\frac{d^3k}{(2\pi)^3}e^{-i(\omega-\bar{\omega})t}\left(\Sigma^{-1}\right)^{ij}(\omega,\boldsymbol{k}) \tag{12}$$

$$\times\left[\frac{i\omega g Q^a v^j}{\omega-\bar{\omega}} + \epsilon^{jkl}k^k B_{0a}^l(\boldsymbol{k}) - \omega D_{0a}^j(\boldsymbol{k})\right].$$

The integral over ω is controlled by the poles of the matrix $\Sigma^{-1}(\omega,\boldsymbol{k})$ which determine the collective modes in the system. Equivalently, these modes are found as solutions of the equation $\det[\Sigma(\omega,\boldsymbol{k})] = 0$. Equation (12) needs to be treated differently for stable and for unstable systems.

3. Stable systems

When the plasma is stable, all modes are damped and the poles of $\Sigma^{-1}(\omega,\boldsymbol{k})$ are located in the lower half-plane of complex ω. Consequently, the contributions to the energy loss corresponding to the poles of $\Sigma^{-1}(\omega,\boldsymbol{k})$ exponentially decay in time. The only stationary contribution is given by the pole $\omega = \bar{\omega} \equiv \boldsymbol{k}\cdot\boldsymbol{v}$ which comes from the current $\boldsymbol{j}_a(\omega,\boldsymbol{k})$. Therefore,

the terms in Eq. (12), which depend on the initial values of the fields, can be neglected, and the energy loss of a fast parton in a stable plasma is

$$\frac{d\overline{E}(t)}{dt} \;=\; -ig^2 C_R v^i v^j \int \frac{d^3k}{(2\pi)^3}\,\bar{\omega}\,\left(\Sigma^{-1}\right)^{ij}(\bar{\omega},\boldsymbol{k})\,, \tag{13}$$

where the bar indicates that averaging over the parton's color state has been performed, and the factor C_R is 4/3 for a quark and 3 for a gluon.

When the plasma is isotropic, the dielectric tensor can be expressed in a standard way through its longitudinal ($\varepsilon_{\rm L}(\omega,\boldsymbol{k})$) and transverse ($\varepsilon_{\rm T}(\omega,\boldsymbol{k})$) components and the matrix $\Sigma^{ij}(\omega,\boldsymbol{k})$ (11) can be easily inverted. The energy loss (13) then equals

$$\frac{d\overline{E}}{dt} = -ig^2 C_R \int \frac{d^3k}{(2\pi)^3}\,\frac{\bar{\omega}}{\boldsymbol{k}^2}\,\left[\frac{1}{\varepsilon_{\rm L}(\bar{\omega},\boldsymbol{k})} + \frac{\boldsymbol{k}^2\boldsymbol{v}^2-\bar{\omega}^2}{\bar{\omega}^2\varepsilon_{\rm T}(\bar{\omega},\boldsymbol{k})-\boldsymbol{k}^2}\right], \tag{14}$$

which corresponds to the standard energy loss due to soft collisions [2].

4. Unstable systems

When the plasma is unstable, the matrix $\Sigma^{-1}(\omega,\boldsymbol{k})$ contains poles in the upper half-plane of complex ω, and the contributions to the energy loss from these poles grow exponentially in time. After a sufficiently long time, the parton's energy loss will be dominated by the fastest unstable mode. For an unstable plasma, the terms in Eq. (12), which depend on the initial values of fields $\boldsymbol{D}$ and $\boldsymbol{B}$, cannot be neglected, as these terms are amplified by a factor exponentially growing in time. Using Eqs. (9) the initial values can be computed as

$$D^i_{0a}(\boldsymbol{k}) \;=\; -igQ^a\bar{\omega}\varepsilon^{ij}(\bar{\omega},\boldsymbol{k})\left(\Sigma^{-1}\right)^{jk}(\bar{\omega},\boldsymbol{k})v^k\,, \tag{15}$$

$$B^i_{0a}(\boldsymbol{k}) \;=\; \int\limits_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{\omega}\left(\boldsymbol{k}\times\boldsymbol{E}_a(\omega,\boldsymbol{k})\right)^i = -igQ^a\epsilon^{ijk}k^j\left(\Sigma^{-1}\right)^{kl}(\bar{\omega},\boldsymbol{k})v^l. \tag{16}$$

Substituting Eqs. (15) and (16) into Eq. (12) and averaging over the parton's colors as before, we obtain

$$\frac{d\overline{E}(t)}{dt} = g^2 C_R v^i v^l \int\limits_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}e^{-i(\omega-\bar{\omega})t}\left(\Sigma^{-1}\right)^{ij}(\omega,\boldsymbol{k})$$
$$\times\left[\frac{\omega\delta^{jl}}{\omega-\bar{\omega}} - \left(k^jk^k-\boldsymbol{k}^2\delta^{jk}\right)\left(\Sigma^{-1}\right)^{kl}(\bar{\omega},\boldsymbol{k}) + \omega\,\bar{\omega}\,\varepsilon^{jk}(\bar{\omega},\boldsymbol{k})\left(\Sigma^{-1}\right)^{kl}(\bar{\omega},\boldsymbol{k})\right]. \tag{17}$$

Equation (17) gives the energy loss of a fast parton flying across the unstable plasma. The key point is that the presence of an unstable mode will produce a time dependent exponential growth of the form $e^{\mathrm{Im}\omega(\boldsymbol{k})t}$.

5. Two-stream system

In order to calculate the energy loss, one must invert the matrix $\Sigma^{ij}(\omega,\boldsymbol{k})$ defined by Eq. (11) to substitute the resulting expression into Eq. (17). For a general anisotropic system this is a tedious calculation, and, therefore, we consider for simplicity the example of the two-stream system which has unstable longitudinal electric modes. We assume that the chromodynamic field is dominated by the longitudinal chromoelectric field and take $\boldsymbol{B}(\omega,\boldsymbol{k})=0$ and $\boldsymbol{E}(\omega,\boldsymbol{k})=\boldsymbol{k}(\boldsymbol{k}\cdot\boldsymbol{E}(\omega,\boldsymbol{k}))/\boldsymbol{k}^2$. Then, $\Sigma^{ij}(\omega,\boldsymbol{k})$ is trivially inverted as

$$\left(\Sigma^{-1}\right)^{ij}(\omega,\boldsymbol{k})=\frac{1}{\omega^2\varepsilon_{\mathrm{L}}(\omega,\boldsymbol{k})}\frac{k^ik^j}{\boldsymbol{k}^2}\,,\qquad \varepsilon_{\mathrm{L}}(\omega,\boldsymbol{k})\equiv\varepsilon^{ij}(\omega,\boldsymbol{k})\frac{k^ik^j}{\boldsymbol{k}^2}\,, \tag{18}$$

and Eq. (17) simplifies to

$$\frac{d\overline{E(t)}}{dt}\;=\;g^2C_R\int\limits_{-\infty+i\sigma}^{\infty+i\sigma}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\frac{e^{-i(\omega-\bar\omega)t}}{\omega^2\varepsilon_{\mathrm{L}}(\omega,\boldsymbol{k})}\frac{\bar\omega^2}{\boldsymbol{k}^2}\left[\frac{\omega}{\omega-\bar\omega}+\frac{\bar\omega}{\omega}\right]. \tag{19}$$

Eq. (19) gives a non-zero energy loss in the vacuum limit when $\varepsilon_{\mathrm{L}}\to 1$. Therefore, we subtract from the formula (19) the vacuum contribution, or equivalently we replace $1/\varepsilon_{\mathrm{L}}$ by $1/\varepsilon_{\mathrm{L}}-1$.

The next step is to calculate $\varepsilon_{\mathrm{L}}(\omega,\boldsymbol{k})$. With the distribution function of the two-stream system in the form

$$f(\boldsymbol{p})=(2\pi)^3n\left[\delta^{(3)}(\boldsymbol{p}-\boldsymbol{q})+\delta^{(3)}(\boldsymbol{p}+\boldsymbol{q})\right], \tag{20}$$

where n is the effective parton density in a single stream, one finds [6]

$$\varepsilon_{\mathrm{L}}(\omega,\boldsymbol{k})=\frac{(\omega-\omega_+(\boldsymbol{k}))(\omega+\omega_+(\boldsymbol{k}))(\omega-\omega_-(\boldsymbol{k}))(\omega+\omega_-(\boldsymbol{k}))}{\left(\omega^2-(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)^2}\,, \tag{21}$$

where $\boldsymbol{u}\equiv\boldsymbol{q}/E_{\boldsymbol{q}}$ is the stream velocity, $\mu^2\equiv g^2n/2E_{\boldsymbol{q}}$ is a parameter analogous to the Debye mass squared, and $\pm\omega_\pm(\boldsymbol{k})$ are the four roots of the dispersion equation $\varepsilon_{\mathrm{L}}(\omega,\boldsymbol{k})=0$ which are

$$\omega_\pm^2(\boldsymbol{k})\;=\;\frac{1}{\boldsymbol{k}^2}\Bigg[\boldsymbol{k}^2(\boldsymbol{k}\cdot\boldsymbol{u})^2+\mu^2\left(\boldsymbol{k}^2-(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)$$
$$\pm\mu\sqrt{\left(\boldsymbol{k}^2-(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)\left(4\boldsymbol{k}^2(\boldsymbol{k}\cdot\boldsymbol{u})^2+\mu^2\left(\boldsymbol{k}^2-(\boldsymbol{k}\cdot\boldsymbol{u})^2\right)\right)}\Bigg]. \tag{22}$$

It is easy to see that $0 < \omega_+(\boldsymbol{k}) \in \mathbb{R}$ for any $\boldsymbol{k}$. For $\boldsymbol{k}^2(\boldsymbol{k}\cdot\boldsymbol{u})^2 \geq 2\mu^2(\boldsymbol{k}^2 - (\boldsymbol{k}\cdot\boldsymbol{u})^2)$, the minus mode is also stable, $0 < \omega_-(\boldsymbol{k}) \in \mathbb{R}$, but for $\boldsymbol{k}\cdot\boldsymbol{u} \neq 0$ and $\boldsymbol{k}^2(\boldsymbol{k}\cdot\boldsymbol{u})^2 < 2\mu^2(\boldsymbol{k}^2 - (\boldsymbol{k}\cdot\boldsymbol{u})^2)$ one finds that $\omega_-(\boldsymbol{k})$ is imaginary which is the well-known two-stream electric instability.

Strictly speaking, the stream velocity $\boldsymbol{u}$ given by the distribution function (20) equals the speed of light. However, the distribution (20) should be treated as an idealization of a two-bump distribution with bumps of finite width. Then, the momenta of all partons are not exactly parallel or antiparallel and the velocity $\boldsymbol{u}$, which enters Eqs. (21, 22), obeys $\boldsymbol{u}^2 \leq 1$.

Equations (19) and (21) determine the energy loss of a parton in the two-stream system. The integral over ω can be computed analytically as it is determined by the six poles of the integrand, located at $\omega = \pm\omega_+(\boldsymbol{k})$, $\pm\omega_-(\boldsymbol{k})$, $\bar{\omega}$ and 0. The remaining integral over $\boldsymbol{k}$ must be done numerically.

Performing the calculations we have redefined all dimensional quantities by multiplying them by the appropriate power of μ to obtain dimensionless variables. We have chosen the following values of the parameters: $g = 1$, $|\boldsymbol{v}| = 1$, $|\boldsymbol{u}| = 0.9$, $C_R = 3$. To take the integral over $\boldsymbol{k}$, cylindrical coordinates with the axis z along the stream velocity $\boldsymbol{u}$ have been used. Since the integral appears to be divergent, it has been taken over a finite domain such that $-k_{\max} \leq k_{\mathrm{L}} \leq k_{\max}$ and $0 \leq k_{\mathrm{T}} \leq k_{\max}$. In Fig. 1, we show the parton's energy loss per unit length as a function of time for a parton flying along the streams for $k_{\max} = 20,\ 50,\ 100$. The energy loss strongly oscillates with amplitude growing in time. However, the results depend on the

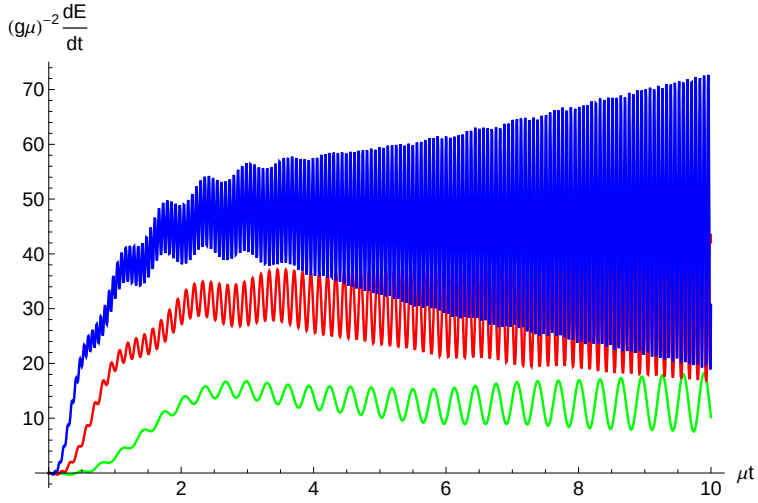

Fig. 1. The energy loss per unit length as a function of time for a parton flying along the direction of streams. The lowest (green) curve corresponds to $k_{\max} = 20$, the middle (red) one to $k_{\max} = 50$ and the most upper (blue) curve to $k_{\max} = 100$.

cut-off parameter $k_{\max}$: the magnitude of energy loss grows and the period of oscillations shrinks when $k_{\max}$ increases. This ultraviolet sensitivity of our results is not very surprising, as our approach is fully classical. In the case of equilibrium (stable) plasma, the energy loss due to soft interactions diverges logarithmically with $k_{\max}$ [1]. The divergence signals a necessity to combine the classical contribution to the energy loss at small wave vectors with the quantum contribution at higher ones. A quantum approach to the parton energy loss in unstable plasma needs to be developed.

Although the two-stream system is not directly relevant to the quark-gluon plasma which is produced in relativistic heavy-ion collisions, let us comment on the potential interest of our study. Choosing $\mu = 200$ MeV, which is a rough estimate of Debye mass in the QGP from nuclear collisions, the units on the horizontal and vertical axis in Fig. 1 are, respectively, fm and 200 MeV/fm. Since the parton's energy loss in the QGP observed in relativistic heavy-ion collisions is of the order of 1 GeV/fm, the magnitude of the energy loss discussed here is certainly of phenomenological interest.

6. Conclusions

We have developed a formalism, where the energy loss of a fast parton in a plasma medium is found as the solution of initial value problem. The formalism allows one to obtain the energy loss in unstable plasma, where some modes exponentially grow in time. In the case of stable plasma, one reproduces correctly the standard energy-loss formula. As an example of an unstable system we have studied a two-stream system. The energy loss per unit length is not constant, as in an equilibrium plasma, but it exhibits strong time dependence.

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REFERENCES

[1] S. Peigne, A.V. Smilga, *Phys. Usp.* **52**, 659 (2009).
[2] M. Le Bellac, *Thermal Field Theory*, Cambridge University Press, Cambridge 2000.
[3] P. Romatschke, M. Strickland, *Phys. Rev.* **D71**, 125008 (2005).
[4] St. Mrówczyński, *Acta Phys. Pol. B* **37**, 427 (2006).
[5] A. Majumder, B. Müller, St. Mrówczyński, *Phys. Rev.* **D80**, 125020 (2009).
[6] St. Mrówczyński, *Phys. Rev.* **D77**, 105022 (2008).
[7] St. Mrówczyński, B. Müller, *Phys. Rev.* **D81**, 065021 (2010).
[8] S.K. Wong, *Nuovo Cim.* **A65**, 689 (1970).